The Australian Mathematical Sciences Institute

PD 2: Multiplicative concepts

Good understanding of place value and multiplicative concepts helps children work well in many areas of mathematics such as fractions.

In this session, we will look at building solid foundations and moving children forward in their thinking.

If you would like a copy of these slides, please have your memory stick ready at the end of the session.

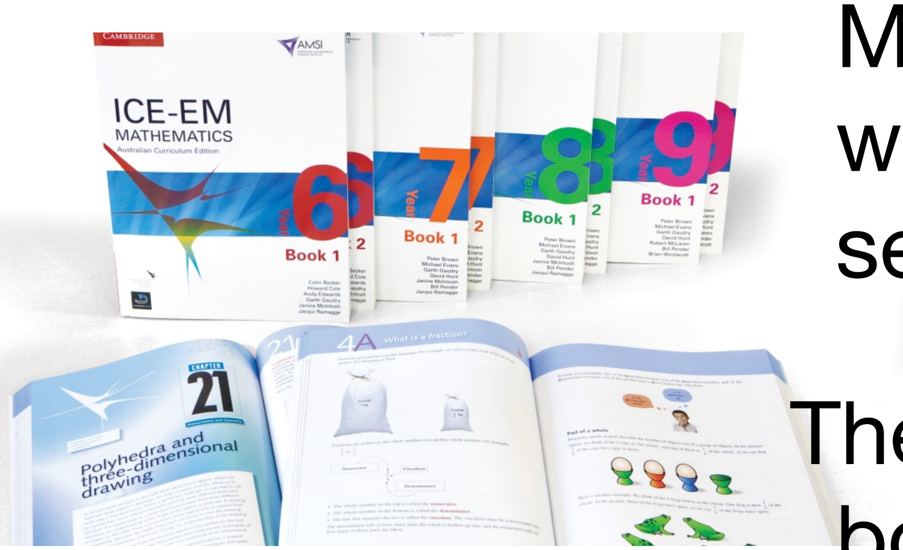
The Australian Mathematical Sciences Institute

AMSI was established in November, 2002

We aim to improve the teaching of mathematics at primary and secondary level by joining with teachers, mathematics teacher associations and government agencies to develop a strategy to address issues such as teacher shortfalls and under-qualified teachers.

AMSI has 34 members including university mathematics departments, the Australian Mathematics Trust, the Australian Bureau of Statistics, the Bureau of Meteorology and CSIRO.

The developers of ICE-EM Mathematics school textbooks for Years 5 to 10



AMSI's education division, the International Centre of Excellence for Education in Mathematics (ICE-EM), has been undertaking wide ranging education programs at primary, secondary and tertiary levels since 2004.

The ICE-EM Mathematics Program provides books, professional development and teacher resource materials.

The Australian Curriculum versions of the books are published by Cambridge University Press.

Outreach

In 2013/14 the Outreach team will work in:

* Gippsland, VIC (DEECD)
* Yarraville/Footscray, VIC (Boeing)
* Geelong, VIC (The William Buckland Foundation)
* Oakey, QLD (Australian Government)
* Warialda, NSW (Australian Government)

Outreach

- One year of funding - to end June 2014
- Professional development and support to local schools

Outreach

What do schools receive?

Access to AMSI online materials.

A new resource portal (website) to be developed this year.

Outreach

What do schools receive?

8 Professional development sessions, one of these specifically for secondary mathematics teachers and careers advisors.

Schools visits

- Scheduled at a time to suit the school.
- Includes modelled lessons, one-to-one sessions, target specific issues such as content or graduate teachers

Outreach

What do schools contribute?

CRT costs are covered by the school.

We will ask participating teachers to complete a survey at the start and again at the end.

Australian Curriculum: Mathematics

Multiplicative concepts

Prep

Focus on Counting, addition and subtraction

Year 1

CD: Develops confidence with number sequences to and from 100 by ones from any starting point. Skip counts by twos, fives and tens starting from zero (ACMNA012) (TIMESNA01)

Australian Curriculum: Mathematics

Multiplicative concepts

Year 2

CD: Investigates number sequences, initially those increasing and decreasing by twos, threes, fives and ten from any starting point, then moving to other sequences (ACMNA026)(TIMESNA01)

CD: Recognises and represents multiplication as repeated addition, groups and arrays (ACMNA031)(TIMESNA03)

CD: Recognises and represents division as grouping into equal sets and solves simple problems using these representations (ACMNA032)(TIMESNA03)

ELAB: Identifies the difference between dividing a set of objects into three equal groups and dividing the same set of objects into groups of three (ACMNA032) (TIMESNA03)

Australian Curriculum: Mathematics

Multiplicative concepts

Year 2
CD: Recognises and interprets common uses of halves, quarters and eighths of shapes and collections (ACMNA033)
ELAB: Recognises that sets of objects can be portioned in different ways to demonstrate fractions (ACMNA033)
ELAB: Relates the number of parts to the size of a fraction (ACMNA033)

Australian Curriculum: Mathematics

Multiplicative concepts

Year 3

CD: Recalls multiplication facts of two, three, five and ten and related division facts (ACMNA056) (TIMESNA03)

CD: Represents and solves problems involving multiplication (eg: writing simple word problems in numerical form and vice versa) using efficient mental and written strategies and appropriate digital technologies (ACMNA057)(TIMESNA03)

CD: Models and represents unit fractions including halves, thirds, quarters and fifths and their multiples to a complete whole(ACMNA058) (TIMESNA14)

Australian Curriculum: Mathematics

Multiplicative concepts

Year 3
CD: Models and represents unit fractions including halves, thirds, quarters and fifths and their multiples to a complete whole(ACMNA058) (TIMESNA14)
ELAB: Partitions areas, lengths and collections to create halves, thirds, quarters and fifths, such as folding the same sized sheets of paper to illustrate different unit fractions and comparing the number of parts with their sizes (ACMNA058) (TIMESNA14)
ELAB: Locates unit fractions on a number line where the numerator is one and the denominator is a whole number eg:1/2 or 1/3. (ACMNA058)

Multiplication

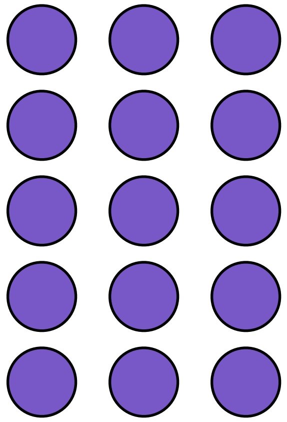
Use a variety of multiplication models as well as the algorithm.

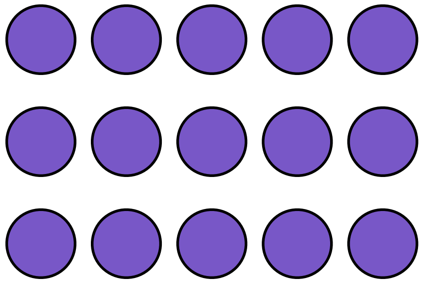
- Number line
- Arrays
- Area

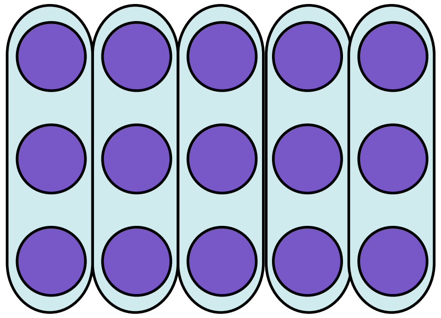
Topping up skills idea:
Understanding and rapid recall of multiplication tables is important.

