# Making connections in number

## PREP

# Making connections in number

Developing automaticity with number facts is the aim of many of the tasks we do with children.

The tasks themselves are not the aim.

To achieve this we need to help children pull apart numbers and put them back together again.

As children grow, tasks that require mental strategies need to become more cognitively demanding.

Children often develop these skills by themselves, other children need some gentle encouragement to do so.

## The Number System

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**Counting numbers or Natural numbers**

$$\{1, 2, 3, 4, 5, 6, 7, \ldots\}$$

Whenever you add or multiply two counting numbers, you get another counting number. This may not be true when you subtract or divide (e.g., 4-7 = -3, and –3 is not a counting number even though 4 and 7 are).

# The number system

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# The number system

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**Whole numbers**

$$\{0, 1, 2, 3, 4, 5, 6, \ldots\}$$

These are the counting numbers, together with 0.

# The number system

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**Integers**

{… -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, …}

Note that you can add, subtract and multiply two integers and you will always get an integer, but you may have trouble with division (e.g., 3 ÷ 6 = 1/2).

# The number system

**Rational numbers**

{m/n, where m and n are integers}

For example, 2/3, 7/11, -34/97, 7, 0, -1000, 0.15 = 15/100, 0.5555… = 5/9

Rational numbers are just fractions.

In decimal form, the rationals are those numbers which terminate (e.g., -5.657, because we can use our understanding of base 10 and decimals to write this as 5 657/1000) or those which have a recurring block (e.g., 4.3333… = 4 2/3 or 14/3, and 2.789789… = 2 263/333 or 929/333).

Division of a rational by a rational always gives a rational (except that you can't divide by 0).

# The number system

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**Irrational numbers**

Cannot be written in the form m/n, i.e, not rational.

For example,

$\pi$ = 3.1415926535879…,

0.101001000100001000001…,

e = 2.71828…,

0.1234567891011121314151617…

$\sqrt{2}$ = 1.4142…

The irrationals are those numbers that cannot be written as a fraction.

In decimal form, the irrational numbers are precisely the non-recurring non-terminating decimals.

There are actually “more” irrational numbers than there are rational numbers.

# The number system

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**Real numbers**

The rational numbers together with the irrational numbers make up the real numbers. These are the numbers we use in everyday life.

**Complex numbers**

[Learned in late high school or university, if at all.]

There are numbers which help us deal with things like $\sqrt{-5}$ (i.e. square roots of negative numbers). The real numbers can't do this, just as, for example, the integers let us take care of the fact that a whole number minus a whole number isn't always a whole number (e.g. 3-5).

# Counting

Early counting was done via one-to-one correspondence.

The word calculus means stone. It is the root of the word calculation because shepherds used pebbles to count their sheep.

Eventually words were used to describe sizes of sets;

| nose | eyes | clover | limbs | hand |
|---|---|---|---|---|
| ↕ | ↕ | ↕ | ↕ | ↕ |
| 1 | 2 | 3 | 4 | 5 |

but objects were still counted by comparing sets, eg by asking are there as many objects as I have limbs? etc

# Numeration

Eventually, it was realised that we can use sets of number-names in order as comparison sets, so

$$\text{nose} \leftrightarrow 1$$
$$\text{nose, eyes} \leftrightarrow 2$$
$$\text{nose, eyes, clover} \leftrightarrow 3$$
$$\text{nose, eyes, clover, limbs} \leftrightarrow 4$$
$$\text{nose, eyes, clover, limbs, hand} \leftrightarrow 5$$

This works because, for example, there are 3 words in the phrase "nose, eyes, clover".

The last number word you say is the comparison set you need.

# Numeration

Number is an abstract concept. *Three* is the concept of threeness.

**Numerals**

A numeral is a symbolic representation of a number.

So 7, 5 + 2, and VII are all numerals for the number we call seven;

10, 23 – 13 and X are all numerals for the number we call ten.

