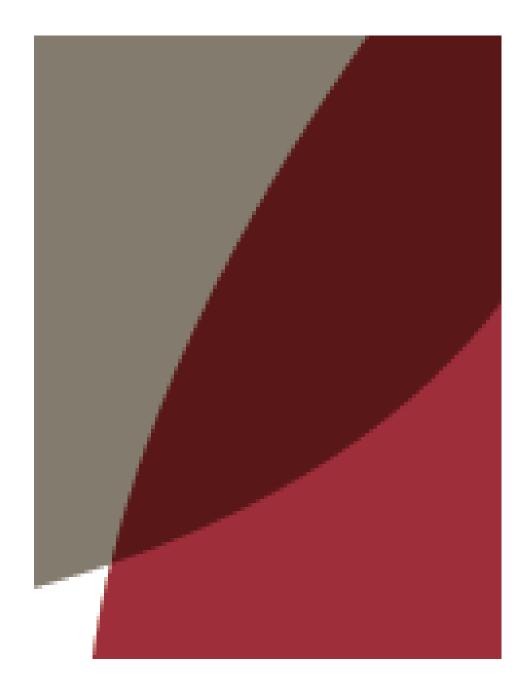


Data, Probability and Statistics

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INTERNATIONAL CENTRE ENCE FOR EDUCATION IN MATHEMATICS



Have you all chosen your biscuit? If not, please do so now.



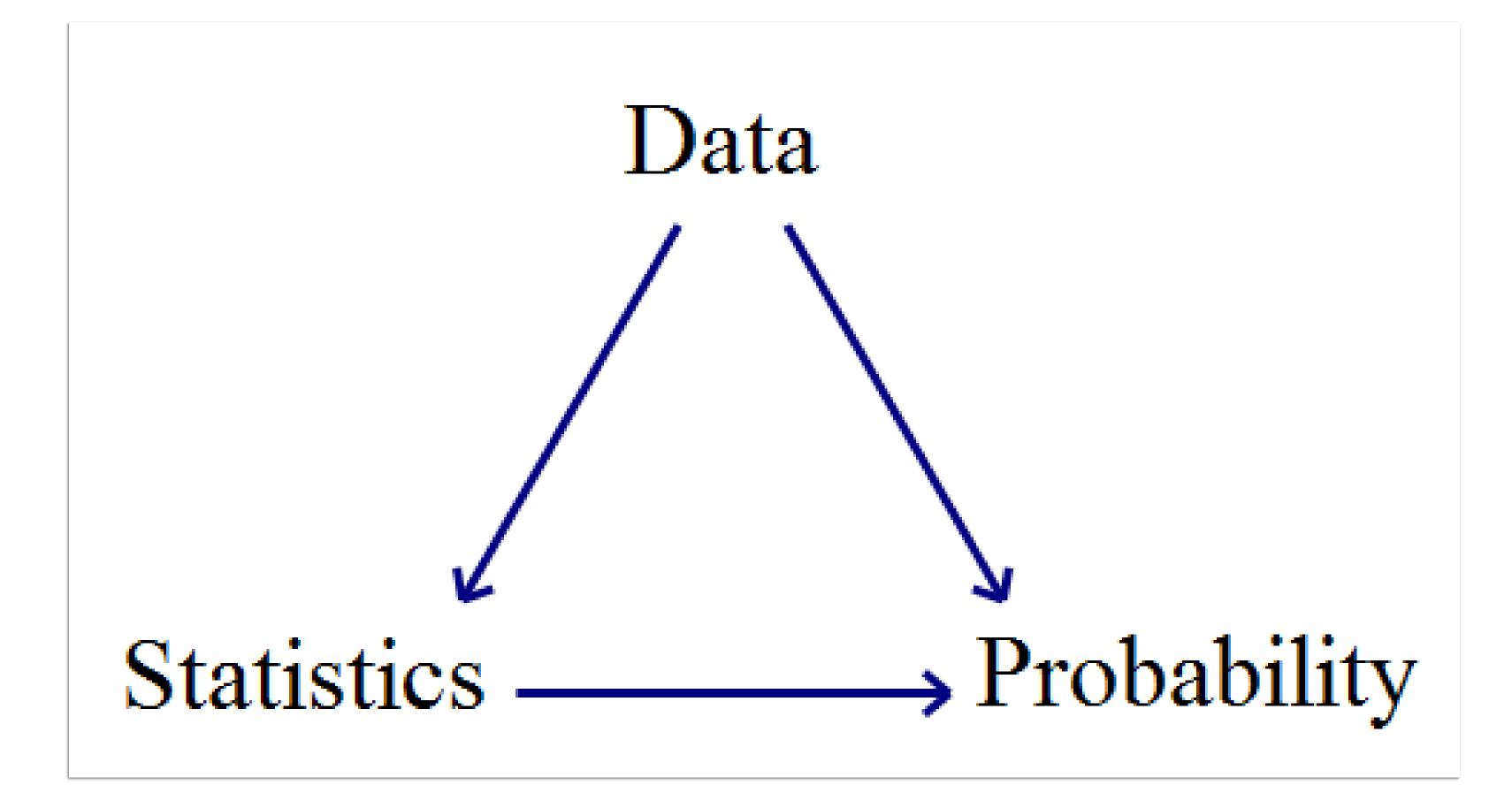
What is it?

Chance – a layman's expression for the possibility of something happening.

Probability – The calculation of the precise likelihood of something happening.



What is it?





What is it?

Data – Information about properties and values associated with an item of interest

Statistics – calculations and graphical representations to make sense of the raw information

Probability – Extrapolation of the data and statistics to make predictions about how likely different outcomes are to occur.



What is it?

Some probabilities are fixed. That is to say, when playing a game with a six sided die, the probability of rolling a six on any one throw is always $\frac{1}{2}$.

Other probabilities change with time and circumstance: for example, forty years ago males who smoked was 45% of the population while in 2010 it was just under 18% (17.9%)



Experiment vs Theory?

Experiment –

Theory -



Experiment vs Theory?

Which of the four rows is not (least likely to be) random?

Т	Т	Η	Т	Η	Т	Т	Т	Η	Т	Т	Т	Т	Т	Т	Η	Т	Т	Т	Т	Т	Т	Т	Η	Н	Η	Т	Т	Т	Т
Т	Т	Т	Т	Т	Т	Т	Н	Т	Т	Н	Т	Т	Η	Т	Т	Т	Т	Н	Т	Т	Н	Т	Т	Н	Т	Т	Т	Н	Н
Η	Н	Т	Н	Т	Н	Т	Т	Н	Н	Т	Η	Т	Η	Т	Н	Т	Н	Т	Н	Т	Н	Т	Н	Т	Н	Т	Η	Т	Н
Н	Н	Н	Т	Т	Η	Η	Η	Η	Н	Т	Η	Н	Η	Н	Τ	Н	Т	Η	Н	Η	Н	Н	Η	Н	Η	Η	Н	Η	Н



Why is it important?

Table 1.3.2 Percentage of current smokers* in Australia, 1945-1976

Year	Male (%)	Fema (%)
1945	72	26
1964	58	28
1969	45	28
1974	45	30
1976	43	33

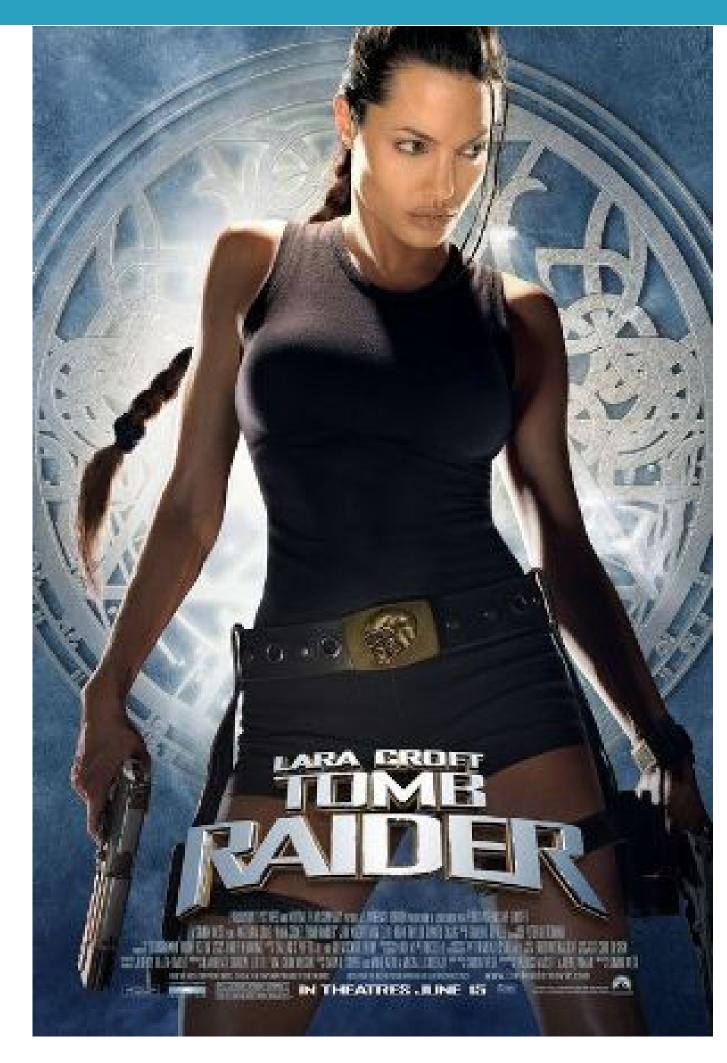
Tobacco in Australia: Facts and Issues (Cancer Council)

http://www.tobaccoinaustralia.org.au/downloads/chapters/Ch1_Prevalence.pdf





Why is it important?



Angelina Jolie

After surgery: 5% probability



BRCA 1 gene from her mother

87% probability of developing breast cancer

Why is it important?

- **Medical Research**
 - Insurance
- Games of Chance
 - + + +



WHY DO WE NEED IT?

Probability is an area of mathematics with many diverse applications.

- For example, it is used in measuring risk in insurance and banking and to set both interest rates and insurance premiums.
- Probability calculations are an essential component of most medical research. Research in genetics often involves complicated probability considerations.

It is also closely linked to mathematical and applied statistics.



Probability WHY DO WE NEED IT?

Statistics and statistical thinking have become increasingly important in a society that relies more and more on information and calls for evidence.

The need to develop statistical skills and thinking across all levels of education has grown and is of core importance in a century which will place even greater demands on society for statistical capabilities throughout industry, government and education.

WHY DO WE NEED IT?

Statistics is the science of variation and uncertainty.

Concepts of probability underpin all of statistics, from handling and exploring data to the most complex and sophisticated models of processes that involve randomness.

Statistical methods for analysing data are used to evaluate information in situations involving variation and uncertainty, and probability plays a key role in that process.

WHY DO WE NEED IT?

- All statistical models of real data and real situations are based on probability models.
- Probability models are at the heart of statistical inference, in which we use data to draw conclusions about a general situation or population of which the data can be considered randomly representative.



How do we teach it?

- **Biscuits and Birthdays**
- What biscuit was most popular?