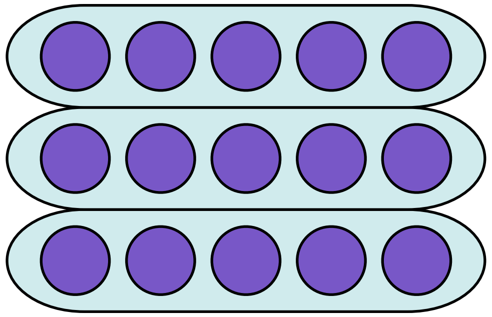
Multiplication- arrays

15



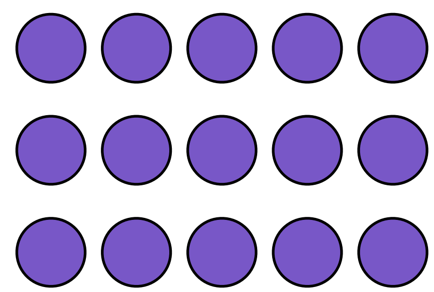


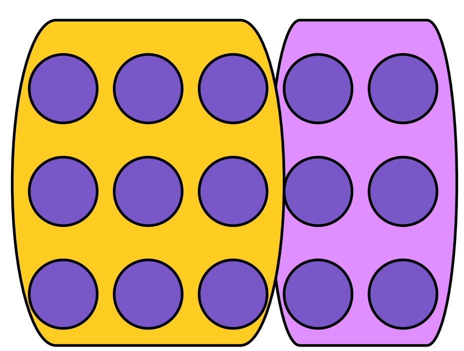


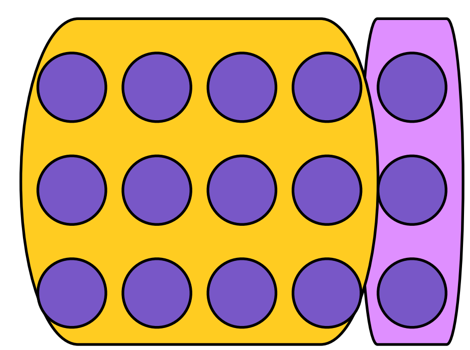


Multiplication- arrays

15

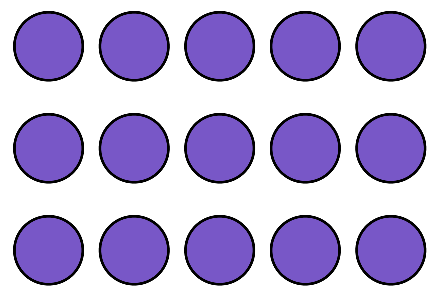


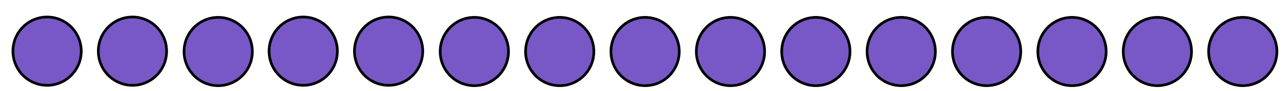




Multiplication- arrays

15

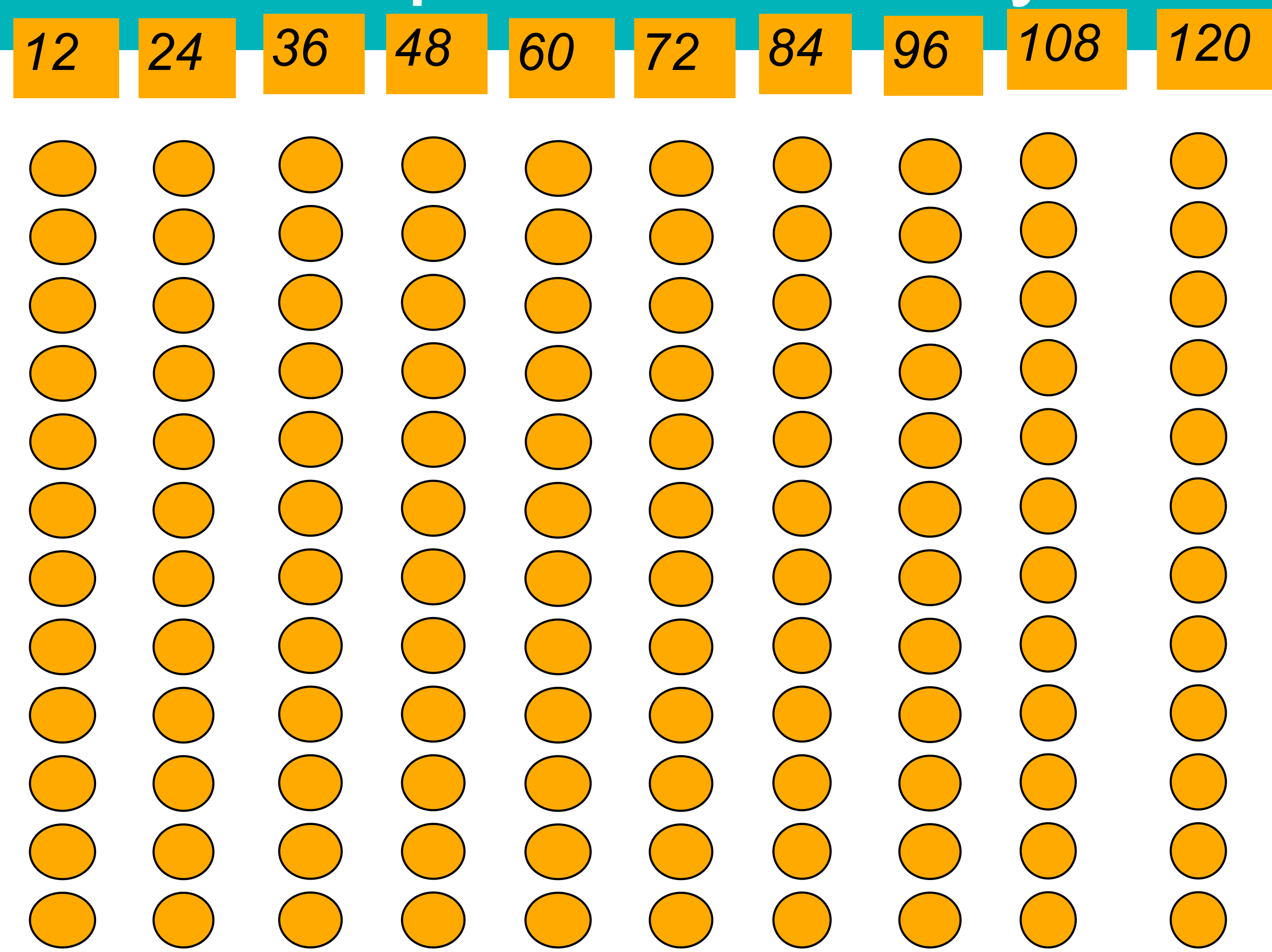




The factors of 15 are
3 and 5

1 and 15

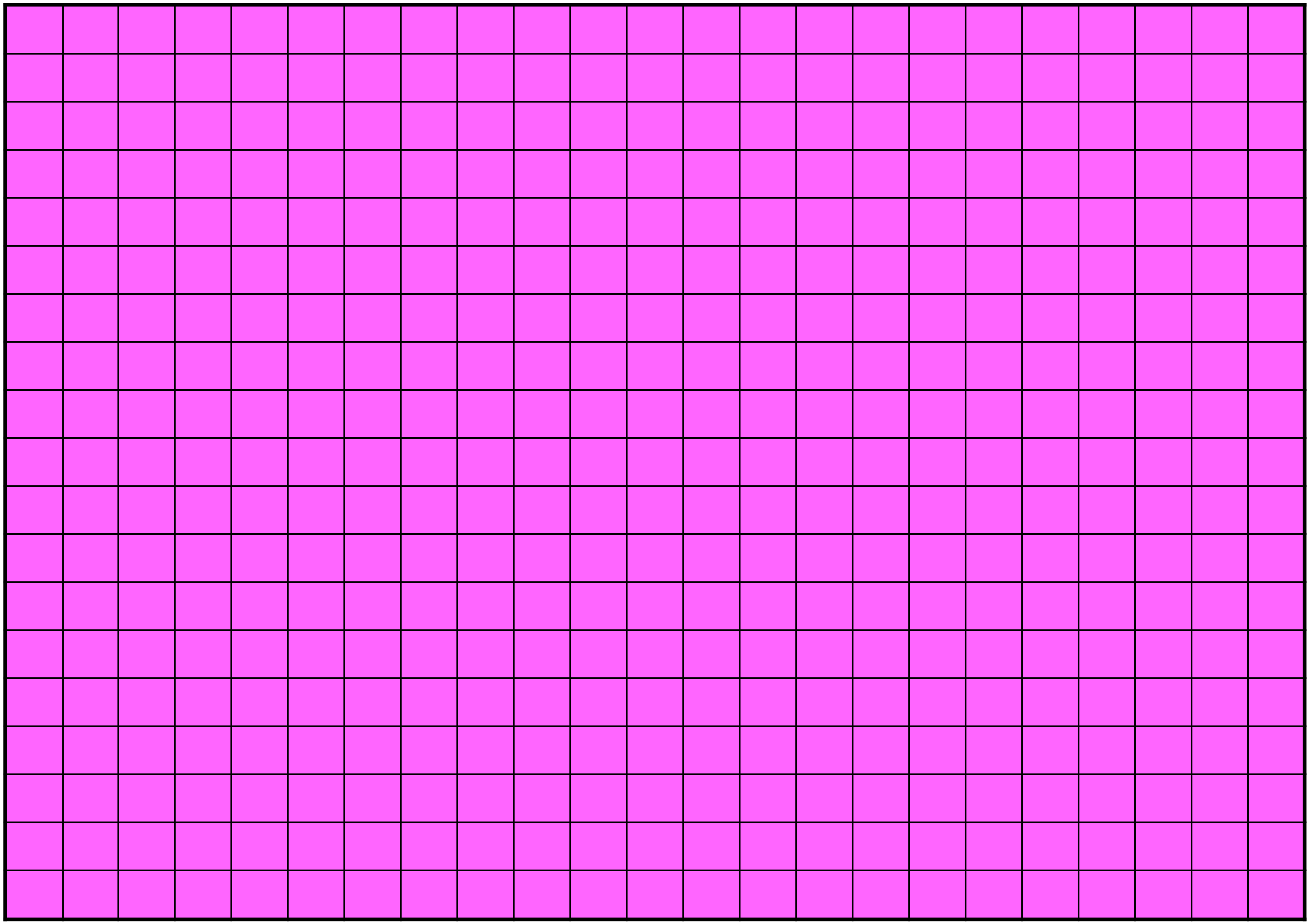
Multiplication- arrays



More than repeated addition

How many pink tiles are there in the next slide?

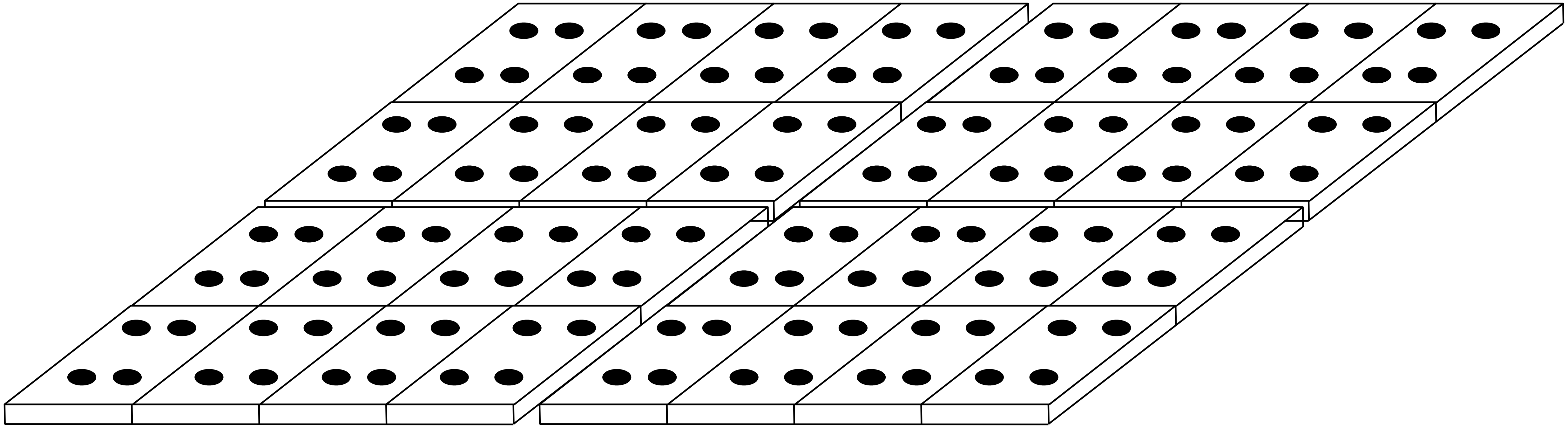
More than repeated addition



More than repeated addition

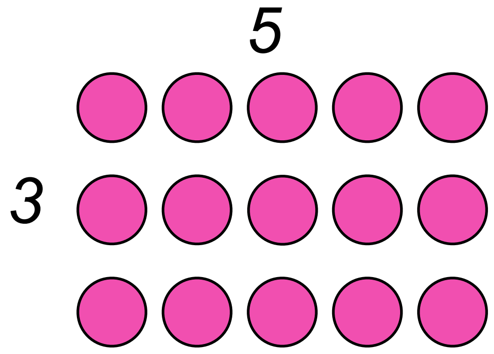
How many dots are there?

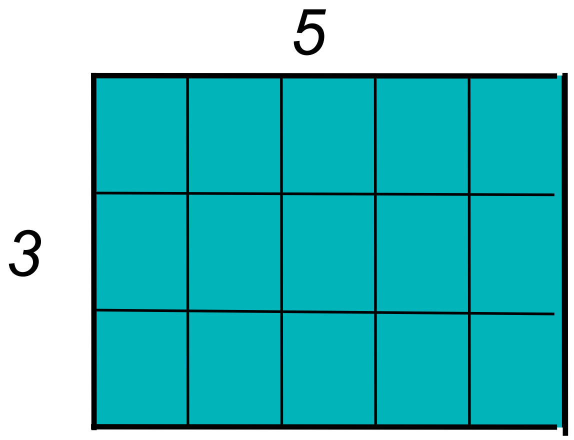
More than repeated addition



Multiplication- arrays

link to area





15

Multiplication- arrays

Prime numbers

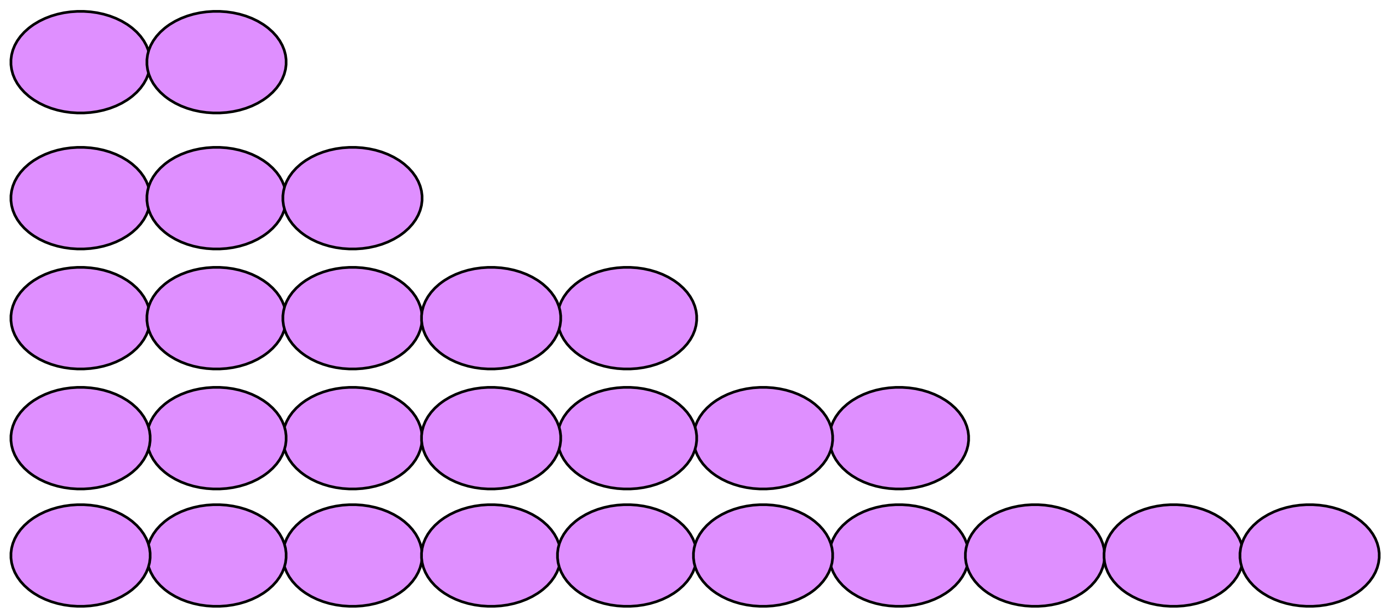
2

3

5

7

11



we can only make arrays that are single rows

Prime numbers

Manhattan

On a unifix 100 board:

•place orange unifix cubes on all the multiples of 2

•place blue unifix cubes on all the multiples of 3

•place red unifix cubes on all the multiples of 4

And so on up to multiples of 10

FreeFoto.com

Prime numbers have no cubes on them

Numbers with lots of factors are the tallest ‘buildings’

With thanks to Mark Richardson,
Williamstown Primary School, Victoria

Prime numbers

The Licorice Factory - MCTP

Prime numbers

Cicada hatchings in the USA

Prime factorisation

Prime factorisation is one of the key building blocks in mathematics.

The prime factorisation for a number is unique to that number.

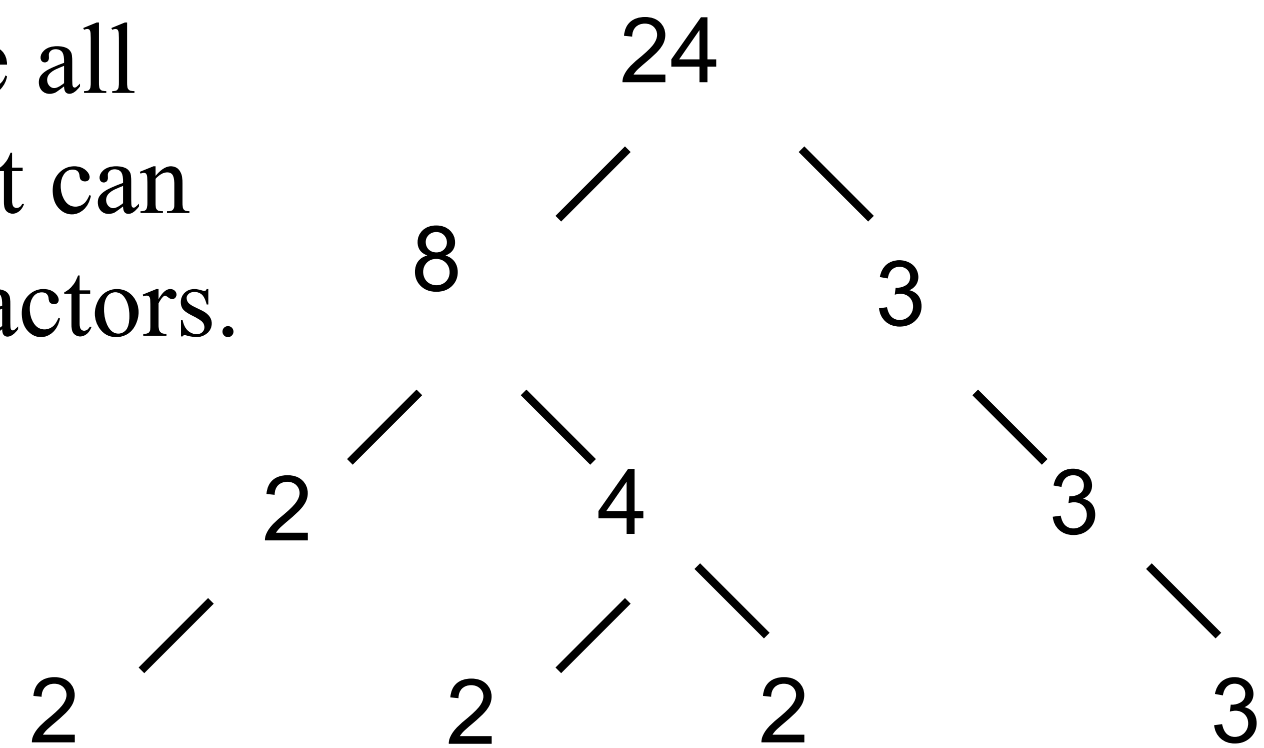
For example:

$140 = 2 \times 2 \times 5 \times 7$

$= 2^2 \times 5 \times 7$

Prime factorisation

Factor trees do not give all factors of a number, but can be used to find prime factors.