**Digits**

We call the symbols 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9 digits. They are numerals consisting of a single symbol.

(It is not a coincidence that the same word means fingers.

Early counting was, and still is, done using fingers.)

# Hindu Arabic Notation

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Hindu-Arabic notation is a place value system based on bundles of 10; so it is a decimal system.

The key to a place value system is the use of a place marker.

A place value system using 9 digits and a space or the word kha (for emptiness) as place marker was used in India the 6th century.

# Hindu Arabic Notation

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By the 9th century the system had made its way to the Arab world (including Persia and Al-Andalus in what is now Spain).

The digit 0 evolved from "·" and was used in both Madhya Pradesh (Northern India) and the Arab world by the 10th century.

Leonardo Fibonacci learned to use the notation from merchants in Africa when he was a boy and wrote a book, Liber Abaci, in 1202 which popularized the system.

# Number with small people

Teachers can provide experiences which help children see that there can be many different relationships between things, just as there is with numbers:

# Number with small people

Helen and Claire (aged 5) are sitting on Claire's outside hammock. Claire has said that she sometimes sits on the hammock at night with her Daddy and looks at the stars.

Helen: Do you try to count the stars?

Claire: Yes.

Helen: And how many stars did you count?

Claire: About eighty-eleven.

Helen: Wow! That many?

[Helen decides that she would like to know more about Claire's counting scheme.]

Helen: And how far can you count?

Claire: [Points across the valley] About over to those mountains over there.

# Number with small people

A child learns to count by experiencing counting in their life. They then count real “things”, concrete objects from the world around them.
Through a series of experiences where the child constructs, de-constructs and re-constructs knowledge about counting they begin to abstract the counting sequence and can generalise the “idea” of counting.
A parent, teacher or life experiences provide opportunity for resolution of misconceptions, a sorting out of the mathematical ideas.
It is not only counting that is learned in this way, imagine learning about flowers without experiencing what one felt, looked and smelled like!

# Number with small people

Classification of groups of objects and manipulating materials with a variety of characteristics. Children put things together in a group based on a characteristic. For example, sets of coloured animals can be classified according to colour, animal, lives inside/outside, or is a favourite.

# Counting

Counting
Children's development of counting ability follows human development of number systems:

- Counting by ones
- Counting using groups and singles
- Counting using five as a benchmark
- Counting using tens and ones

You might design activities that force children to do this. My friend the Martian makes piles of five icy pole sticks and bundles them with a rubber band, can you count how many he has? etc.

# Counting - the song

My preschooler can count twice as high as your preschooler can, but does that mean s/he really understands more about maths?

In truth, s/he has merely memorized a sequence of words. Although children can't learn maths unless they know how to count, counting is only one aspect of maths.

# Counting – the song

Rhys McIntosh 'nearly two'

Learning the rhythmic progression of counting begins to take place from very early in the child's life.

Listening to a heartbeat is the first experience of that "song" before any melody is put to it.

Then, being walked in the arms of an adult and the rocking and patting that soothes a young baby continue the lesson in beat.

A toddler counts how many steps holding the hand of a parent, and a two-year-old's favourite number is two until they have a birthday, when they quickly learn what comes next in the sequence.

# Counting – the song

Children start by rote counting, they learn the song and sometimes in the beginning they get the words of the song in the wrong order.

Then children recite numeral names in order.

As the child gets older s/he attaches a numeral name to a number of things.

Children learn that each number symbol represents an amount, gradually putting recognition of numerals together with counting.

# Counting

Different arrangements for using objects to help with counting

Arrange the counters differently. See if children know where they started counting so they can decide where to stop.