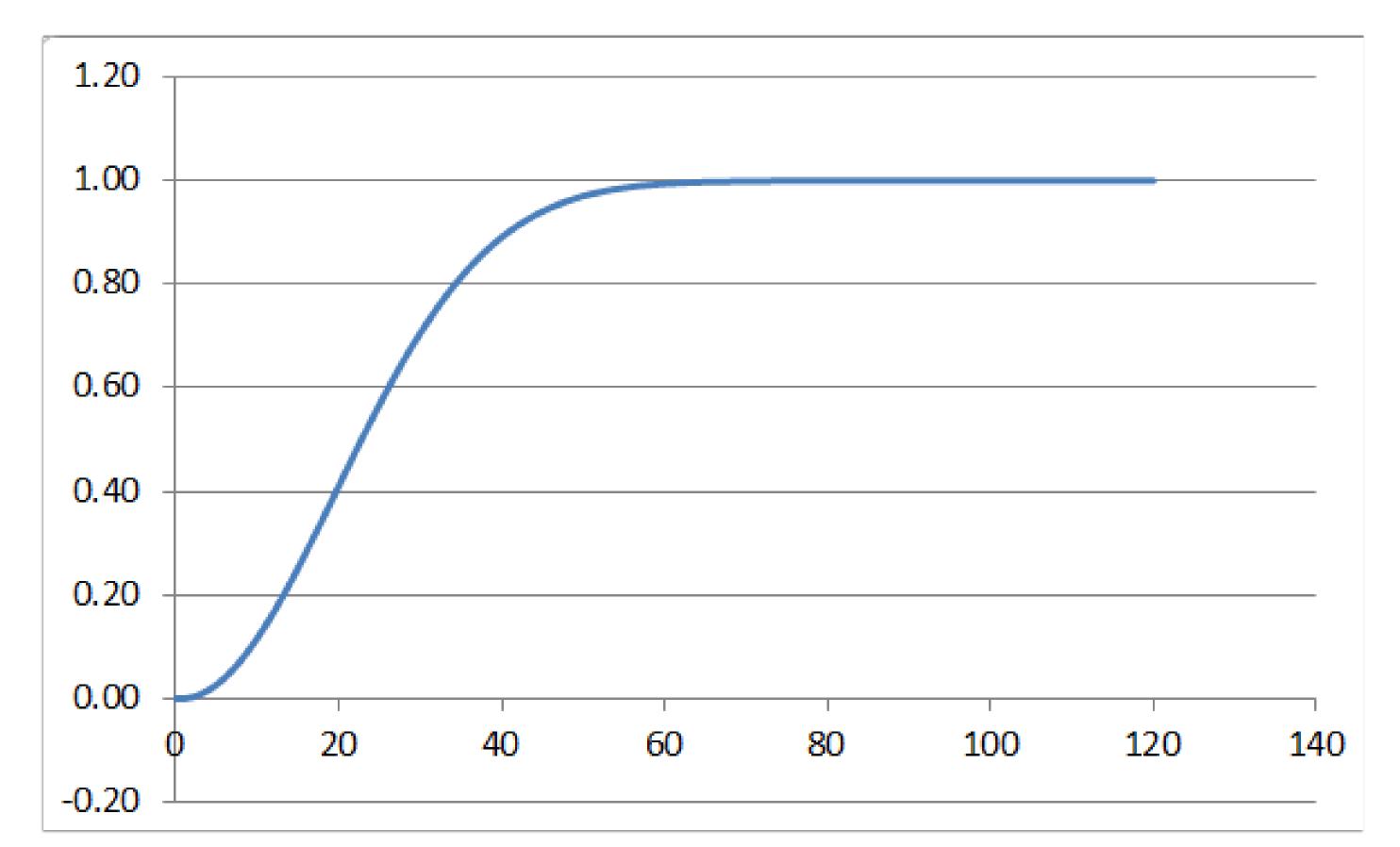
When attending an AMSI Cluster PD, what is the probability of a random participant having a Turkish **Delight Tim Tam?**

What is the probability that two people in this room share the same birthday?



How do we teach it?

Sharing a Birthday







365!(365-*n*)!×365^{*n*} P(n)=1-

How do we teach it?

Not all teaching can be so immediate or tasty.

ACCEPT that not everything can be contextualised,

BUT give them glimpses of how it will affect their lives now AND IN the future.



Applications

It is worth having the conversation with students that until very recently probability and statistics have been interesting...

...but a not very useful or accessible to most people.

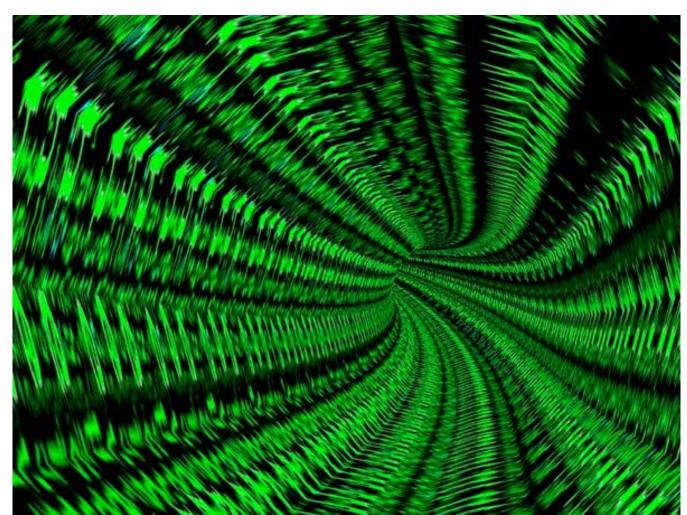


Applications

It is only now, with

- the power of modern computers and
- the ability to collect enormous amounts of data that the power of probability is becoming evident.





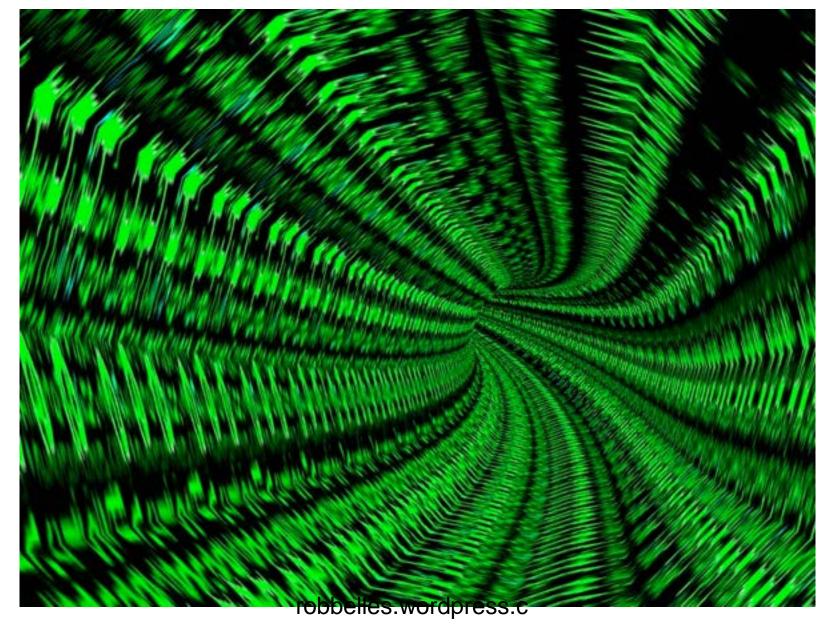
robbelles.wordpress.c om

Applications

In the 21st Century we use statistical analysis to

- understand incredibly complex structures and patterns (bioinformatics, wine)
- make more accurate decisions (stockmarket, sports statistics)





Applications

Every industry can now take advantage of this:

Some examples include: Biology, Business, Criminology, Defence, Environment, Epidemiology, Government, Health, Market Research, Meteorology, Teaching. Transport



Computing, Energy, Finance, Insurance, **Operations Research**,

Applications

Now, for perhaps the first time in history,

Laypeople can name an industry and ask,

"Where can we use mathematics?"

instead of trying to think of industry situations where is mathematics is useful.

Every decision we can make in relation to data, we can now pose as a mathematical problem to be solved.



Applications

We are used to checking the weather forecast at the end of the TV news bulletin every evening.

On most occasions the presenter will confidently state "It will rain heavily tomorrow" or "A cool change will arrive at about 10pm tonight".





How do we teach it?

AND NOW FOR THE CONTENT



How do we teach it?

Probability is about counting

1) How many possible outcomes an event has AND

2) How many of those outcomes are desirable

How many biscuits are there and how many do I like?



- In order to count effectively we need an appropriate vocabulary. This includes:
- experiment
- sample space
- outcome
- equally likely outcome
- probability

- total probability
- subset
- complement
- event

Experiment for example: rolling a die flipping a coin drawing a card from the deck choosing a biscuit

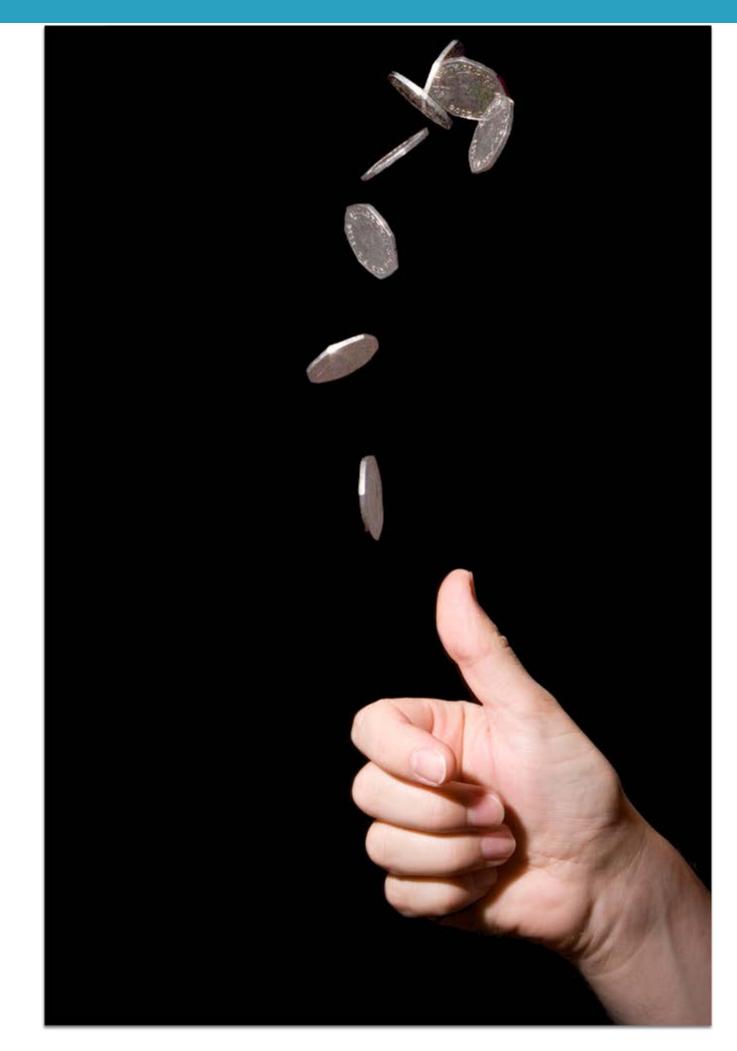


Outcome

the possible results in the experiment:

heads or tails

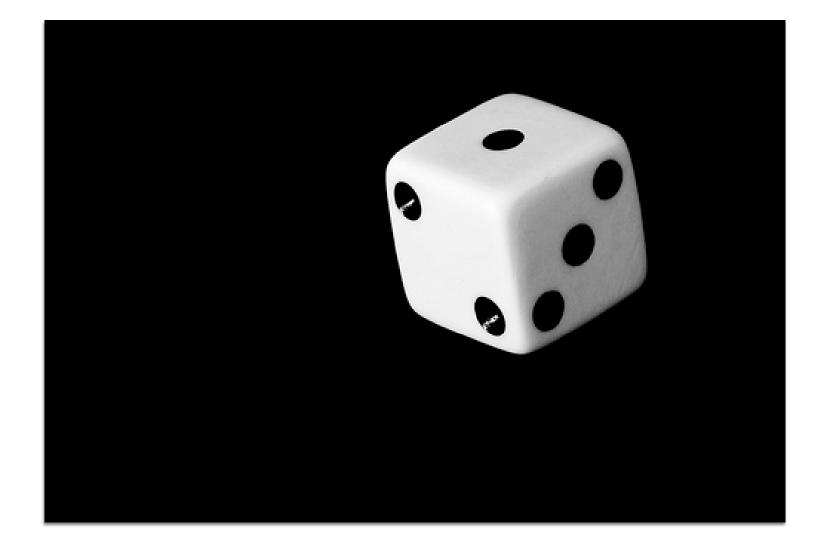
1,2,3,4,5 or 6



sample space

the complete set of possible outcomes:

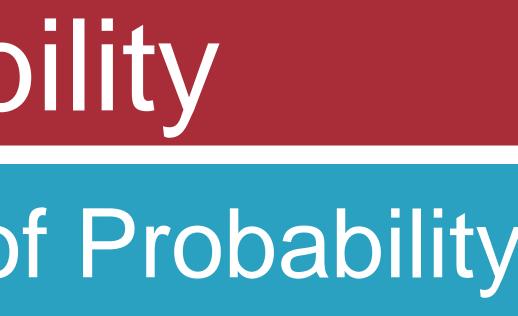
$S = \{1, 2, 3, 4, 5, 6\}$



The Language of Probability

equally likely outcome

when no outcome should occur more frequently than another





probability

the chance, possibility or likelihood of a particular outcome occurring



total probability

there are a finite number of possible outcomes, so the sum of the probability of these outcomes is 1.