Prime factorisation

Division by prime numbers gives prime factors.

2. | 24
2 | 12
3. | 6
4. | 3
1

Prime factorisation

Consider 10! using prime factorisation.

$10! = 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2$ (x 1 but not shown)

$= 2 \times 5 \times 3^2 \times 2^3 \times 7 \times 2 \times 3 \times 5 \times 2^2 \times 3 \times 2$

$= 7 \times 5^2 \times 3^4 \times 2^8$

Now we can ask questions such as how many zeroes there would be at the end of such a number.

Try 90! It doesn’t take as long as you think!

Lowest common multiple

Using prime factorisation.

For example, find the lowest common multiple of 24 and 18.

$24 = 2^3 \times 3$ $\qquad$ $18 = 2 \times 3^2$

For the LCM, take the factors with the highest power from each.

LCM $= 2^3 \times 3^2$

$= 8 \times 9$

$= 72$

When am I going to use this?

Needed for fractions and algebra

Highest common factor

Using prime factorisation.

For example, find the highest common factor of 24 and 18.

$24 = 2^3 \times 3$ $\qquad$ $18 = 2 \times 3^2$

For the HCF, take the factors with the lowest power from each.

HCF $= 2 \times 3$

$= 6$

Prime factorisation- alternative method

Korean method- (Christian Brothers QLD method)

To find the lcm and hcf of 24 and 36:

2.	24	36
2	12	18
3.	6	9
	2	3

HCF $= 2^2 \times 3$
$= 12$

LCM $= 2^2 \times 3 \times 2 \times 3$
$= 72$

Square root

Square roots by prime factorisation.

To find the square root of 1296, first find the prime factors.

$$\sqrt{1296} = \sqrt{2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3}$$

$$= \sqrt{2^4 \times 3^4}$$

$$= \sqrt{(2^2 \times 3^2) \times (2^2 \times 3^2)}$$

$$= 2^2 \times 3^2$$

$$= 36$$

- only need to go up to the square root of the number to find the factors...

VOCAB

- Dinosaurs
- then lazy
- pre-loading?

Strengthening multiplication tables skills

- Rapid recall of the times tables to 12 is essential to operating efficiently on numbers.
- Ask students to think carefully about how well they know each fact, can they recall them very, very quickly off the top of their head?
- If the answer is ‘no’ then that student needs to work on that fact.
- Draw up a 12 x 12 chart and as students achieve each step, they colour in those facts if they are not already coloured in.
- The facts that are not coloured in are the ones that this student needs to work on.

Start with a times tables grid

·	1	2	3	4	5	6	7	8	9	10	11	12	13
1	1	2	3	4	5	6	7	8	9	10	11	12	13
2	2	4	6	8	10	12	14	16	18	20	22	24	26
3	3	6	9	12	15	18	21	24	27	30	33	36	39
4	4	8	12	16	20	24	28	32	36	40	44	48	52
5	5	10	15	20	25	30	35	40	45	50	55	60	65
6	6	12	18	24	30	36	42	48	54	60	66	72	78
7	7	14	21	28	35	42	49	56	63	70	77	84	91
8	8	16	24	32	40	48	56	64	72	80	88	96	104
9	9	18	27	36	45	54	63	72	81	90	99	108	117
10	10	20	30	40	50	60	70	80	90	100	110	120	130
11	11	22	33	44	55	66	77	88	99	110	121	132	143
12	12	24	36	48	60	72	84	96	108	120	132	144	156
13	13	26	39	52	65	78	91	104	117	130	143	156	169

Colour in all of the duplicated facts

·	1	2	3	4	5	6	7	8	9	10	11	12	13
1	1	2	3	4	5	6	7	8	9	10	11	12	13
2	2	4	6	8	10	12	14	16	18	20	22	24	26
3	3	6	9	12	15	18	21	24	27	30	33	36	39
4	4	8	12	16	20	24	28	32	36	40	44	48	52
5	5	10	15	20	25	30	35	40	45	50	55	60	65
6	6	12	18	24	30	36	42	48	54	60	66	72	78
7	7	14	21	28	35	42	49	56	63	70	77	84	91
8	8	16	24	32	40	48	56	64	72	80	88	96	104
9	9	18	27	36	45	54	63	72	81	90	99	108	117
10	10	20	30	40	50	60	70	80	90	100	110	120	130
11	11	22	33	44	55	66	77	88	99	110	121	132	143
12	12	24	36	48	60	72	84	96	108	120	132	144	156
13	13	26	39	52	65	78	91	104	117	130	143	156	169

You already know how to multiply by 1

·	1	2	3	4	5	6	7	8	9	10	11	12	13
1	1	2	3	4	5	6	7	8	9	10	11	12	13
2	2	4	6	8	10	12	14	16	18	20	22	24	26
3	3	6	9	12	15	18	21	24	27	30	33	36	39
4	4	8	12	16	20	24	28	32	36	40	44	48	52
5	5	10	15	20	25	30	35	40	45	50	55	60	65
6	6	12	18	24	30	36	42	48	54	60	66	72	78
7	7	14	21	28	35	42	49	56	63	70	77	84	91
8	8	16	24	32	40	48	56	64	72	80	88	96	104
9	9	18	27	36	45	54	63	72	81	90	99	108	117
10	10	20	30	40	50	60	70	80	90	100	110	120	130
11	11	22	33	44	55	66	77	88	99	110	121	132	143
12	12	24	36	48	60	72	84	96	108	120	132	144	156
13	13	26	39	52	65	78	91	104	117	130	143	156	169

And 2!

·	1	2	3	4	5	6	7	8	9	10	11	12	13
1	1	2	3	4	5	6	7	8	9	10	11	12	13
2	2	4	6	8	10	12	14	16	18	20	22	24	26
3	3	6	9	12	15	18	21	24	27	30	33	36	39
4	4	8	12	16	20	24	28	32	36	40	44	48	52
5	5	10	15	20	25	30	35	40	45	50	55	60	65
6	6	12	18	24	30	36	42	48	54	60	66	72	78
7	7	14	21	28	35	42	49	56	63	70	77	84	91
8	8	16	24	32	40	48	56	64	72	80	88	96	104
9	9	18	27	36	45	54	63	72	81	90	99	108	117
10	10	20	30	40	50	60	70	80	90	100	110	120	130
11	11	22	33	44	55	66	77	88	99	110	121	132	143
12	12	24	36	48	60	72	84	96	108	120	132	144	156
13	13	26	39	52	65	78	91	104	117	130	143	156	169

Anything multiplied by 10 ends in 0

·	1	2	3	4	5	6	7	8	9	10	11	12	13
1	1	2	3	4	5	6	7	8	9	10	11	12	13
2	2	4	6	8	10	12	14	16	18	20	22	24	26
3	3	6	9	12	15	18	21	24	27	30	33	36	39
4	4	8	12	16	20	24	28	32	36	40	44	48	52
5	5	10	15	20	25	30	35	40	45	50	55	60	65
6	6	12	18	24	30	36	42	48	54	60	66	72	78
7	7	14	21	28	35	42	49	56	63	70	77	84	91
8	8	16	24	32	40	48	56	64	72	80	88	96	104
9	9	18	27	36	45	54	63	72	81	90	99	108	117
10	10	20	30	40	50	60	70	80	90	100	110	120	130
11	11	22	33	44	55	66	77	88	99	110	121	132	143
12	12	24	36	48	60	72	84	96	108	120	132	144	156
13	13	26	39	52	65	78	91	104	117	130	143	156	169

Multiplying by 5, ends in 5 or 0

·	1	2	3	4	5	6	7	8	9	10	11	12	13
1	1	2	3	4	5	6	7	8	9	10	11	12	13
2	2	4	6	8	10	12	14	16	18	20	22	24	26
3	3	6	9	12	15	18	21	24	27	30	33	36	39
4	4	8	12	16	20	24	28	32	36	40	44	48	52
5	5	10	15	20	25	30	35	40	45	50	55	60	65
6	6	12	18	24	30	36	42	48	54	60	66	72	78
7	7	14	21	28	35	42	49	56	63	70	77	84	91
8	8	16	24	32	40	48	56	64	72	80	88	96	104
9	9	18	27	36	45	54	63	72	81	90	99	108	117
10	10	20	30	40	50	60	70	80	90	100	110	120	130
11	11	22	33	44	55	66	77	88	99	110	121	132	143
12	12	24	36	48	60	72	84	96	108	120	132	144	156
13	13	26	39	52	65	78	91	104	117	130	143	156	169

Squash the square numbers

·	1	2	3	4	5	6	7	8	9	10	11	12	13
1	1	2	3	4	5	6	7	8	9	10	11	12	13
2	2	4	6	8	10	12	14	16	18	20	22	24	26
3	3	6	9	12	15	18	21	24	27	30	33	36	39
4	4	8	12	16	20	24	28	32	36	40	44	48	52
5	5	10	15	20	25	30	35	40	45	50	55	60	65
6	6	12	18	24	30	36	42	48	54	60	66	72	78
7	7	14	21	28	35	42	49	56	63	70	77	84	91
8	8	16	24	32	40	48	56	64	72	80	88	96	104
9	9	18	27	36	45	54	63	72	81	90	99	108	117
10	10	20	30	40	50	60	70	80	90	100	110	120	130
11	11	22	33	44	55	66	77	88	99	110	121	132	143
12	12	24	36	48	60	72	84	96	108	120	132	144	156
13	13	26	39	52	65	78	91	104	117	130	143	156	169

Throw out the threes

·	1	2	3	4	5	6	7	8	9	10	11	12	13
1	1	2	3	4	5	6	7	8	9	10	11	12	13
2	2	4	6	8	10	12	14	16	18	20	22	24	26
3	3	6	9	12	15	18	21	24	27	30	33	36	39
4	4	8	12	16	20	24	28	32	36	40	44	48	52
5	5	10	15	20	25	30	35	40	45	50	55	60	65
6	6	12	18	24	30	36	42	48	54	60	66	72	78
7	7	14	21	28	35	42	49	56	63	70	77	84	91
8	8	16	24	32	40	48	56	64	72	80	88	96	104
9	9	18	27	36	45	54	63	72	81	90	99	108	117
10	10	20	30	40	50	60	70	80	90	100	110	120	130
11	11	22	33	44	55	66	77	88	99	110	121	132	143
12	12	24	36	48	60	72	84	96	108	120	132	144	156
13	13	26	39	52	65	78	91	104	117	130	143	156	169

File the fours

·	1	2	3	4	5	6	7	8	9	10	11	12	13
1	1	2	3	4	5	6	7	8	9	10	11	12	13
2	2	4	6	8	10	12	14	16	18	20	22	24	26
3	3	6	9	12	15	18	21	24	27	30	33	36	39
4	4	8	12	16	20	24	28	32	36	40	44	48	52
5	5	10	15	20	25	30	35	40	45	50	55	60	65
6	6	12	18	24	30	36	42	48	54	60	66	72	78
7	7	14	21	28	35	42	49	56	63	70	77	84	91
8	8	16	24	32	40	48	56	64	72	80	88	96	104
9	9	18	27	36	45	54	63	72	81	90	99	108	117
10	10	20	30	40	50	60	70	80	90	100	110	120	130
11	11	22	33	44	55	66	77	88	99	110	121	132	143
12	12	24	36	48	60	72	84	96	108	120	132	144	156
13	13	26	39	52	65	78	91	104	117	130	143	156	169

Elevens to 110 are next to go

·	1	2	3	4	5	6	7	8	9	10	11	12	13
1	1	2	3	4	5	6	7	8	9	10	11	12	13
2	2	4	6	8	10	12	14	16	18	20	22	24	26
3	3	6	9	12	15	18	21	24	27	30	33	36	39
4	4	8	12	16	20	24	28	32	36	40	44	48	52
5	5	10	15	20	25	30	35	40	45	50	55	60	65
6	6	12	18	24	30	36	42	48	54	60	66	72	78
7	7	14	21	28	35	42	49	56	63	70	77	84	91
8	8	16	24	32	40	48	56	64	72	80	88	96	104
9	9	18	27	36	45	54	63	72	81	90	99	108	117
10	10	20	30	40	50	60	70	80	90	100	110	120	130
11	11	22	33	44	55	66	77	88	99	110	121	132	143
12	12	24	36	48	60	72	84	96	108	120	132	144	156
13	13	26	39	52	65	78	91	104	117	130	143	156	169

Nail the nines!

·	1	2	3	4	5	6	7	8	9	10	11	12	13
1	1	2	3	4	5	6	7	8	9	10	11	12	13
2	2	4	6	8	10	12	14	16	18	20	22	24	26
3	3	6	9	12	15	18	21	24	27	30	33	36	39
4	4	8	12	16	20	24	28	32	36	40	44	48	52
5	5	10	15	20	25	30	35	40	45	50	55	60	65
6	6	12	18	24	30	36	42	48	54	60	66	72	78
7	7	14	21	28	35	42	49	56	63	70	77	84	91
8	8	16	24	32	40	48	56	64	72	80	88	96	104
9	9	18	27	36	45	54	63	72	81	90	99	108	117
10	10	20	30	40	50	60	70	80	90	100	110	120	130
11	11	22	33	44	55	66	77	88	99	110	121	132	143
12	12	24	36	48	60	72	84	96	108	120	132	144	156
13	13	26	39	52	65	78	91	104	117	130	143	156	169

There aren't so many facts to learn now.