- a line
- a circle
- random

# Counting

**100 charts**

Make tables in word

Number Ladder

Make puzzles out of them

Start at different numbers

Go over the hundred

Fill in the blanks

Extend to thousands

# Counting

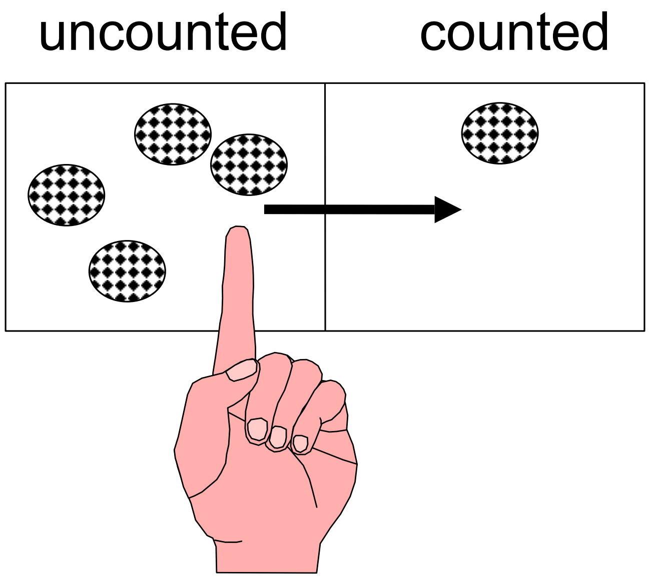
**Try really hard**
Telling children to “try really hard” increases accuracy (there have been studies to show this!) also ask children to “get their brain ready for maths in much the same way we do a reading session:

What might we do?
What might we say?
What might we use?

**Mr Mixup**
A puppet that makes mistakes counting. “Can you explain to Mr. Mixup why it is a mistake?”

# Counting

Counting mat



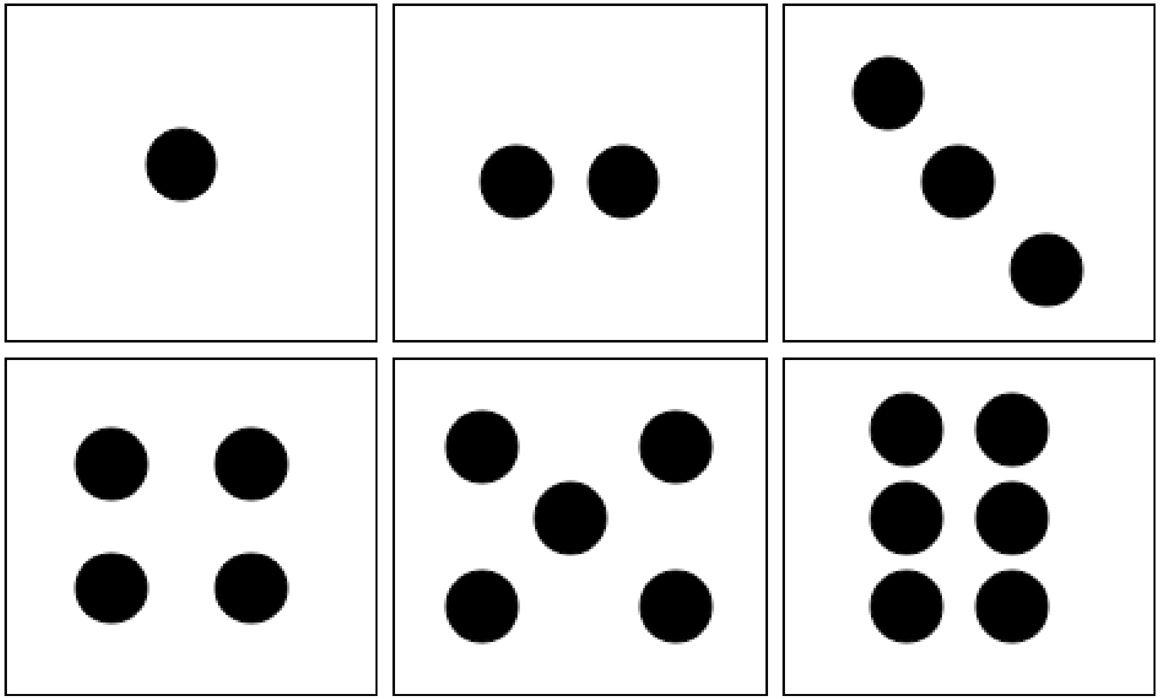
# Subitizing

Subitizing is the ability to identify quickly the numbers in a set without counting.