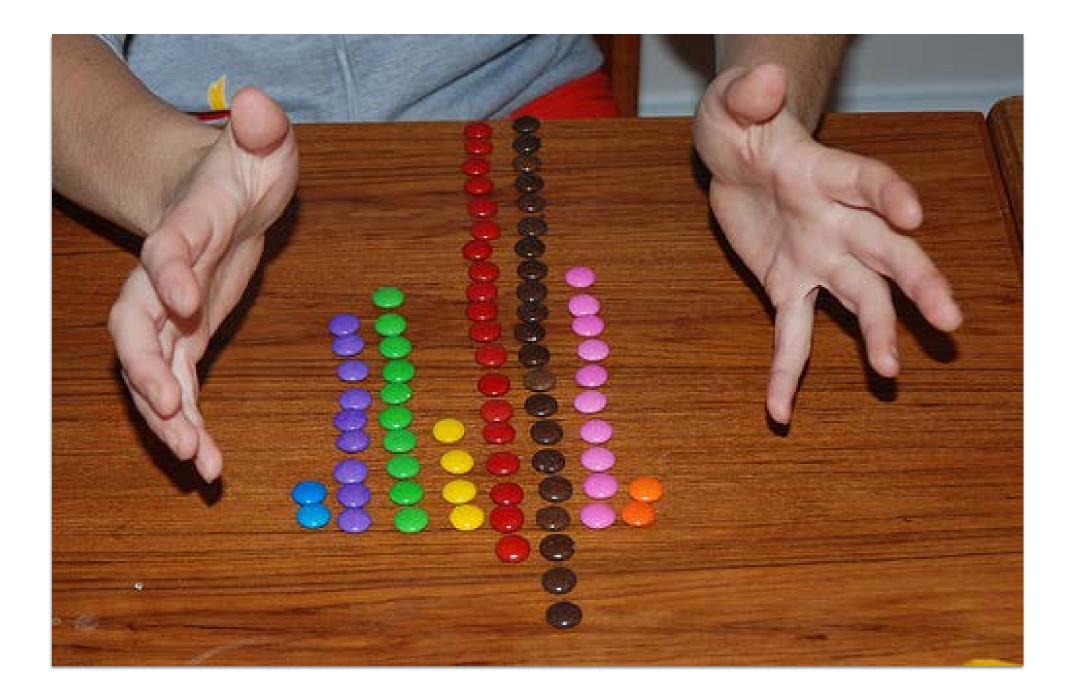


subset

a set containing some, all or none of the elements of the sample space e.g.

red smarties

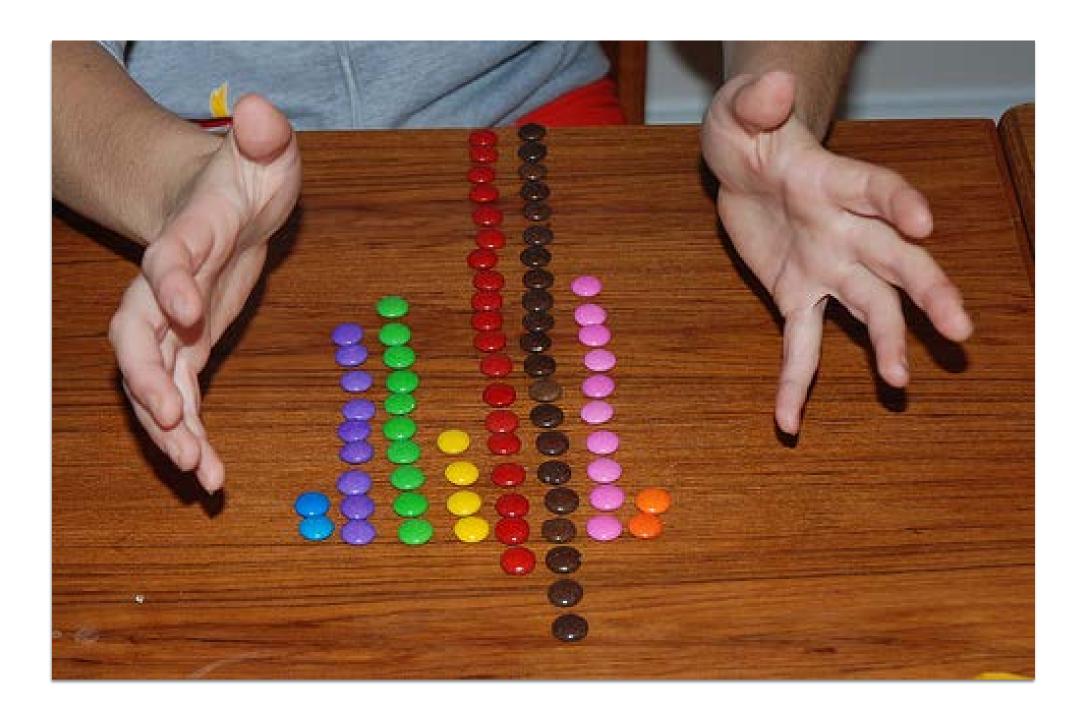
Α



complement

all the other elements in the sample space that are not in the in a subset A e.g.

all other smarties AC



event

a subset of the sample space e.g.

an even number two heads a red smartie 3 scissors





Expressing the probability of an event

Probability of an event = <u>number of favourable outcomes</u> total number of outcomes



Expressing the probability of an event

- Experiment A die is tossed once to obtain an even number
 - **Probability P** The event that we are looking for is shown in brackets next to the P
- e.g. P(even) There are 3 favourable outcomes i.e. 2,4 and 6 There are a total of 6 possible outcomes i.e. 1,2,3,4,5 and 6



Expressing the probability of an event

Probability of an event = <u>number of favourable outcomes</u>

 $P(even) = \frac{3}{6}$ 2





total number of outcomes

The Australian Curriculum

Year	Content D
1	Identifies outcomes of familiar even
	describes them using everyday la
	'won't happen' or 'might happen'
2	Identifies practical activities and e
	chance. Describes outcomes as 'I
	events as 'certain' or 'impossible'

Description

ents involving chance and inguage such as 'will happen',

everyday events that involve likely' or 'unlikely' and identify some

The Australian Curriculum

Year	Content De
3	Conducts chance experiments (toss
	a bag), identifies and describes pos
	variation in results
4	Describes possible everyday events
	occurring
	Identifies everyday events where on
	happens (eg: weather, if it is dry, it c
	Identifies events where the chance of
	occurrence of the other (new baby is
	depend on the sex of the previous b



escription

sing and coin or drawing a ball from ssible outcomes and recognise

s and order their chances of

ne cannot happen if the other cannot be wet)

of one will not be affected by the is either a boy or a girl does not baby) (Independence)

The Australian Curriculum

Content Description

- 5 and represents probabilities of those outcomes using fractions
 - Recognises that probabilities range from 0 to 1
- Describe probabilities using fractions, decimals and percentages 6 Conduct repeated trials of chance experiments, identifying the variation between trials and realising the results tend to the prediction with larger numbers of trials

Compare observed frequencies across experiments with expected frequencies and predict likely outcomes from a run of chance events and distinguish these from surprising results

Year

- Lists outcomes of chance experiments involving equally likely outcomes

The Australian Curriculum

Content Description

Construct sample spaces for single-step experiments with equally likely outcomes. Discuss the meaning of probability terminology (eg: probability, sample space, favourable outcomes, trial)

Assign probabilities to the outcomes of events and determine probabilities for events. Express probabilities in common and decimal fractional and percentage forms

Year

The Australian Curriculum

Year	Content De
8	Identify complementary events and
	solve problems
	Describe events using language of 'a
	inclusive 'or' (A or B or both) and 'an
	Represent such events in two-way ta
	solve related problems



escription

use the sum of probabilities to

'at least', exclusive 'or' (A or B), nd'

tables and Venn diagrams and

The Australian Curriculum

Content Description

- List all outcomes for two-step chance experiments, both with and 9 without replacement using tree diagrams or arrays. Assign probabilities to outcomes and determine probabilities for events
 - Calculate relative frequencies from given or collected data to estimate probabilities of events involving 'and' or 'or'

Investigate reports of surveys in digital media and elsewhere for information on how data were obtained to estimate population means and medians

Year

The Australian Curriculum

Content Description

10 Describe the results of two- and three-step chance experiments, both with and without replacements, assign probabilities to outcomes and determine probabilities of events. Investigate the concept of independence. Recognise that some sets of chance events are dependent on a previous result and others are not, that this distinction is important when calculating probabilities, and that events are independent if $P(A) \times P(B) = P(A \text{ and } B)$ Use the language of 'ifthen, 'given', 'of', 'knowing that' to investigate conditional statements and identify common mistakes in interpreting such language

Year

The Australian Curriculum

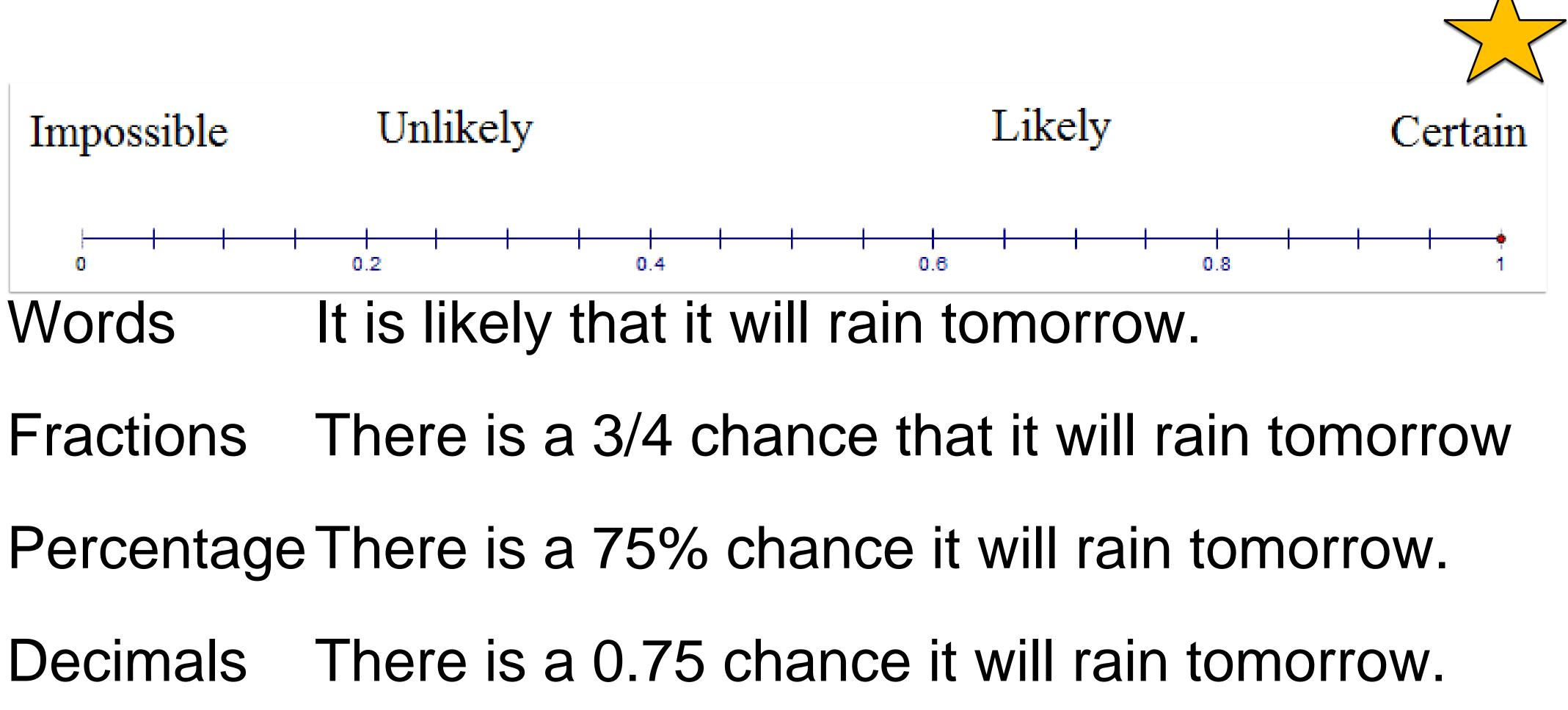
Content Description

10 A Investigate reports of studies in digital media and elsewhere for information on the planning and implementation of such studies, and the reporting of variability. Evaluate media reports that refer to data from a range of contexts.