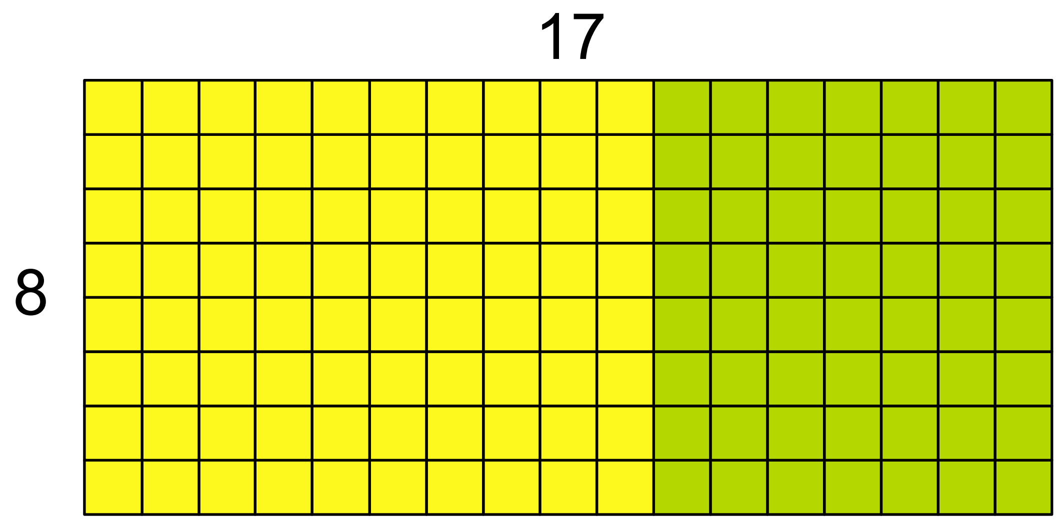
·	1	2	3	4	5	6	7	8	9	10	11	12	13
1	1	2	3	4	5	6	7	8	9	10	11	12	13
2	2	4	6	8	10	12	14	16	18	20	22	24	26
3	3	6	9	12	15	18	21	24	27	30	33	36	39
4	4	8	12	16	20	24	28	32	36	40	44	48	52
5	5	10	15	20	25	30	35	40	45	50	55	60	65
6	6	12	18	24	30	36	42	48	54	60	66	72	78
7	7	14	21	28	35	42	49	56	63	70	77	84	91
8	8	16	24	32	40	48	56	64	72	80	88	96	104
9	9	18	27	36	45	54	63	72	81	90	99	108	117
10	10	20	30	40	50	60	70	80	90	100	110	120	130
11	11	22	33	44	55	66	77	88	99	110	121	132	143
12	12	24	36	48	60	72	84	96	108	120	132	144	156
13	13	26	39	52	65	78	91	104	117	130	143	156	169

Multiplication

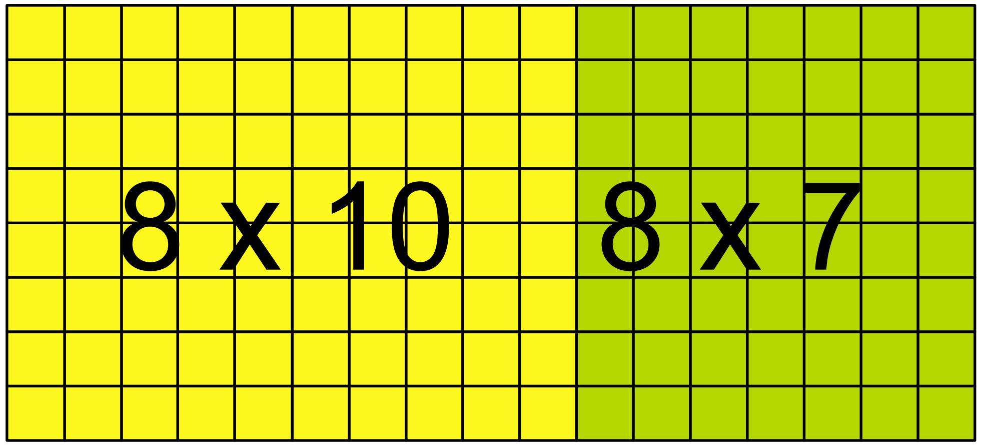
Linking arrays and areas with the multiplication algorithm.

For example, 8×17

Multiplication



Multiplication



8 x 17 = 8 x 10 + 8 x 7

= 80 + 56

= 136

Multiplication

$$\begin{array}{r} {}^{5}1\ 7 \\ \cdot \quad\quad 8 \\ \hline 1\ 3\ 6 \\ \hline \end{array}$$

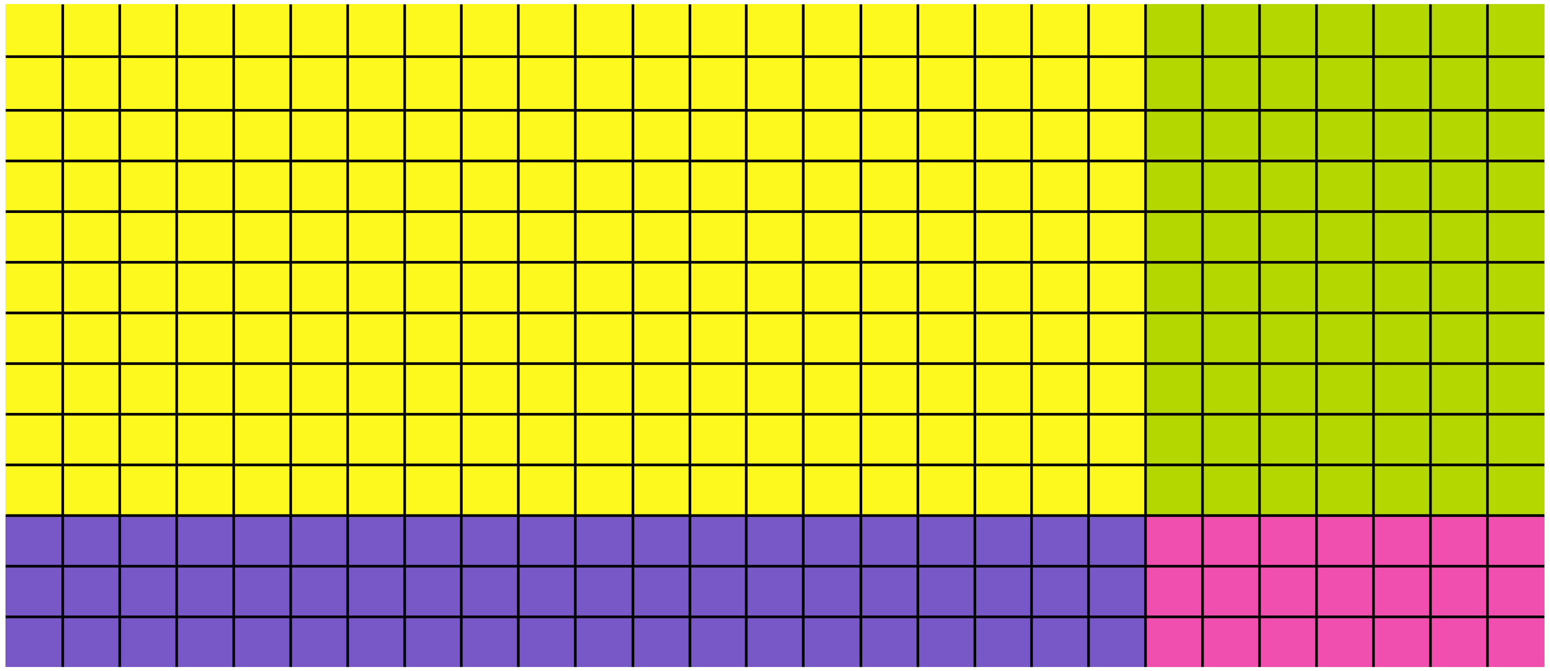
Multiplication

Linking arrays and areas with the long multiplication algorithm.

For example, 27 x 13

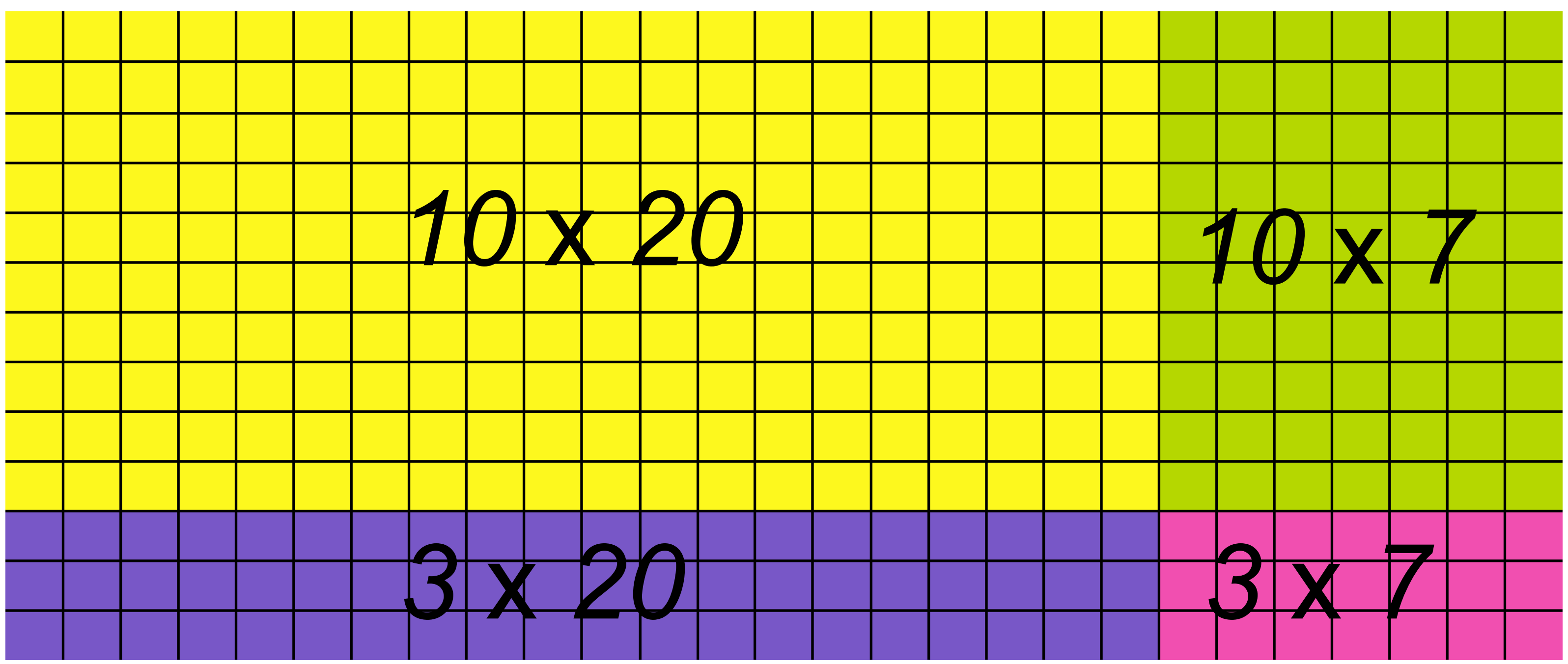
Multiplication

Draw an array



Multiplication

Highlight the chunks



Multiplication

Algorithm

$$
\begin{array}{r}
{}^{2}27 \\
\underline{13} \\
81 \\
\underline{270} \\
\underline{351}
\end{array}
$$