For small children this is usually limited to two or three dots when the objects are randomly arranged. Children as young as two can do this, some say babies can do it too.

As numbers increase in magnitude older children and adults start to try to group them in some way or look for a pattern. There are some common and easy to recognise patterns. For example, dice patterns. Dice are an inexpensive and fun teaching tool, let your little ones play with a handful and observe what they already know about number.

# Subitizing



# Subitizing

Beach ball activities

# WHISPER Counting

Whisper Counting is a precursor to skip counting.

For example: "One, two, THREE, four, five, SIX, seven, eight, NINE.

When Whisper Counting by three's, you say every third number loudly. Gradually say the skip counted numbers louder and louder and the other numbers softer and softer. Increase the speed of counting until the child can count entirely in skip count mode (3, 6, 9, 12, 15...). You can also clap (hop, ring a bell, etc.) instead of whispering as you move to skip counting. Give children time to develop and feel successful with one number before moving to the next one.

# Number line

The number line can be used as a model for number from the first years of primary school through to the early years of secondary schooling. It is a model that can be consistently applied in most areas of number.

Comparing

Modelling

Ordering

Operations

# Number line

Introducing the number line

Mark in zero and one other reference point

Convention of negative numbers to the left, zero in the middle and positive numbers to the right

Move towards children drawing their own

# Number line

Use

- Masking tape on the floor
- String across the room
- Chalk in the playground
- Magnetized numbers on a blackboard or whiteboard
- Cash-register rolls
- Number ladder

# Number ladder

- Look at place value
- Highlight patterns
- Forward and backwards counting
- Skip counting
- Predict where you will be…

# Zero

Zero

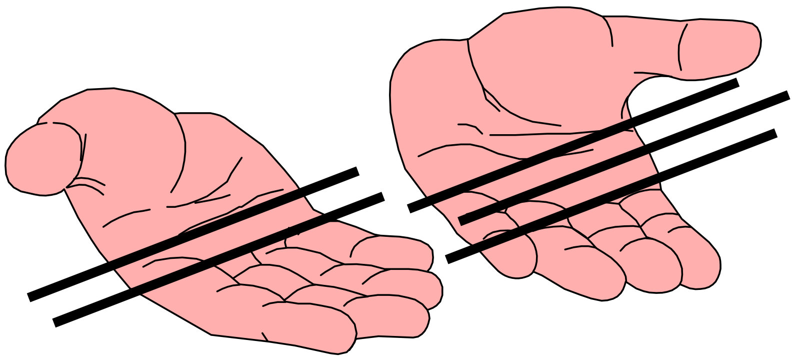
Children use the concept of zero from an early age.

The language of zero, that is the many different words that mean zero, can be used freely in the classroom setting.

- Empty container
- I have nothing left on my plate
- If I take away your toy you won’t have any