Year

Evaluate whether graphs in a report could mislead, and whether graphs and numerical information support the claims

Expressing the probability of an event





Dice Games

Greedy Pig

First Down the Mountain

Cat and Mouse

Mebourne Cup

Don't forget the Maths!



Introduction to Probability

Greedy Pig

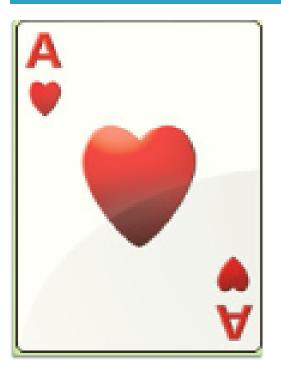
Equipment: an ordinary 6-sided die Each turn of the game consists of one or more rolls of the die. You keep rolling until you decide to stop, or until you roll a 1. You may choose to stop at any time. If you roll a 1, your score for that turn is 0. If you choose to stop rolling before you roll a 1, your score is the sum of all the numbers you rolled on that turn. The player with the highest score wins. Each player has 10 turns.

Introduction to Probability

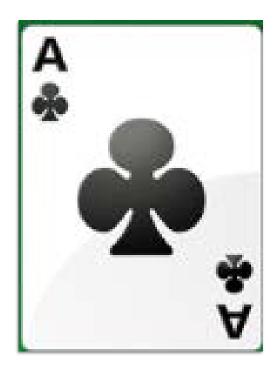
First Down the Mountain

Available from http://www.blackdouglas.com.au/taskcentre/087first.htm

Cards



Choose a team









Cards

- Standard packs have 52 cards of 4 suits and 13 cards in each suit.
- Many students these days do not know this.
- Repetitions and familiarisation. 5 to 10 minutes regularly
- Use card games and card tricks "Pick a card, any card"
- Use suits (Hearts, Diamonds, Clubs, Spades) to choose teams. Count how many in each team.

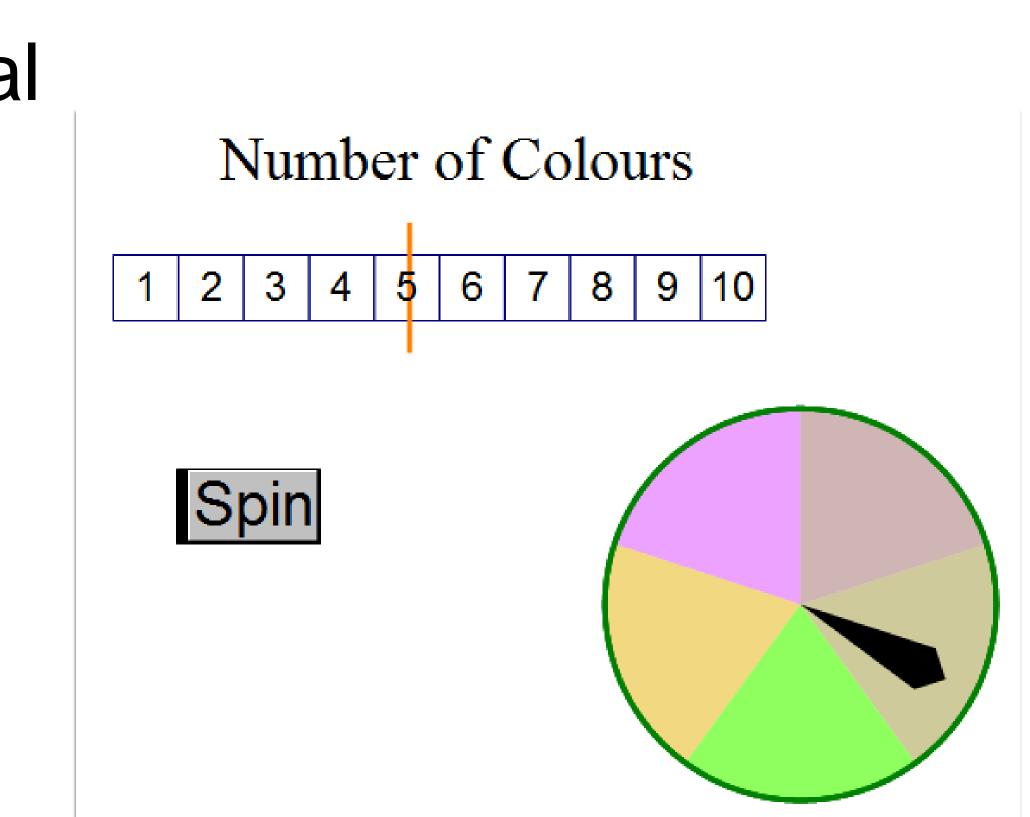


Spinners

Spinners can be physical or digital

They are useful for probabilities where you want the number of outcomes to be non-standard.





Bag of Cubes

	Red	Blue	Green	Yellow	Orange
Bag I		4	2	2	6
Bag 2	3	6			4
Bag 3	3	3	3	3	3
Bag 4	5	2	2	4	2



Probability Mr O'Connor's Die of Death!

Used for randomly selecting students to answer questions in class

It started out as a physical die and then turned into a spread sheet

Keep track of how many times each student is chosen in a lesson. Discuss randomness and fairness, etc



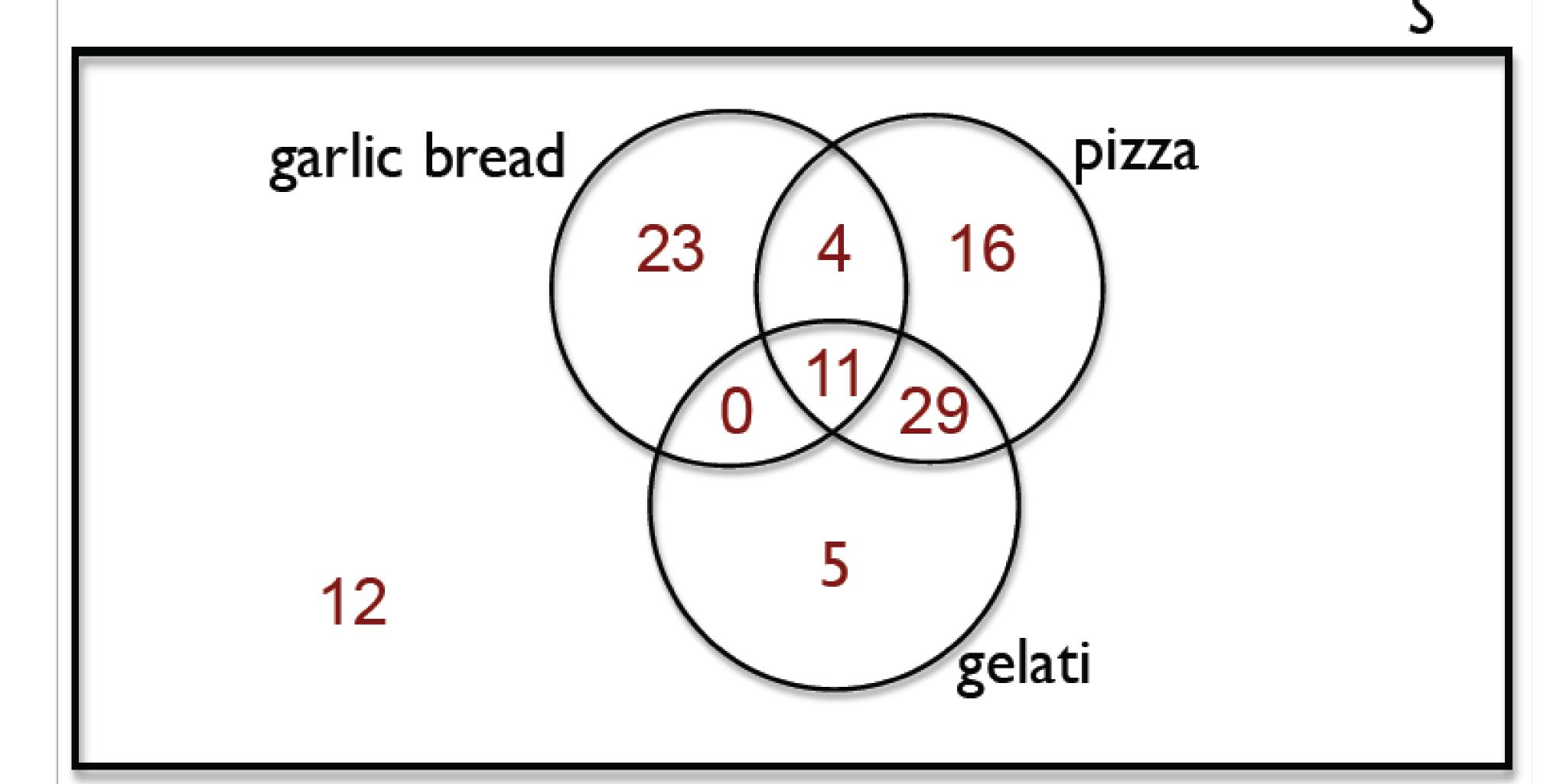


The Italian Restaurant

- 100 diners were asked if they had ordered garlic bread, pizza or gelati.
- 11 people ordered all three items.
- 29 people ordered pizza and gelati but no garlic bread.
- 16 people ordered pizza only.
- There were a total of 60 orders for pizza. Nobody ordered just garlic bread and gelati.
- There were a total of 45 gelati orders.
- 12 people did not order any of these items. How many people ordered garlic bread only?



Venn Diagrams





Resources

Introduction to Probability - TIMES Project, ICE-EM

SAM – Middle Years

maths300 - Education Services Australia

mathsonline -http://www.mathsonline.co.uk/nonmembers/ resource/prob/



Birthdays

We have already looked at sharing birthdays.

What about celebrating birthdays in your class with games that allow for counting and finding probabilities?



Hands, Arms and Legs

- Cross your arms.
- Did you go right over left or left over right?
 - Fold your hands
- Which index finger is on top: left or right?
 - Cross your legs.
- Did you go right over left or left over right?



Rolling two dice and adding

Suppose we have two dice, one red and one blue. We roll both dice simultaneously and then add their face values.



Rolling two dice and adding

Do 5 trials.

What did you get?



Rolling two dice and adding

What are the possible outcomes?