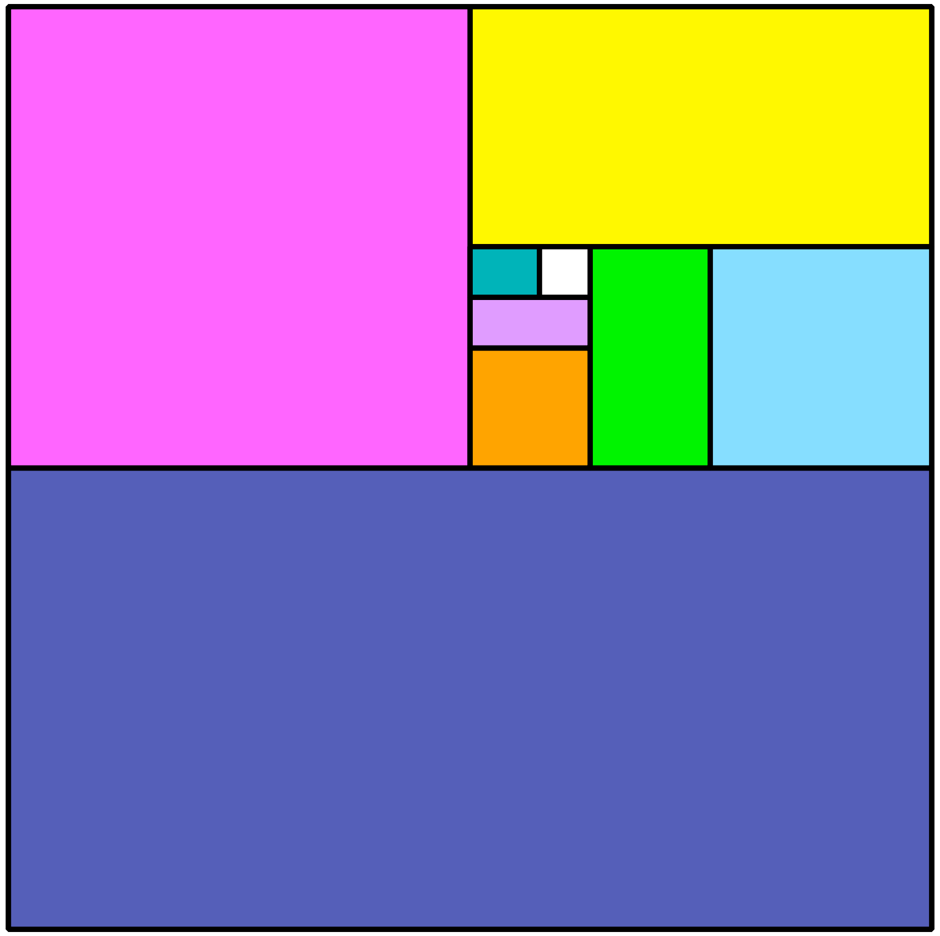
10 times 7 is 70

10 times 20 is 200

3 times 7 is 21

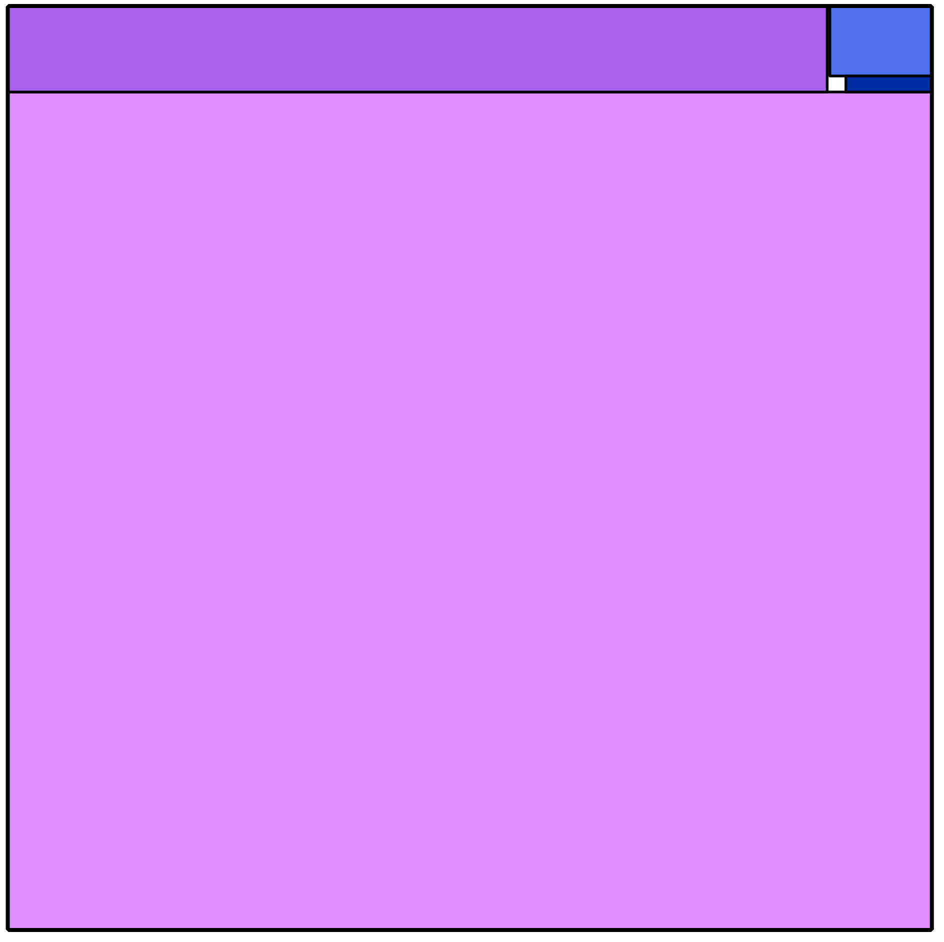
3 times 20 is 60 plus 20 is 80

Fractions - Area



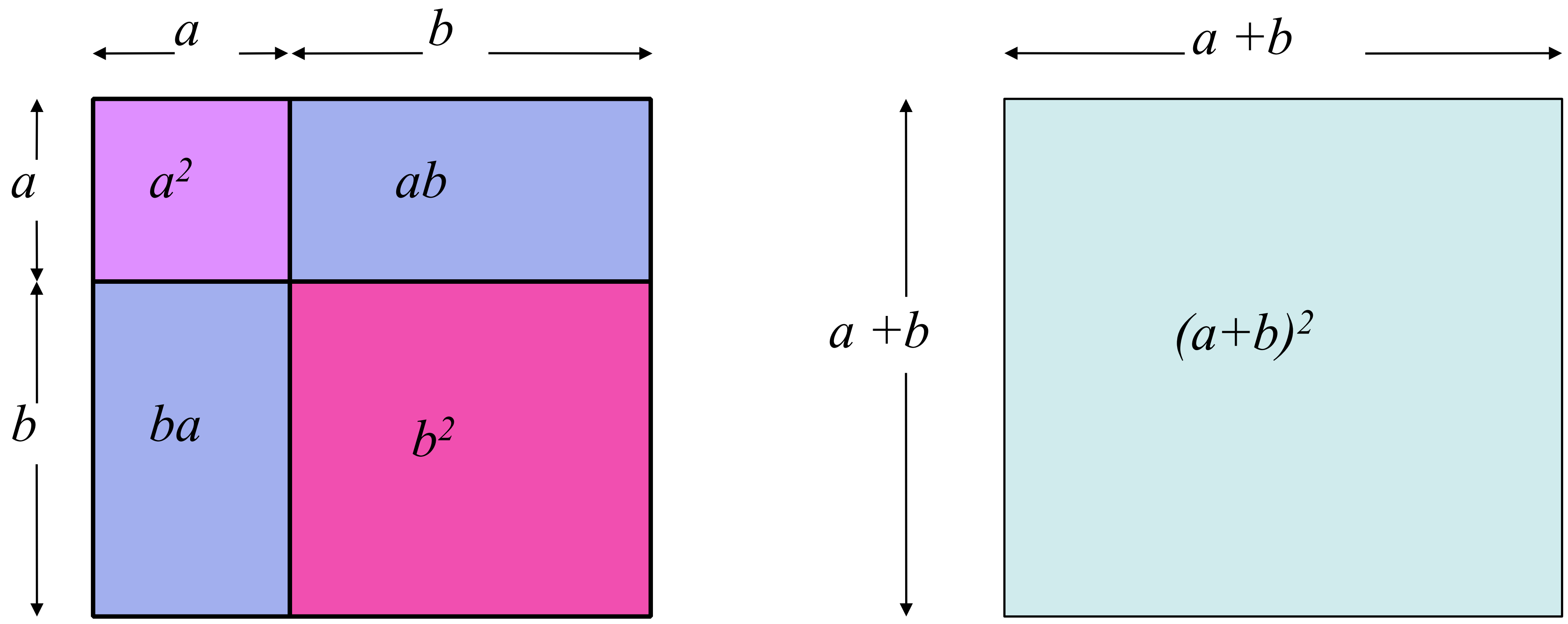
1/2 + 1/4 + 1/8 + 1/16 + 1/32 + 1/64 + …

Decimals - Area



0.9 + 0.09 + 0.009 + 0.0009...