# Early partitioning

**Stick in hands**



# Early partitioning

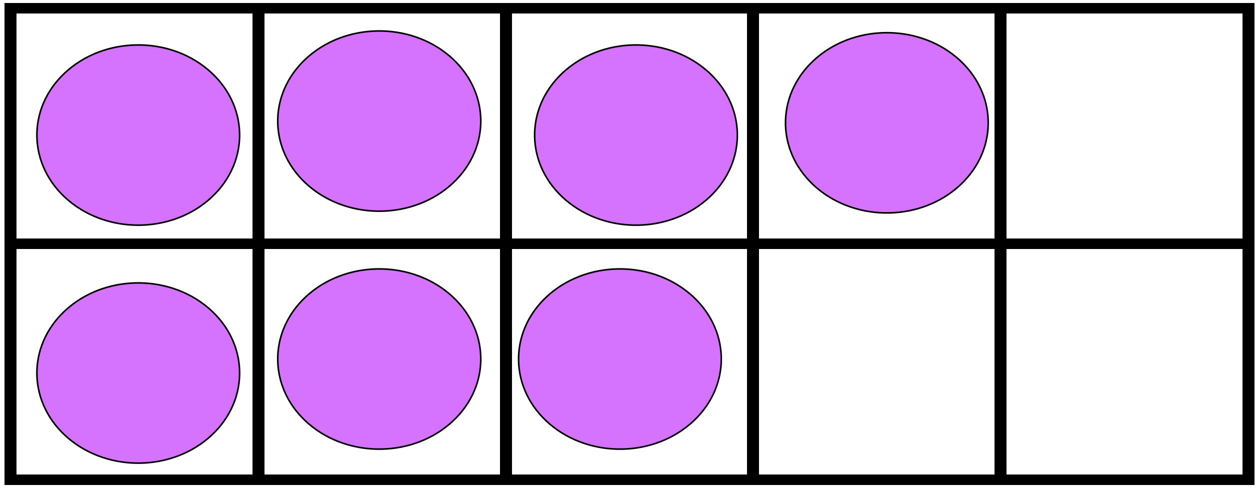
Tens frames can be used to explore part whole concepts and provides a strong conceptual base for addition and subtraction strategies. You can use an egg carton with two of the hollows cut off. Try using counters, sultanas or Smarties on an individual ten frame that has been photocopied and laminated. How many different ways can you show what seven looks like on the ten frame?

Seven is

two, two, two and one
three and four, four and three
five and two, two and five
six and one, one and six
seven and none, zero and seven

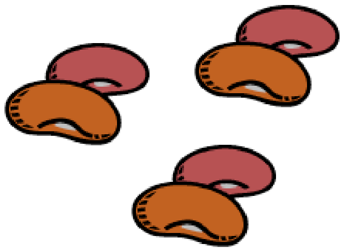
# Early partitioning

- Ten frame



# Early partitioning

Magic beans
(Lima beans sprayed gold on one side)

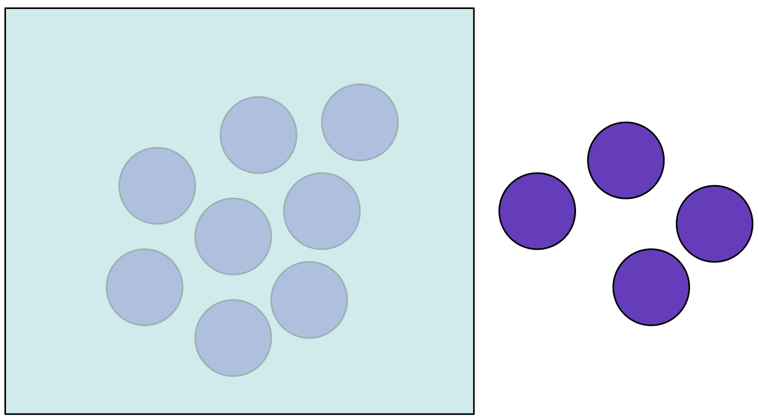


Take a handful.
Throw them.
Talk about the number of gold and white.
Leads to additive statements.

# Addition

Cover up
Encourages ‘counting on’

Tell the child
“I have four buttons here and 8 more under the card.”



# Fractions

At this level we are only concerned with real objects, sharing & cutting up.
There is no bigger half!
Experience of parts is the pre-cursor to working with fractions. Children learn wholes have parts. Later they learn that parts are fractions of wholes.
Experiencing things that
- have parts such as bodies, cars,
- have sets such as puzzles, tea sets, toys
-divide wholes such as cakes, packets of biscuits, blocks of chocolate
sets the scene for later work with fractions and introduces some of the language of fractions.