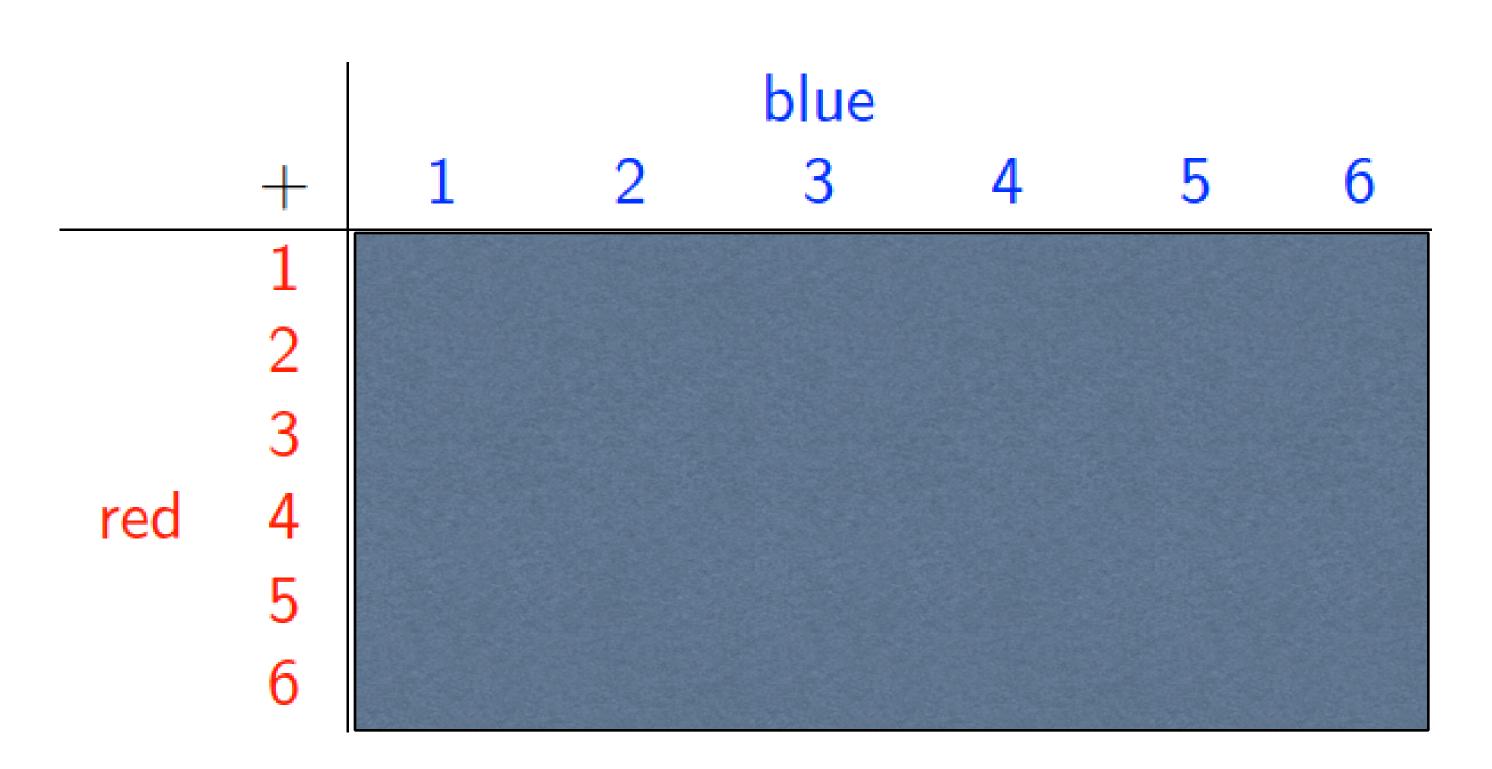


Rolling two dice and adding

The possibilities can be tabulated:



Rolling two dice and adding





Rolling two dice and adding

		blue					
	+	1	2	3	4	5	6
	1	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
red	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12



Rolling two dice and adding

There are 36 equally likely rolls but there are only 11 outcomes in our sample space:

2; 3; 4; 5; 6; 7; 8; 9; 10; 11 and 12.



Misconceptions

Outcomes of an experiment need not be equally likely!

Assuming that each outcome is equally likely is the most common misconception in probability.

Often people say, "Either this will happen or that will happen, so there is a 50% chance that this will happen"; even when the comment is contradicted by their own experience.



Always, Sometimes, Never

https://www.ncetm.org.uk/resources/

History

In order to agree to a fair contract it is essential to quantify the risk involved.

Shipping insurance contracts date back to Roman times (200 BC – 600 AD). This means that probabilities were being empirically calculated over two thousand years ago. Premiums for ships plying their trade in the eastern Mediterranean could be as high as 50% for a single trip!

Compare this with modern car premiums that range from 2% to 4% for "safe drivers" for a whole year.





History

Astragals

Sheep knuckles.

Not all sides are equally likely to roll face up





History

1654 Chevalier de Mere inspired Pascal and Fermat to correspond on games of chance (particularly on how to split the pot of money in a game that has been interrupted)



History

In the mid-seventeenth century, Pascal and Fermat discussed through correspondence, the basics of probability theory and solved the dividing the wager problem using the binomial theorem, and of course, Pascal's Triangle.

(By the way, Pascal's triangle or the 'arithmetical triangle' was discovered at least six hundred years earlier by an Arabic mathematician.)

History

- The earliest significant record of probabilities being exactly calculated date from the late middle ages in Europe.
- Games of chance had been played since the earliest times (at least five thousand years ago) but detailed analysis of, for example, the fifty-six ways three dice could land and their different probabilities was not recorded before about 1400.
- Another early discussion concerned the fair (equitable) way to split the stakes in an incomplete game.

History





Christiaan Huygens

Pierre de Fermat



History

"It seems that to make a correct conjecture about any event whatever, it is necessary only to calculate exactly the number of possible cases and then to determine how much more likely it is that one case will occur than another. But here at once our main difficulty arises, for this procedure is applicable to only a very few phenomena.... What mortal, I ask, could ascertain the number of diseases, counting all possible cases ... and say how much more likely one disease is to be fatal than another...? Or who could enumerate the countless changes that the atmosphere undergoes every day and from that predict what the weather will be a month or even a year from now?"

History

JACOBI BERNOULLI, Profett Batil. & utrinique Societ. Reg. Scienciae Gall. & Pruff. Social. MATHEMATICE CREESSRAIMS,

ARS CONJECTANDI,

OPUS POSTHUMUM.

Acredit.

TRACTATUS

DE SERIEBUS INFINITIS,

Et Ecurrou . Gallice feripte DE LUDO PILÆ RETICULARIS.



BASILEE, Impendis THURNISIORUM, Frateum. db bee zitt.



History



Pierre de Fermat



History

The first book on probability was written by Christian Huygens in 1657. Most of his ideas were written in terms of expectation, which he thought of as the average amount that one would win if a game was played many times. It is essentially the average - the average score when a die is thrown is 3 1/2.

The next such book did not appear for over fifty years when James (Jakob) Bernoulli published Ars Conjectandi in 1713, cited earlier.

History

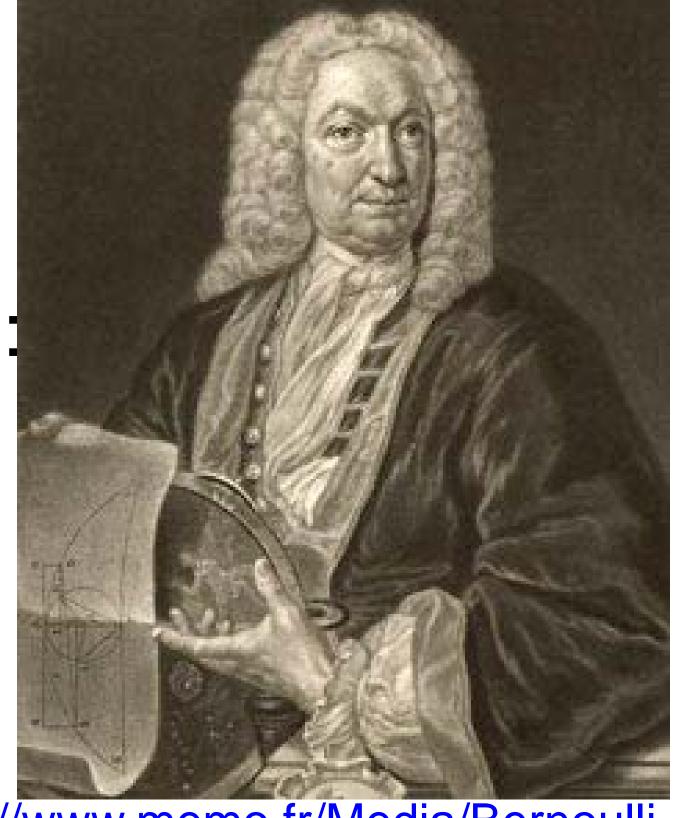
Christiaan Huygens



History

Bernoulli introduced a number of novel ideas:

- the law of large numbers and
- the concept of a confidence interval.



http://www.memo.fr/Media/Bernoulli. **j**pg

History Bernoulli, quite arbitrarily, chose a probability of 0.999 to mean that an event was a moral certainty and conversely if the probability of the event was less than 0.001 then the event was morally impossible. As an example, Bernoulli considered an urn containing 3000 white and 2000 black pebbles. By analysing sums of terms in a binomial expansion, he showed that by sampling the urn 25 550 times one would know with moral certainty that the proportion was within 1/50 of the true proportion of 3/5 to 2/5

Clearly this is incredibly sophisticated compared to the elementary counting of Pascal. only fifty years earlier.

History

Perhaps the most important statistical method of the nineteenth century was that of least squares. This method provided one of the primary tools for what was called "the combination of observations". A typical problem is to find the line of best fit, y = a + bx, given k data points $(x_1, y_1), (x_2, y_2) \dots (x_k, y_k)$.

Many such problems arose in astronomy and Laplace, Legendre and Gauss developed the method of least squares for determining (or estimating) a and b in the line of best fit.

History

Gauss used the method of least squares to predict the position of the asteroid Ceres from the limited observational data he had available from astronomers.

And it worked!!!!!

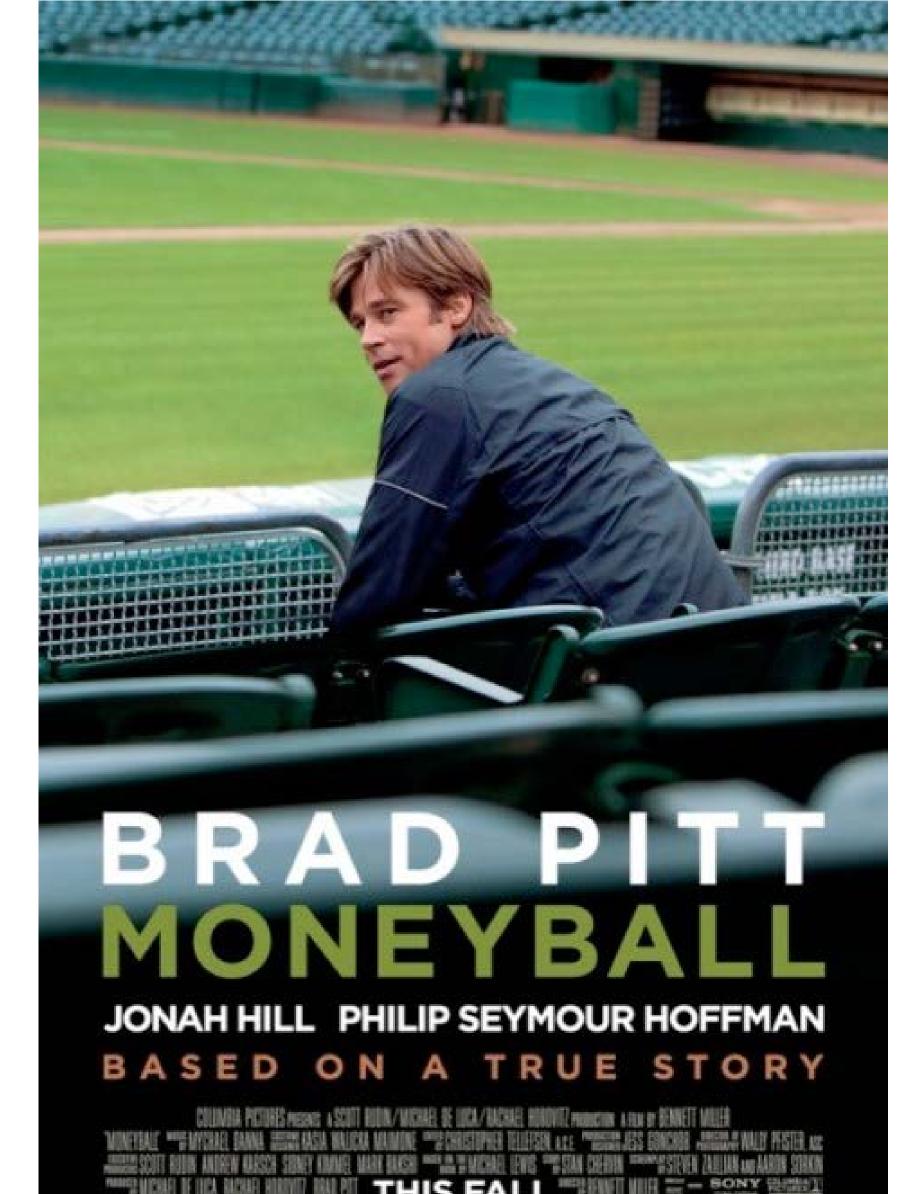
More Applications - The Value of Wine

Bordeaux wines have been made in much the same way for centuries. The variability in the quality and prices of Bordeaux vintages is predicted by the weather that created the grapes.