Algebra - Area

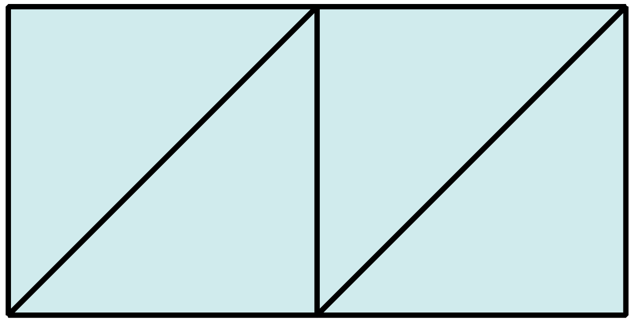


$$(a+b)^2 = a^2 + 2ab + b^2$$

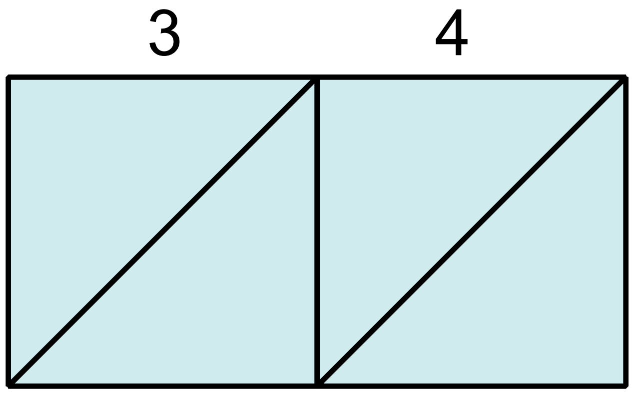
Lattice Multiplication

34 x 6

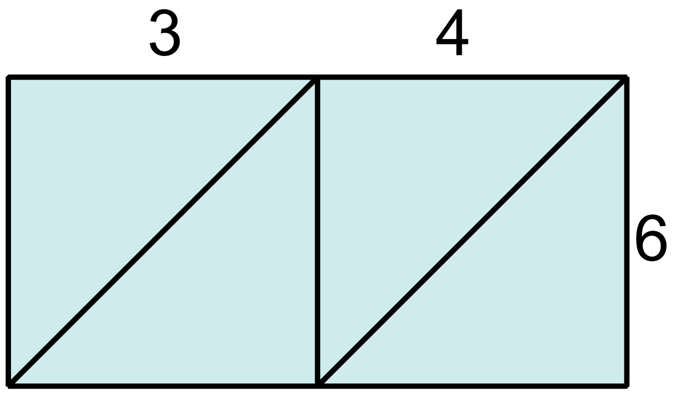
Lattice Multiplication



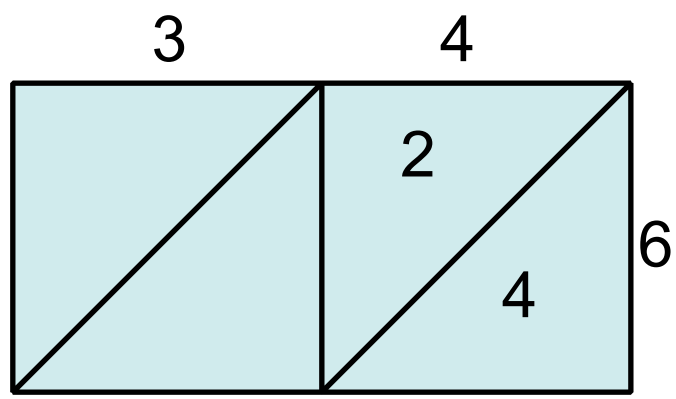
Lattice Multiplication



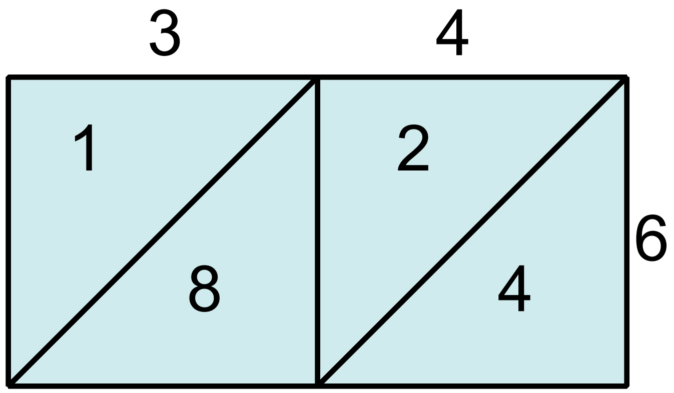
Lattice Multiplication



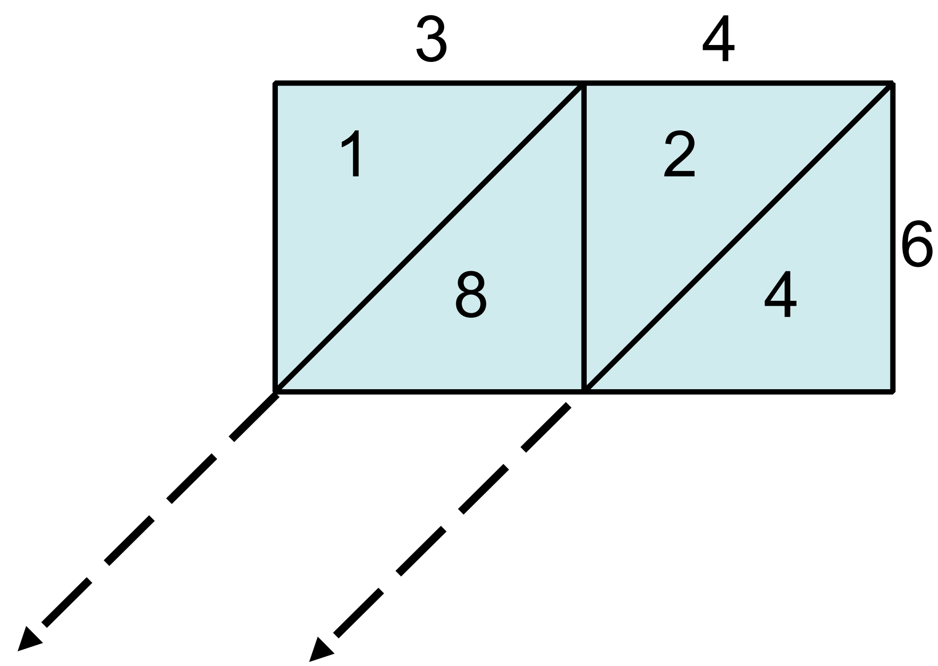
Lattice Multiplication



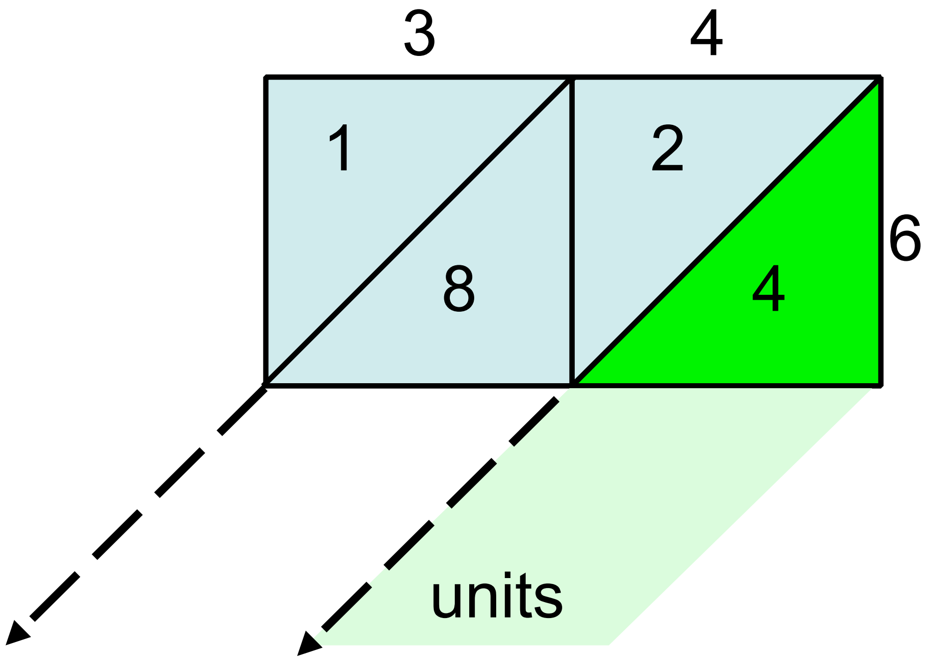
Lattice Multiplication



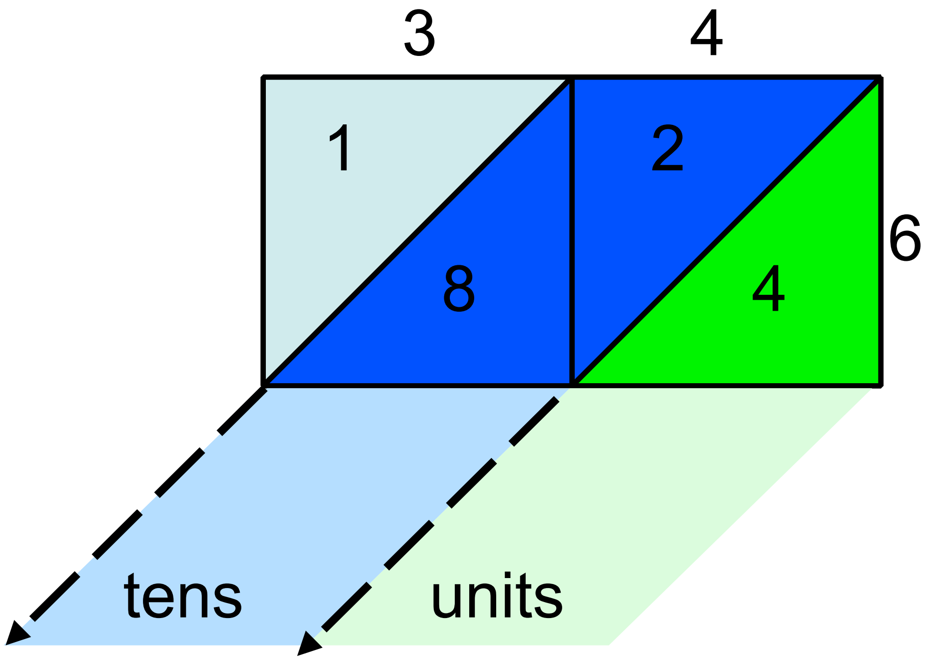
Lattice Multiplication



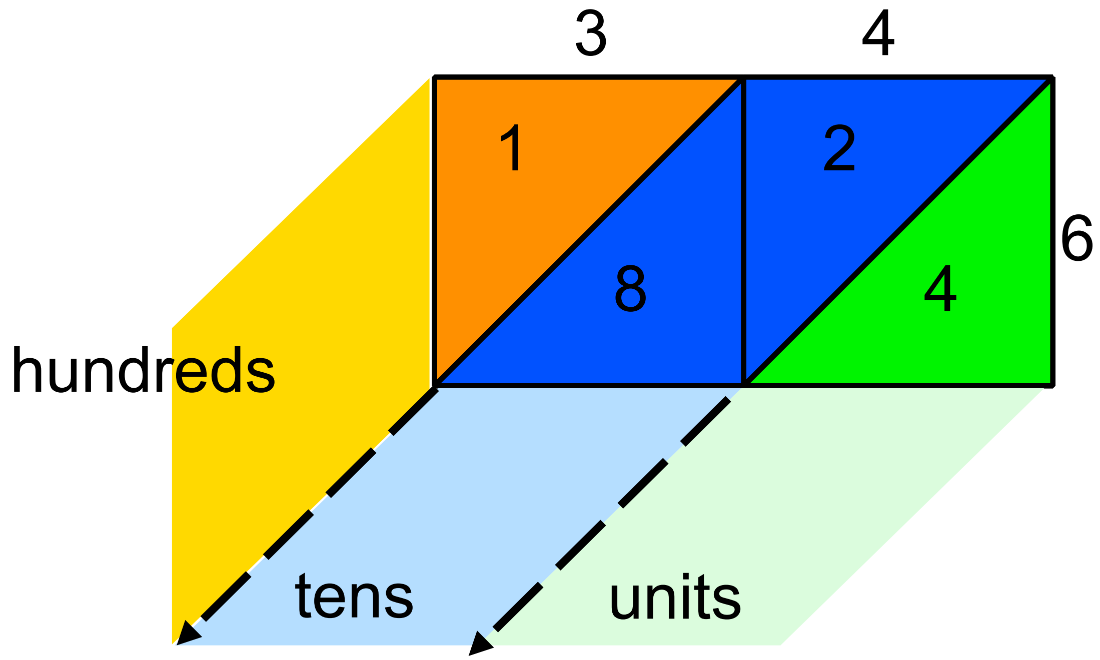
Lattice Multiplication



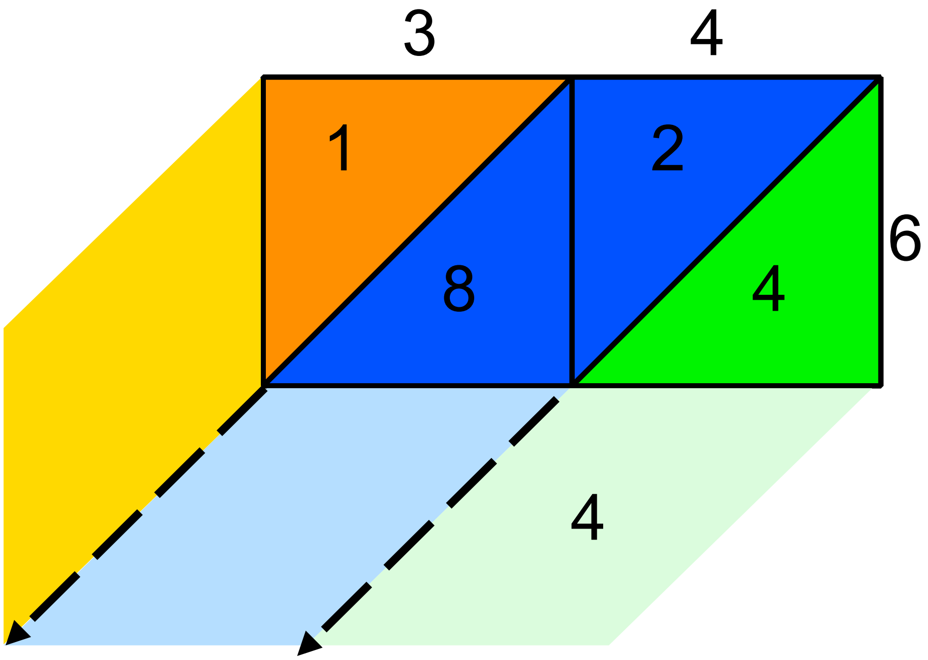
Lattice Multiplication



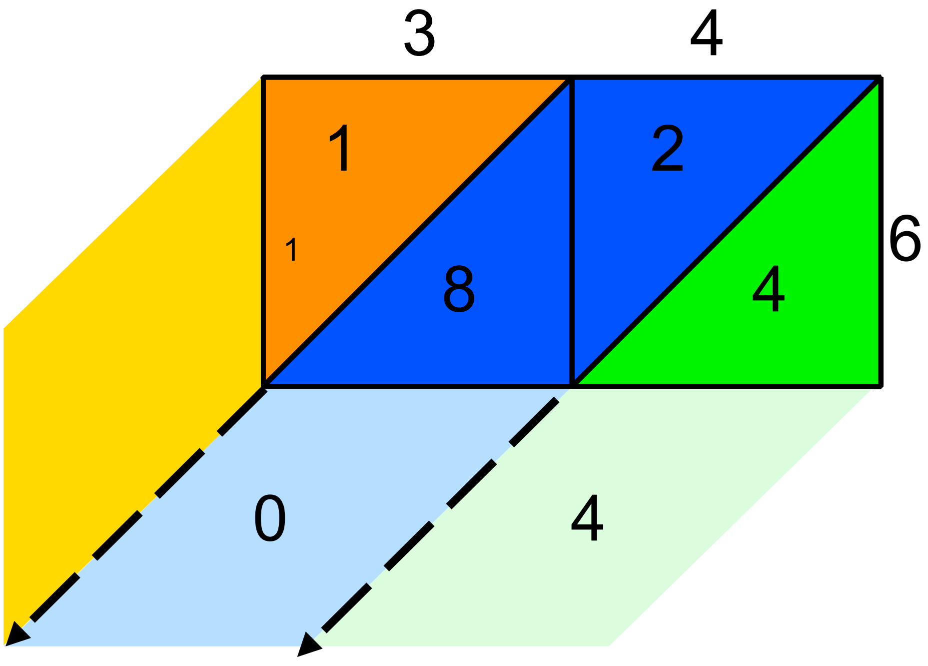
Lattice Multiplication



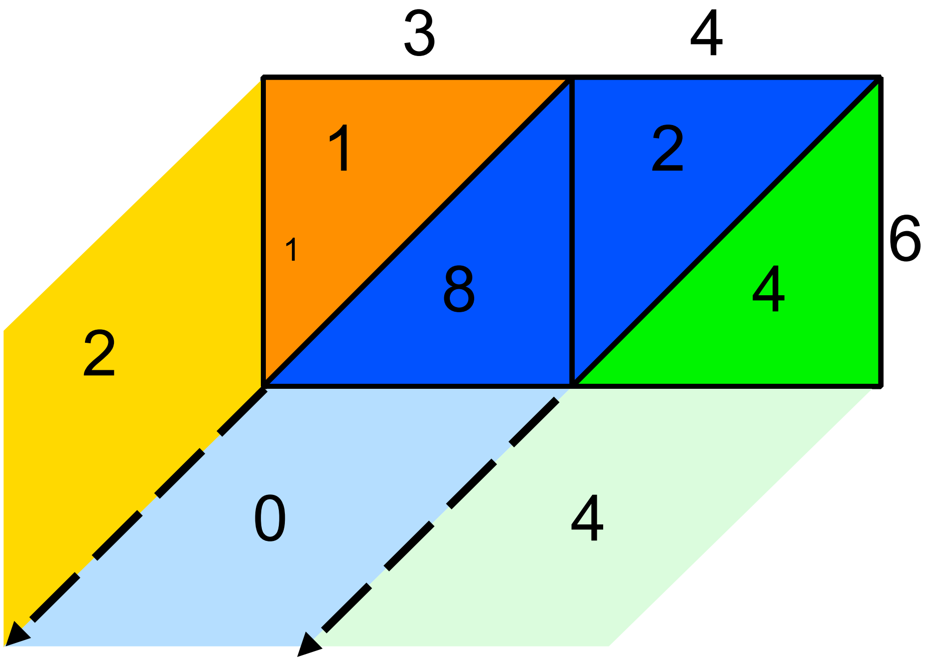
Lattice Multiplication



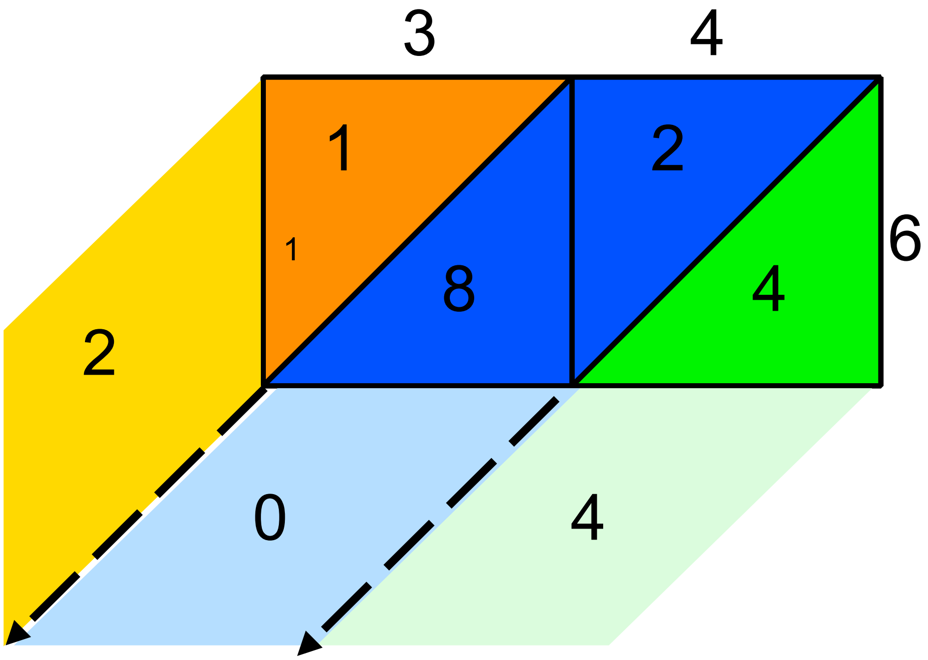
Lattice Multiplication



Lattice Multiplication

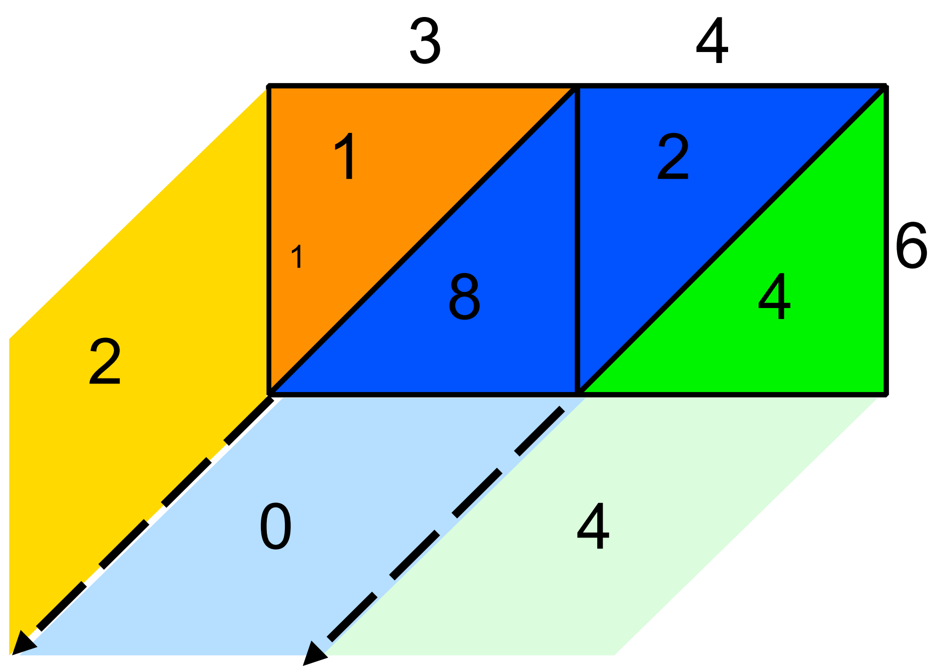


Lattice Multiplication



34 x 6 = 204

Lattice Multiplication

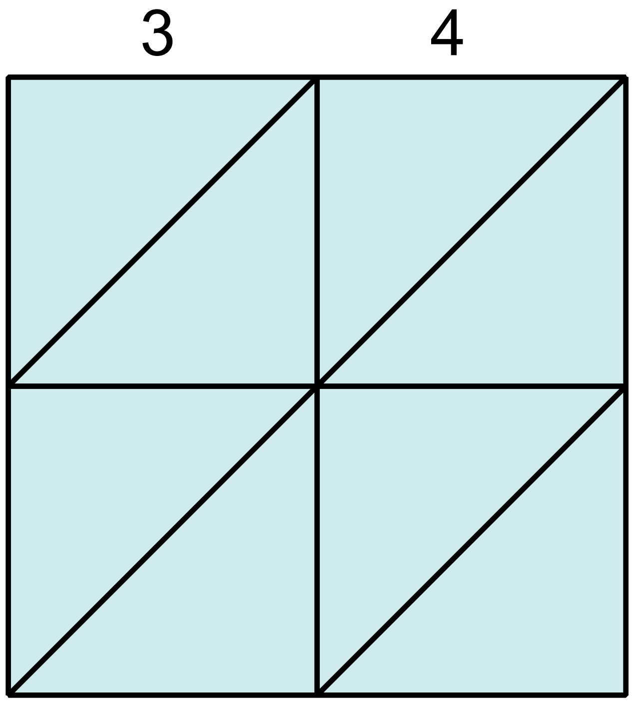


34 x 6 = 204

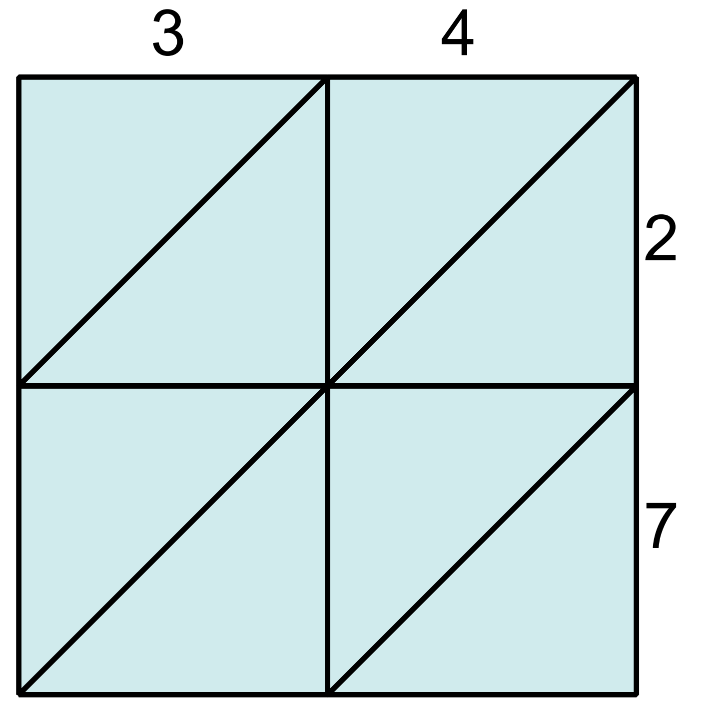
$$\begin{array}{r} {}^{2}3\ 4 \\ \times\quad 6 \\ \hline 2\ 0\ 4 \\ \hline \end{array}$$

$$\begin{aligned} 34 \times 6 &= (30 + 4) \times 6 \\ &= (30 \times 6) + (4 \times 6) \\ &= 180 + 20 + 4 \\ &= 204 \end{aligned}$$