The analysis provides a useful basis for assessing market inefficiency, the effect of climate change on the wine industry and the role of expert opinion in determining wine prices.



http://wallpaper.dig.ru/40 Bordeaux Red Wine Glasse s.htm



Applications - Super Bowl

moneyball/

Probability

http://blogamole.tr3s.com/2011/09/23/review-show-me-the-

Applications - World Series Baseball

- 'Moneyball' is a movie about Oakland A's general manager Billy Beane's successful attempt to put together a baseball club on a budget by using computer-generated analysis to draft his players.
- Billy Beane had the lowest salary constraint in baseball. He found a competitive advantage using probability.
- Billy turned baseball on its ear when he used statistical data to analyse and place value on the players he picked for the team.



Applications – Gambling so as to lose as little as possible

- A group of MIT Maths students spend their weekends in Las Vegas
- And break the bank
- Based on a True story
- The DVD has an extra where the actors explain the probability and the mathematics behind the movie.
- It also has Monty Hall



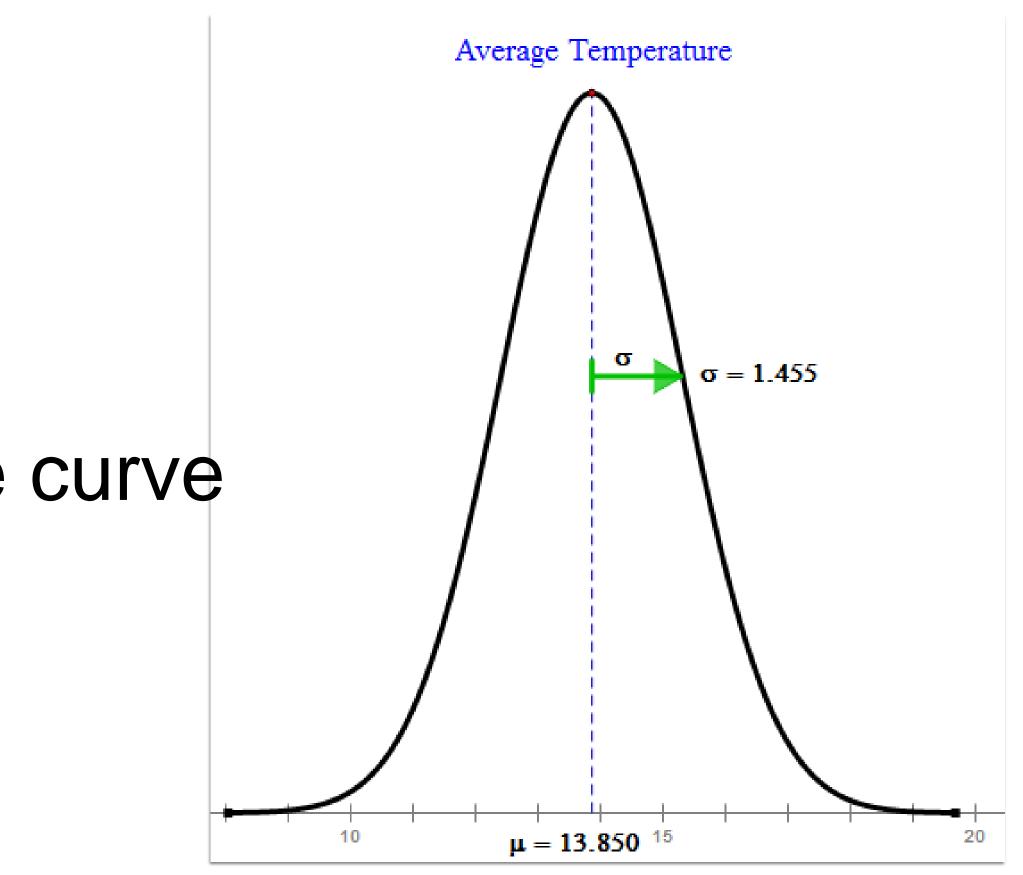




Applications – Explaining Climate Change

What is my one? The area under the bell curve. Statistics gives us the shape. Probability is the area under the curve

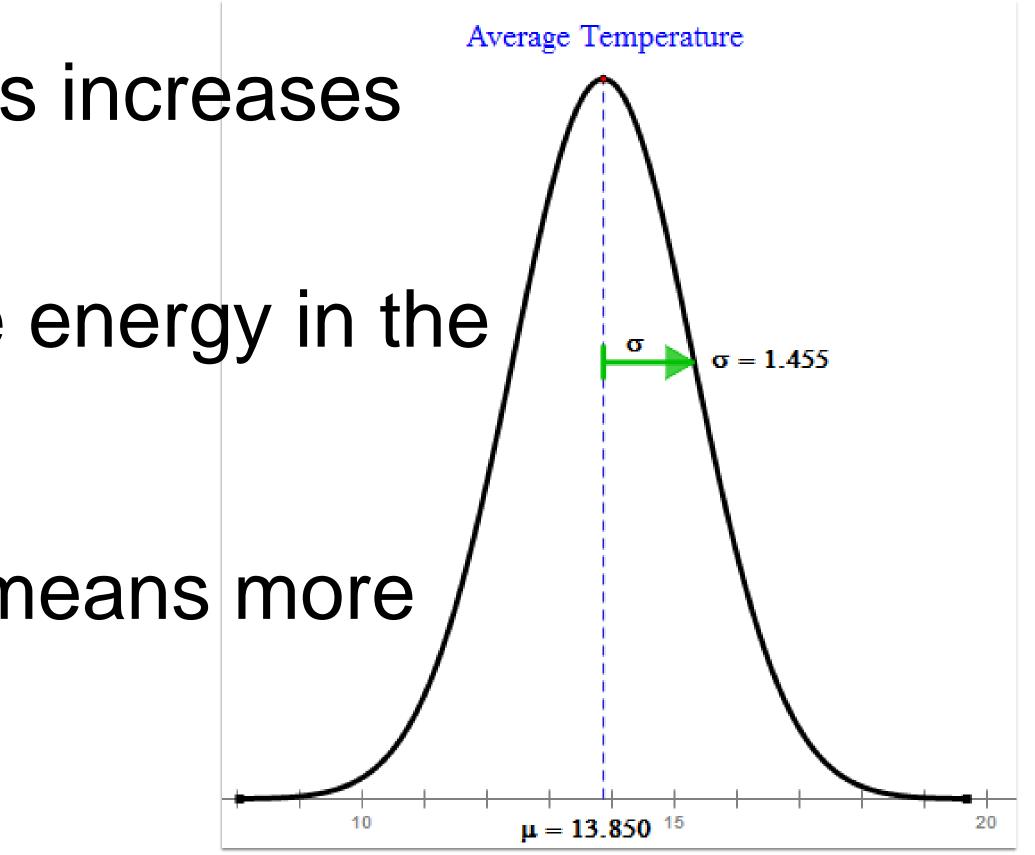




Applications – Explaining Climate Change

- What happens when average temperature increases?
- Probability of higher temperatures increases
- Higher temperatures mean more energy in the atmosphere
- More energy in the atmosphere means more violent storms





Applications - Finance

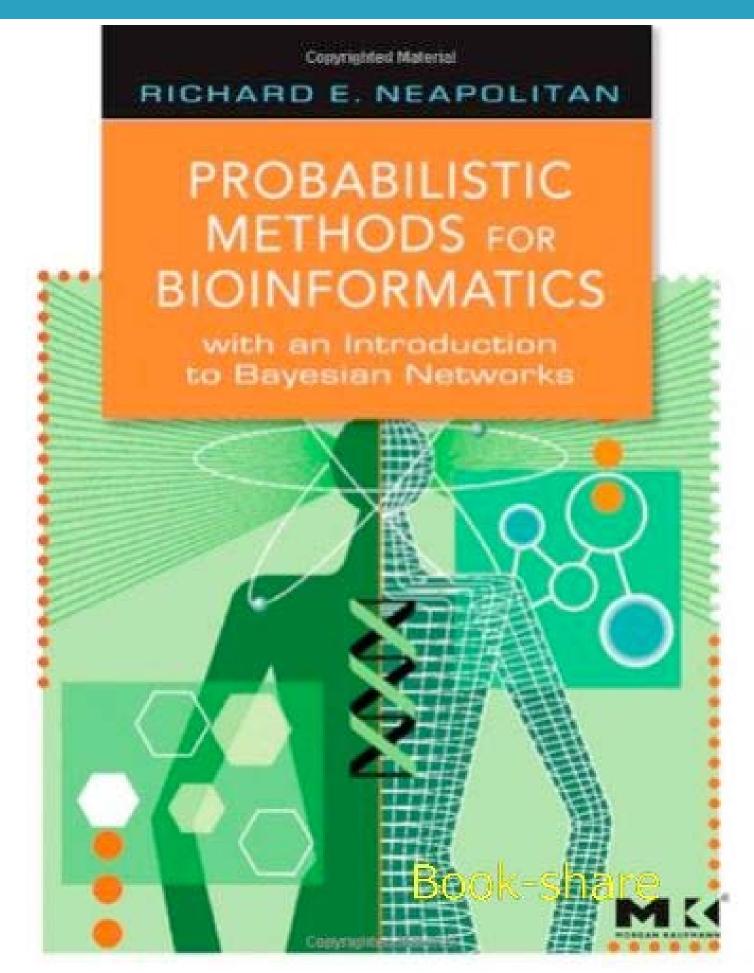
Two thirds of all money traded on the stock exchange is now done by computers using statistical models.

Access to these models is difficult due to the highly secret nature of the research - there is alot of money to be made!





Applications - Medical Research



research.

http://www.downeu.net/s/Social+Networks+and+Health++Models+Methods+and+Applications



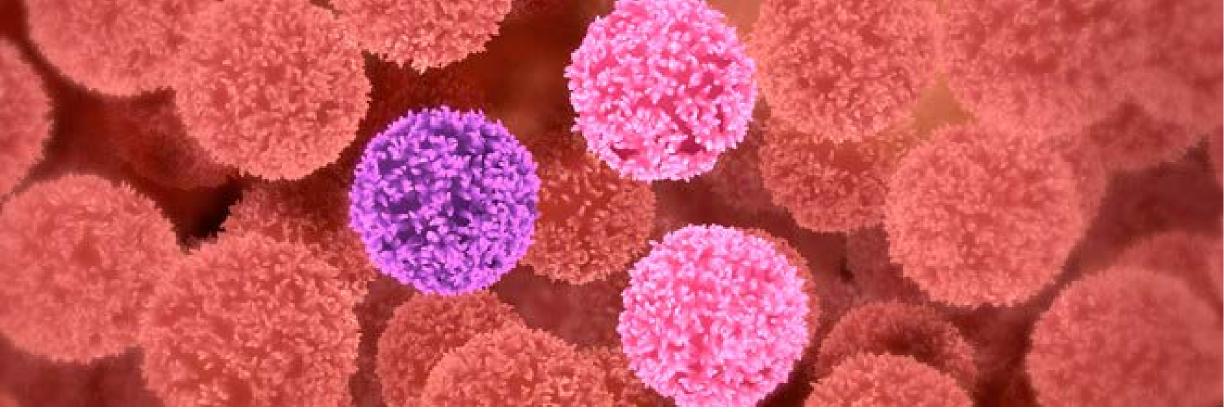
Probabilities enter almost every aspect of medical treatment and medical

Bioinformatics

For students with maths and programming skills there is a new career that might interest them...

Bioinformatics mixes mathematics, biology and programming.

The demand is very high.



http://www.wehi.edu.au/faculty/disease_research/





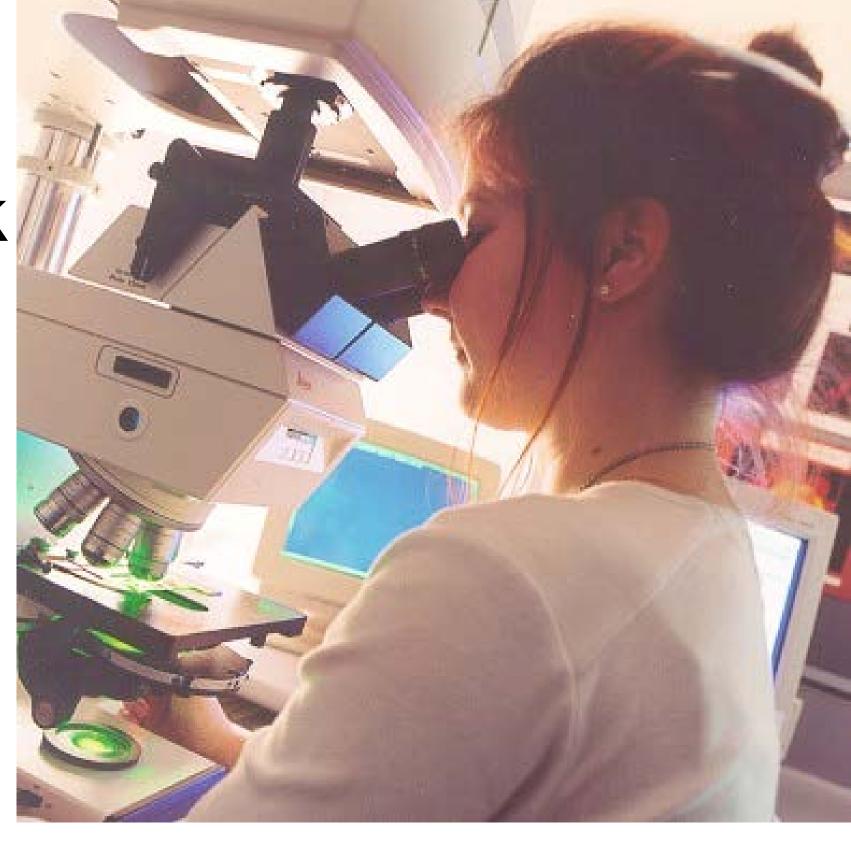
Bioinformatics

Through the UROP program, undergraduate students are employed in biomedical research, preparing 'work ready' graduates in this emerging field.

They work with real scientists and mathematicians throughout their course, gaining useful employment, experience and a taste of what the work is like.

http://www.bio21.com.au/pages/urop





Bioinformatics

The Walter and Eliza Hall Institute (WEHI) at the University of Melbourne and the Royal Melbourne Hospital has a Bioinformatics Division that investigates:

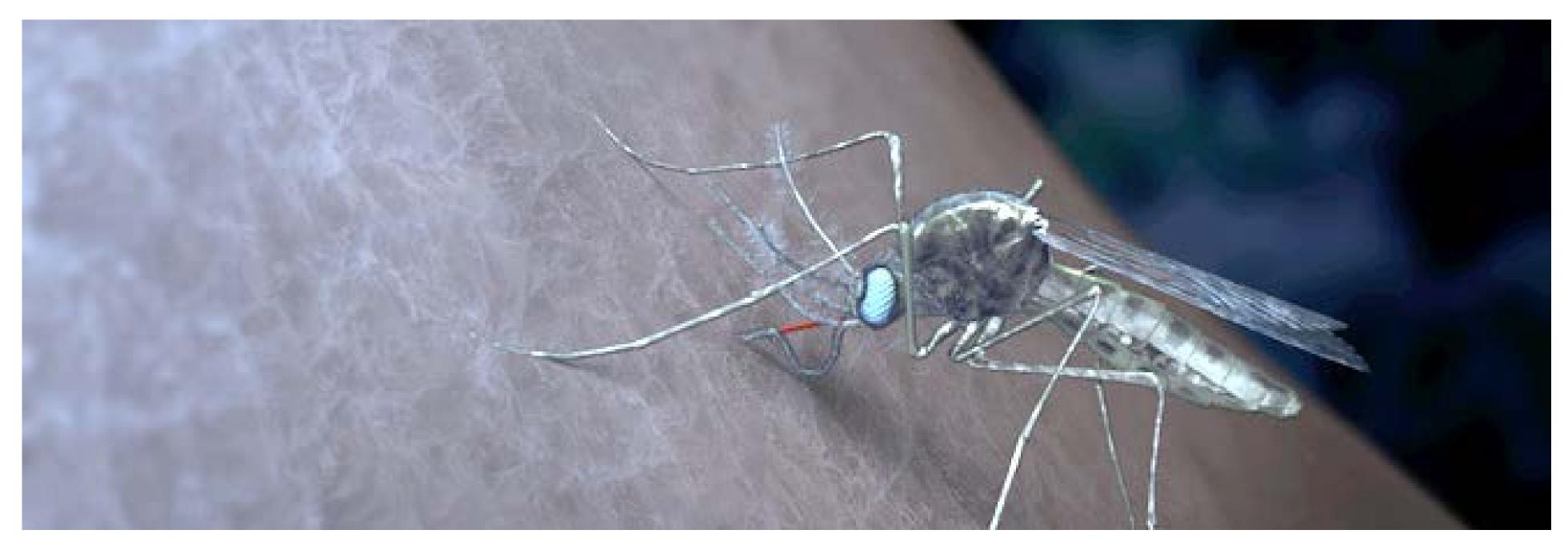
Cancer - Breast Cancer, Leukaemia and Lymphoma, Multiple Myeloma **<u>Chronic inflammatory diseases</u>** - Coeliac Disease, Type 1 Diabetes, Rheumatoid Arthritis, Transplantation Infectious diseases - Malaria, Chronic Infectious Diseases

www.wehi.edu.au/faculty/disease_research/



Bioinformatics and Malaria

A child dies of malaria every 15 seconds and the total number of deaths every year is 1.5 to 2.7 million.



http://www.wehi.edu.au/faculty/disease_research/



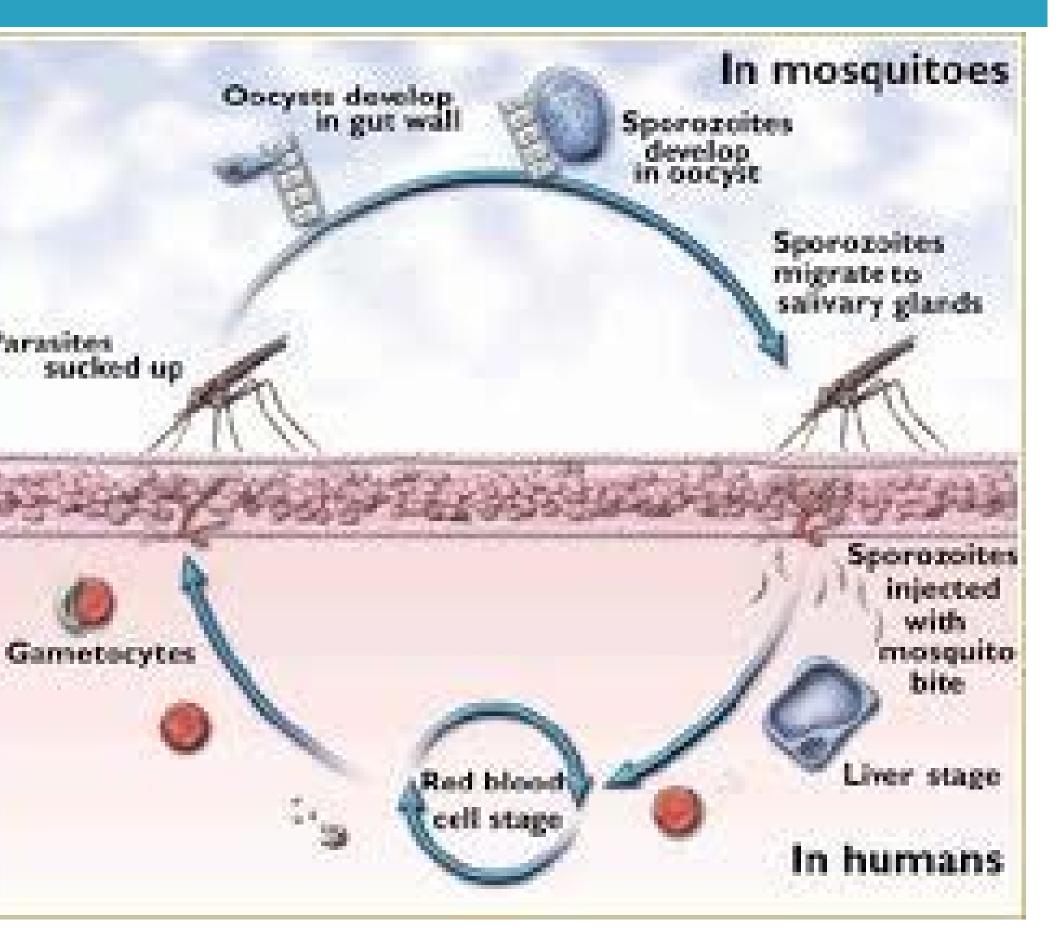


Bioinformatics and Malaria

By comparing the protein sequences in two similar parasites we can know more about how each of them works.

http://www.malariaworld.org/category/continent/africa?page=4





Bioinformatics and Malaria

Plasmodium falciparum is a protozoan parasite, one of the species of Plasmodium that cause malaria in humans.

http://en.wikipedia.org/wiki/File:Plasmodium_falciparum_01.png



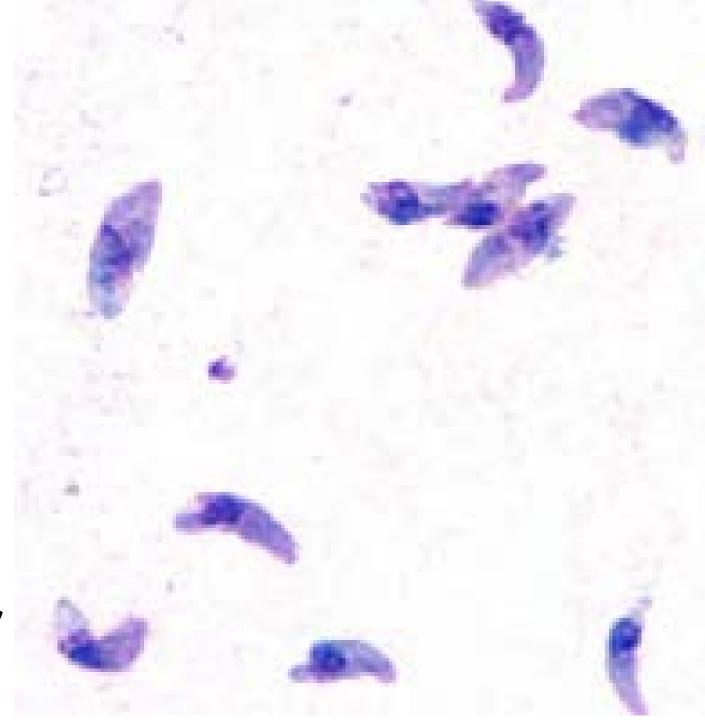


Bioinformatics and Malaria

Toxoplasma gondii is a species of parasitic protozoa.

Usually transmitted by cats, **Toxoplasmosis** is usually minor and selflimiting but can have serious or even fatal effects on a fetus whose mother first contracts the disease during pregnancy or on an immunocompromised human or cat.

http://en.wikipedia.org/wiki/Toxoplasma_gondii

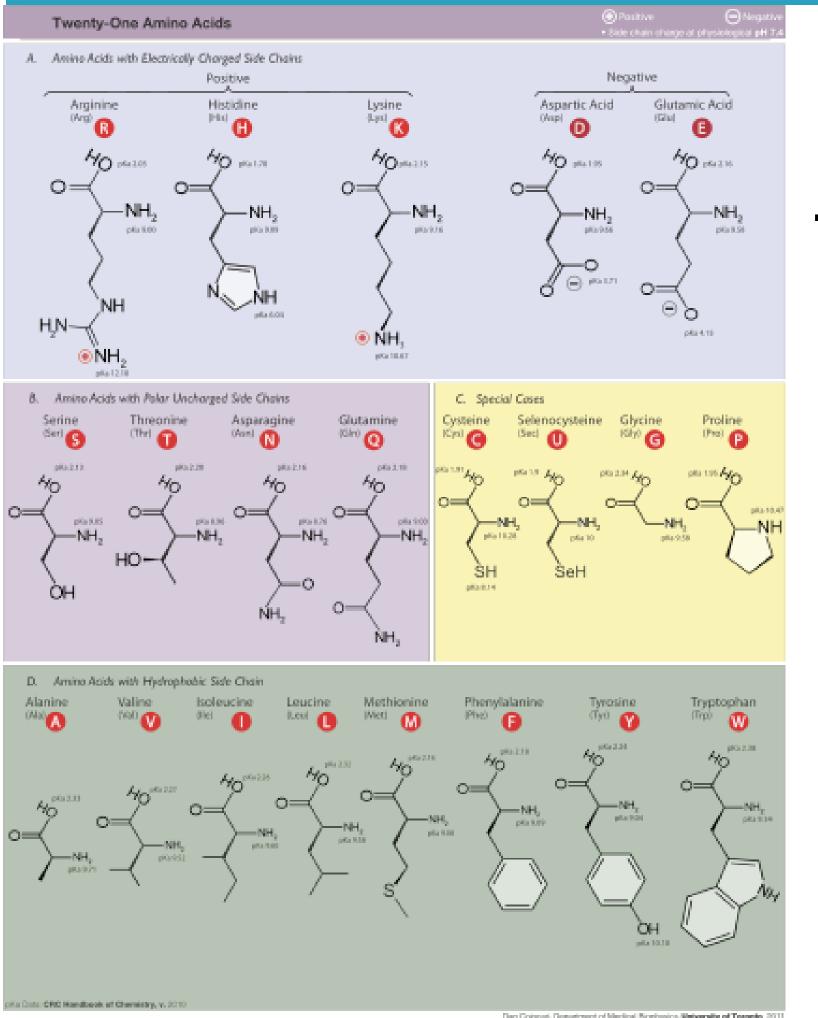


Bioinformatics and Malaria

- Hopefully some of what we know about toxoplasma can help us understand more about malaria.
- Everything in a cell is made of proteins. Proteins are used for transport within cells and are part of the cell structure.
- Knowing about the similar pathway proteins shared by these two species, we can use similar methods to transport drugs into a malaria cell.



Bioinformatics and Malaria



genetic material.

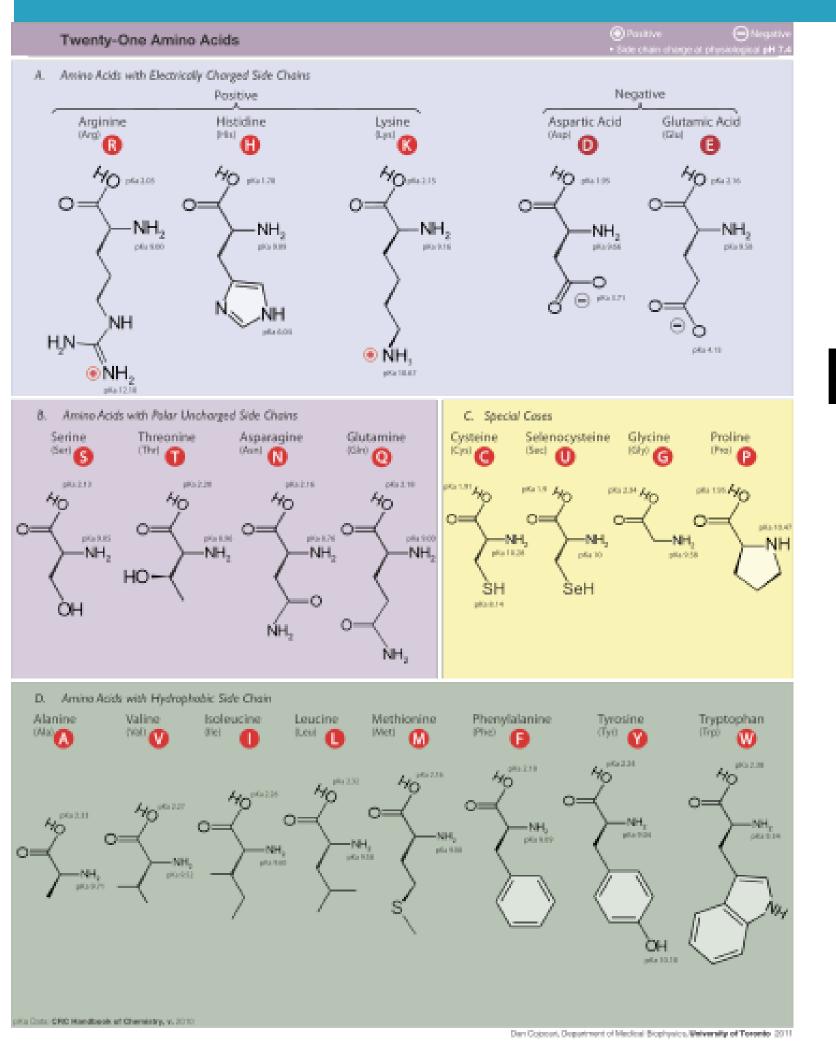


There are 21 Proteins that make up all

Denoted by the letters:

RHKDESTNQCUGPAVILMFYW

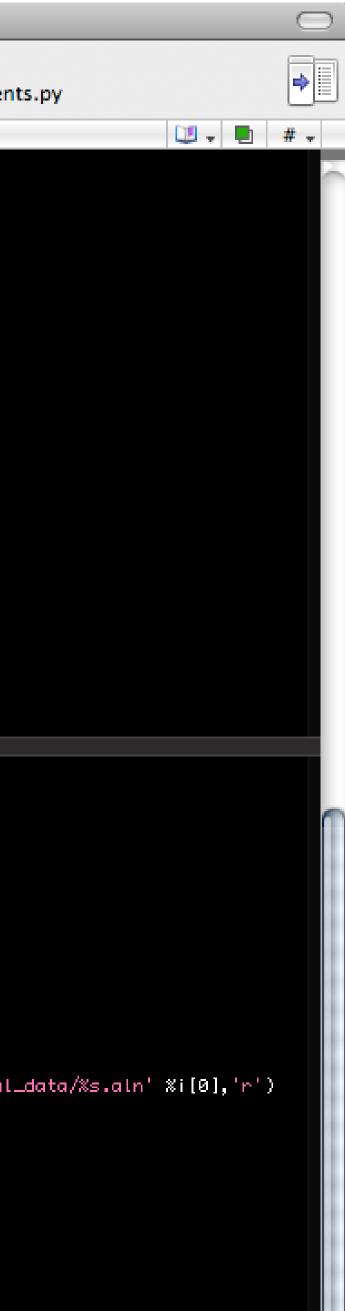
Bioinformatics and Malaria





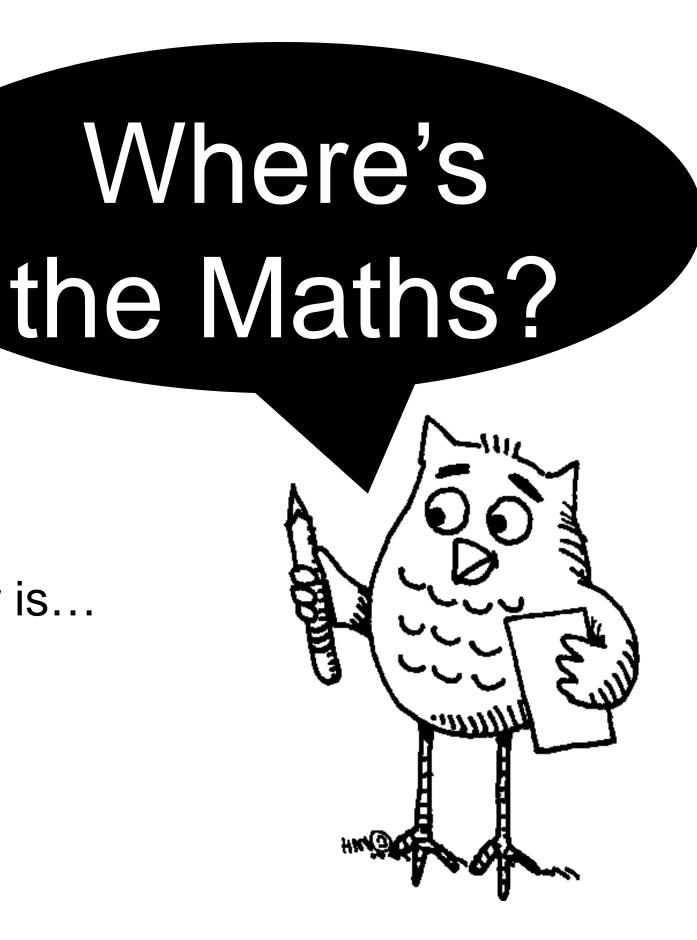
Predicting the likelihood of finding the same arrangement of proteins... using Python Programming software.

```
make_alignments.py
\bigcirc \bigcirc \bigcirc \bigcirc
                  6
                                   Last Saved: 11/29/11 3:44:24 PM
                            TEXT
            Ta
                                  File Path: ~/Documents/proj1b/make_alignments.py
make_alignments.py $ FastNW $
     return scormat, arrow
def Backtrace( arrow, seq1, seq2):
    st1, st2 = ","
     v,h = arrow.shape
     ok = 1
     v -=1
     h-=1
     while ok:
         if arrow[v,h] == 0:
              st1 += seq[v-1]
              st2 += '-
              v -=1
         elif arrow[v,h] == 1:
st1 += '-'
              st2 += seq2[h-1]
              h-=1
         elif arrow[v,h] == 2:
              st1 += seq1[v-1]
              st2 += seq2[h-1]
              v-=1
              h–=1
         if v == 0 and h == 0:
              ok = 0
     #reverse the strings
     st1 = st1[::-1]
     st2 = st2[::-1]
     return st1,st2
 ef FastNW(subvalues
def align(seqs,protein_seq):
     from numpy import *
    return None
def print_nicely():
    data = open('/Users/Imcintosh/Documents/proj1b/DATA.txt','r')
    data = open('/Users/Imcintosh/Documents/proj1b/DATA.txt','r')
     output = open('/Users/Imcintosh/Documents/proj1b/output.txt','w')
     count = 0
     for row in data:
         i = row.split('\t')
if i[6] == 'y':
              count +=1
         if (count == 200 and i[6]=='y'):
    protein_seq = open('/Users/lmcintosh/Documents/proj1b/clustal_data/%s.aln' %i[0],'r')
              length = int(find_start(i[0]))
              seqs = i[9].strip('"').split(',')
              j = 0
              line1 = []
              line2 = []
              line3 = []
              for line in protein_seq:
                   j +=1
                   #first output is on fourth line
                   #=> everything on 4+3+n4 is useless
                   if j > 3:
                       if (i-4) \otimes 4 == 0:
```



What is a mathematics curriculum?

We already knew... We remembered... We used equipment... We need to find out... It was interesting when... The tricky bit was... We didn't know that... It was cool when... The important thing to remember is... A new word we learnt was... Our group worked well when... We discovered... Congratulations to... The strategy we used was...



We thought... We said... We wrote... We saw... We heard... We know... We drew... We said... We asked... We felt... We liked... We learnt... We didn't like... We found out...