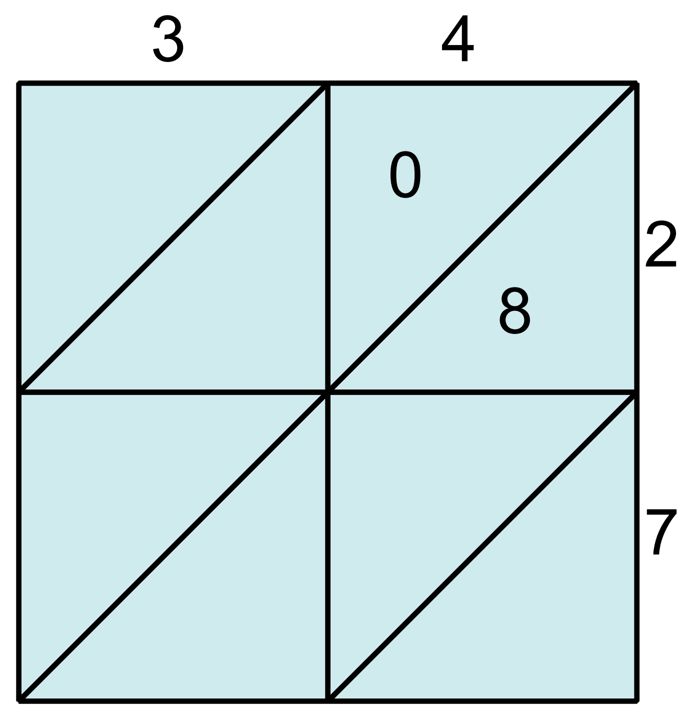
Lattice Multiplication



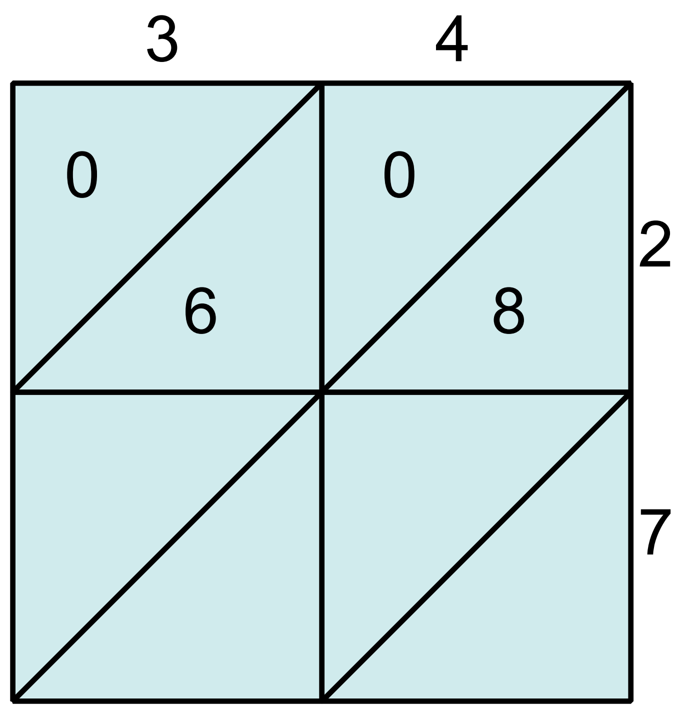
Lattice Multiplication



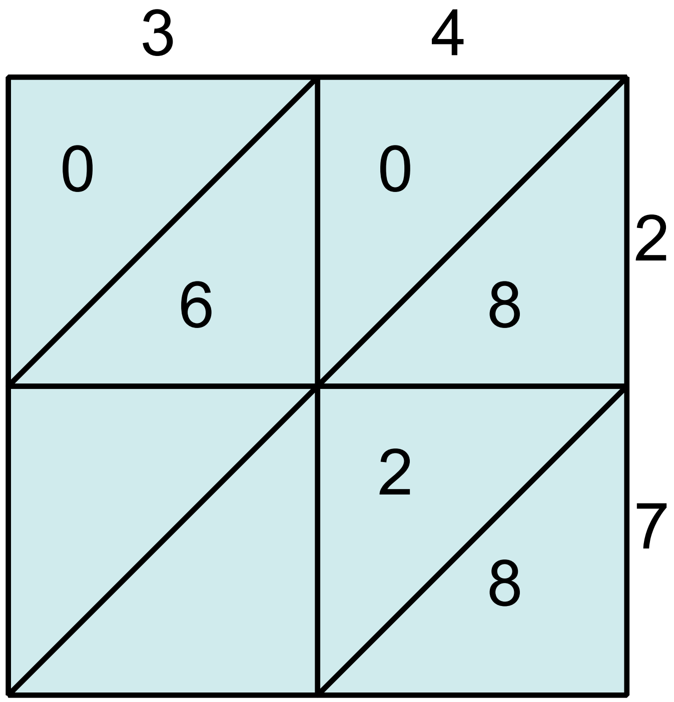
Lattice Multiplication



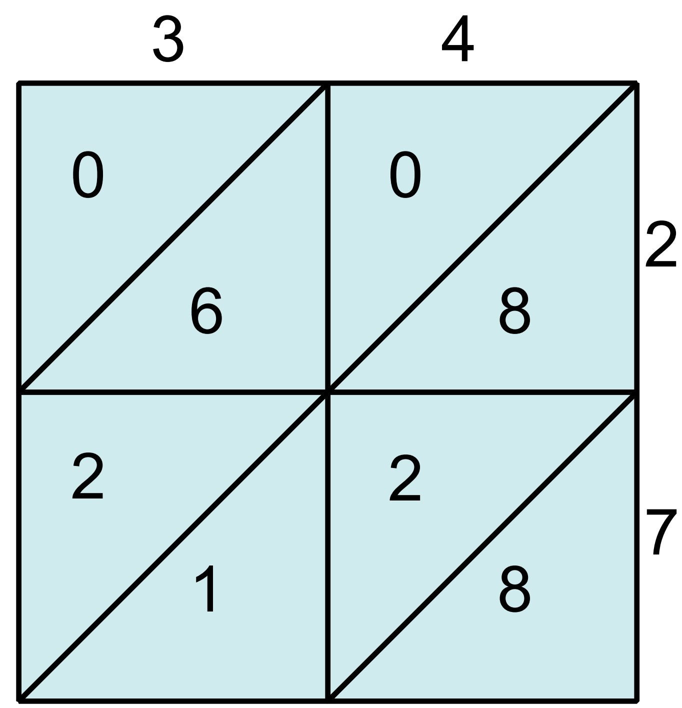
Lattice Multiplication



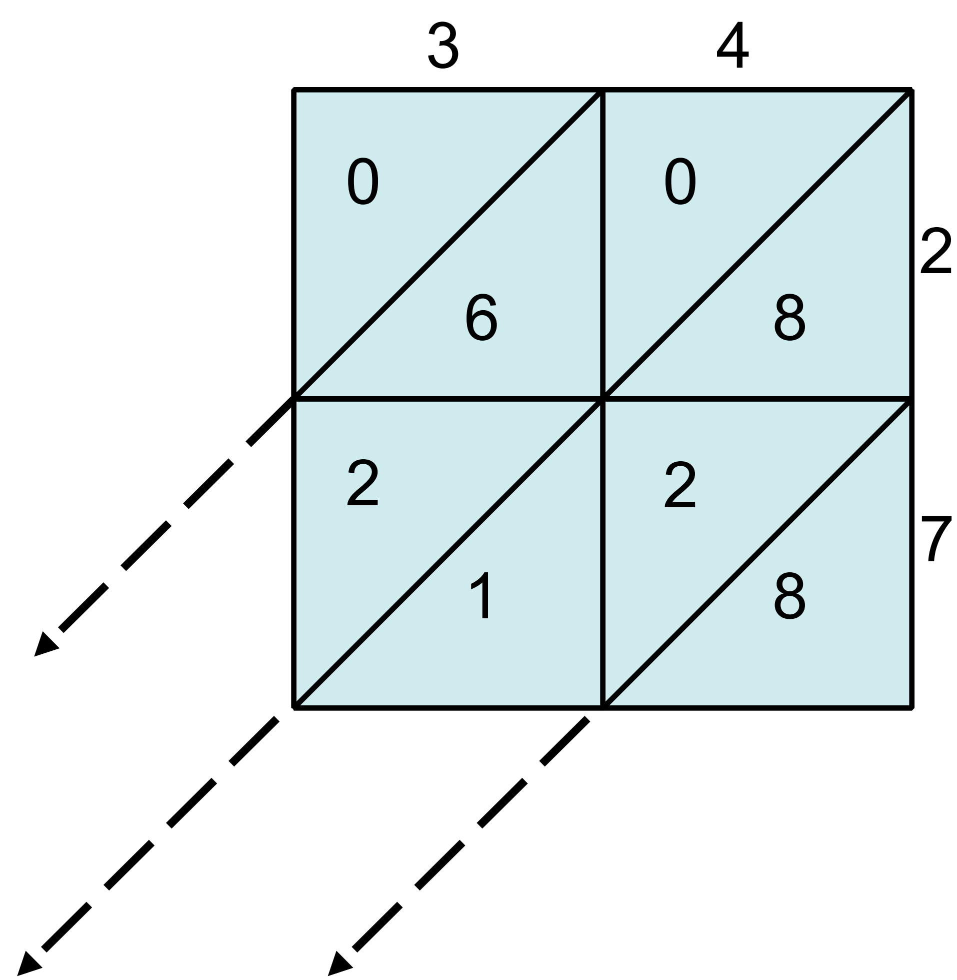
Lattice Multiplication



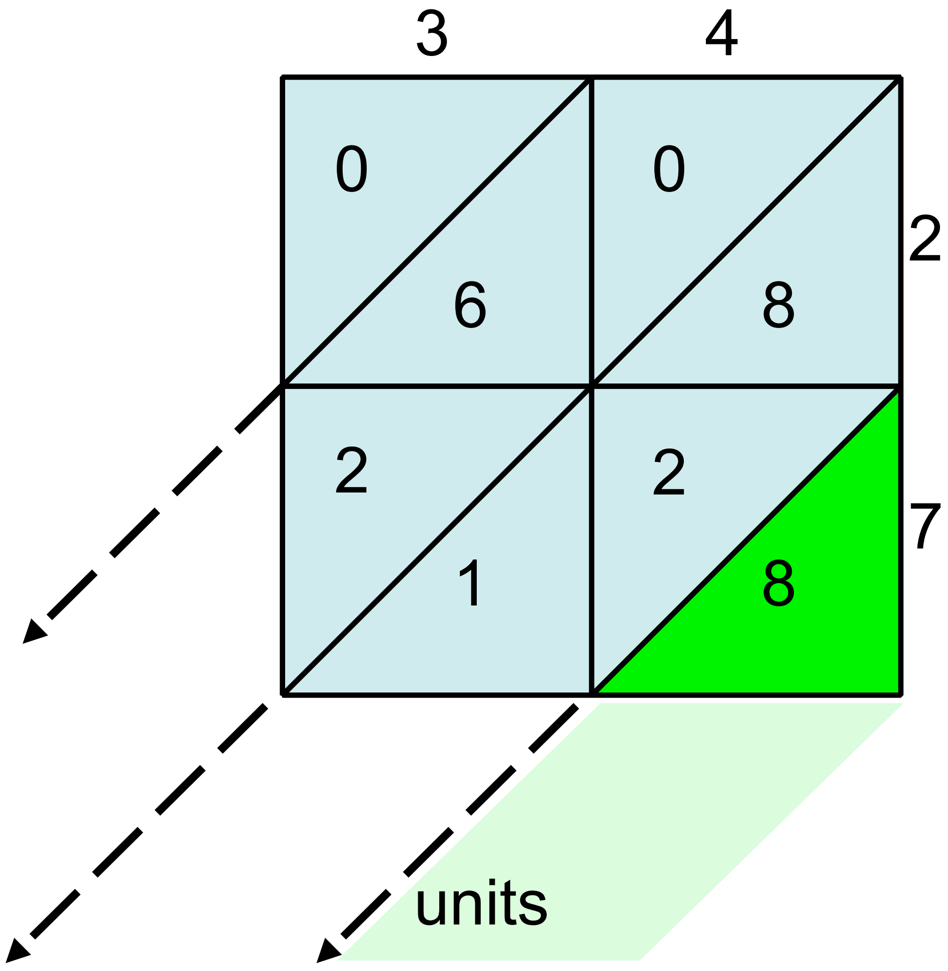
Lattice Multiplication



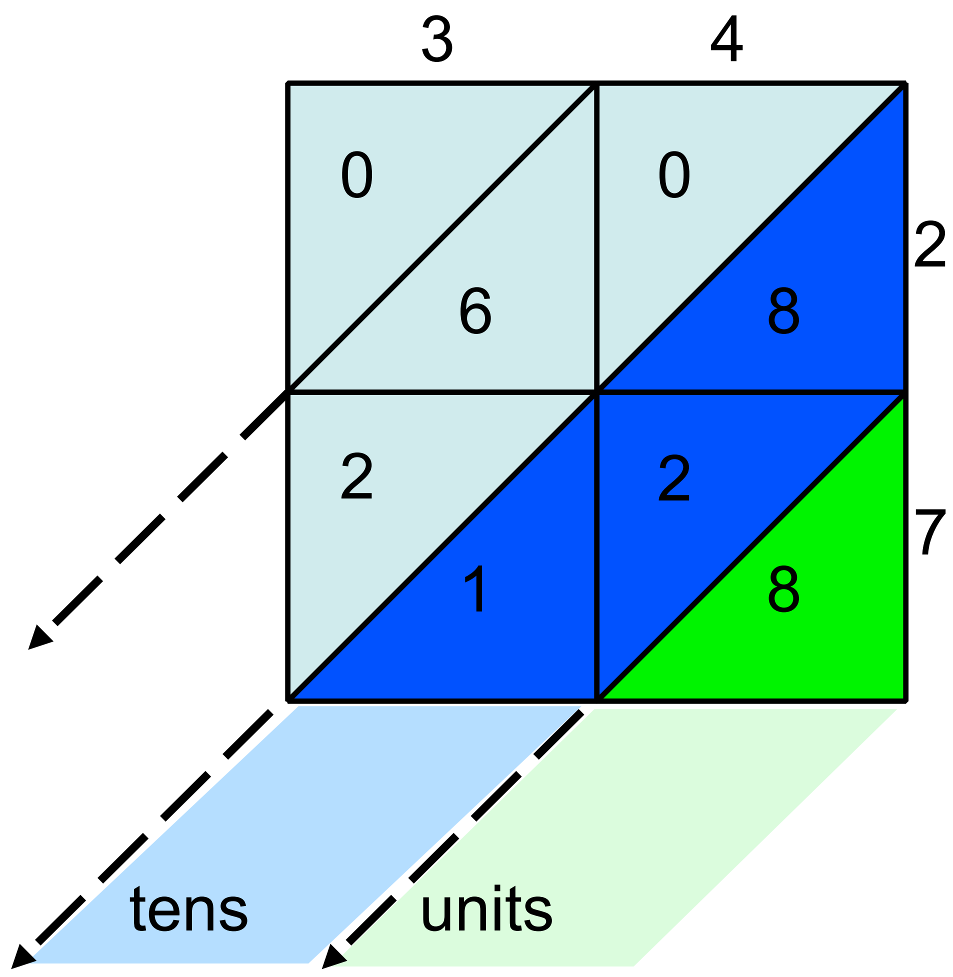
Lattice Multiplication



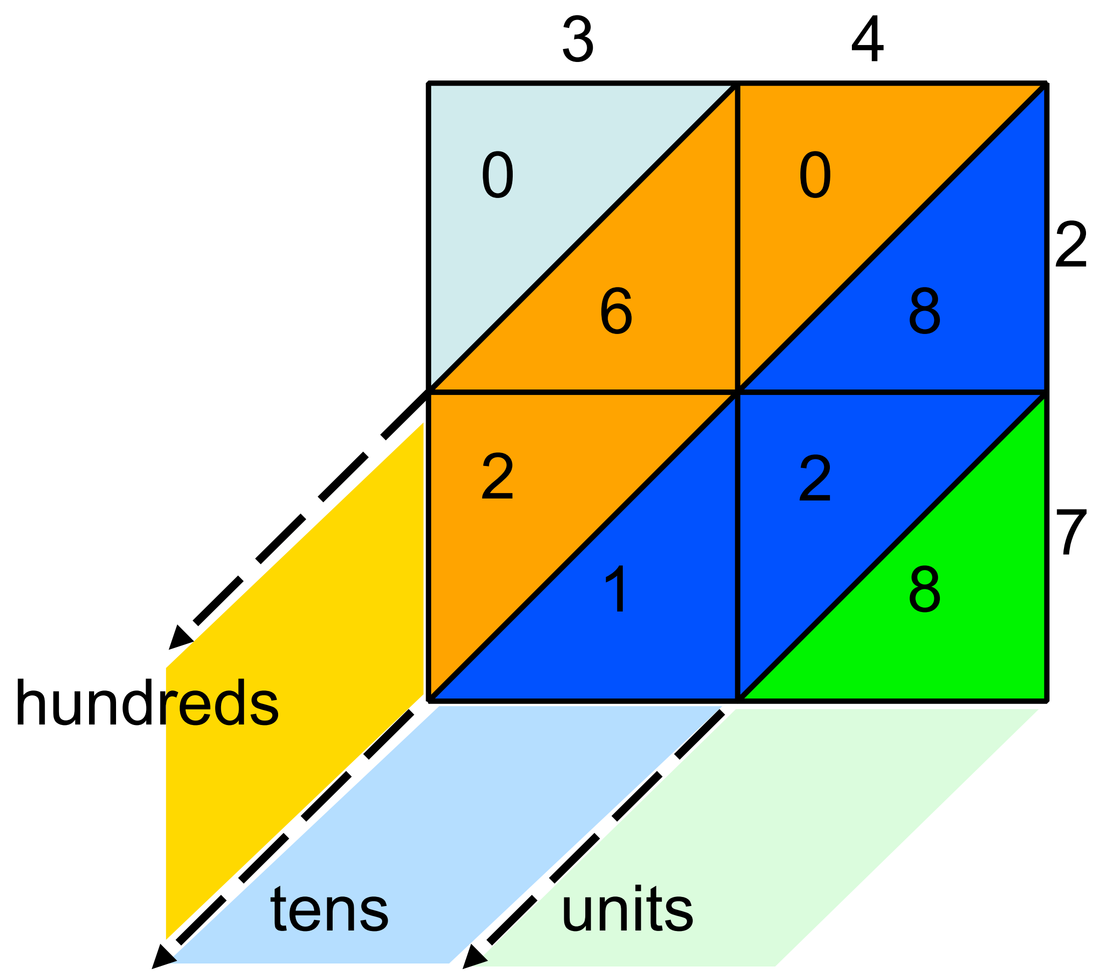
Lattice Multiplication



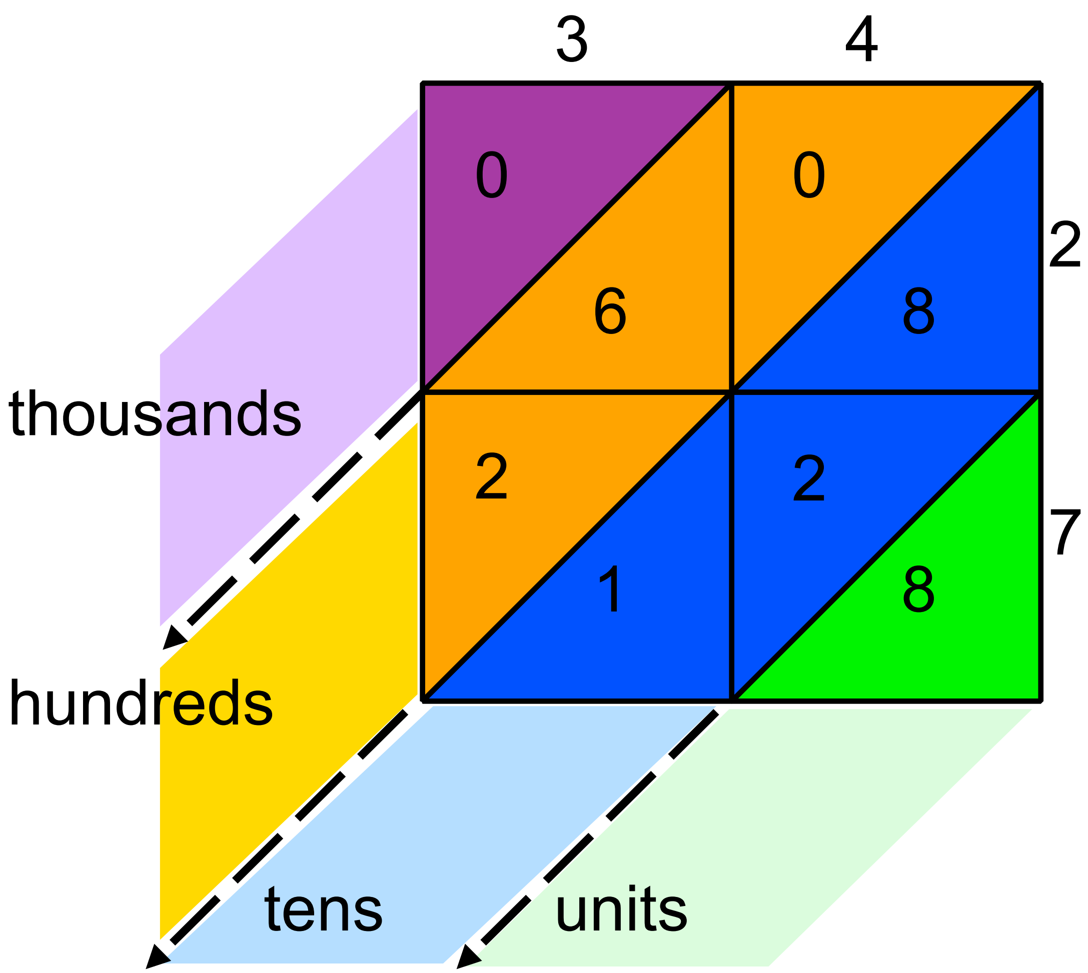
Lattice Multiplication



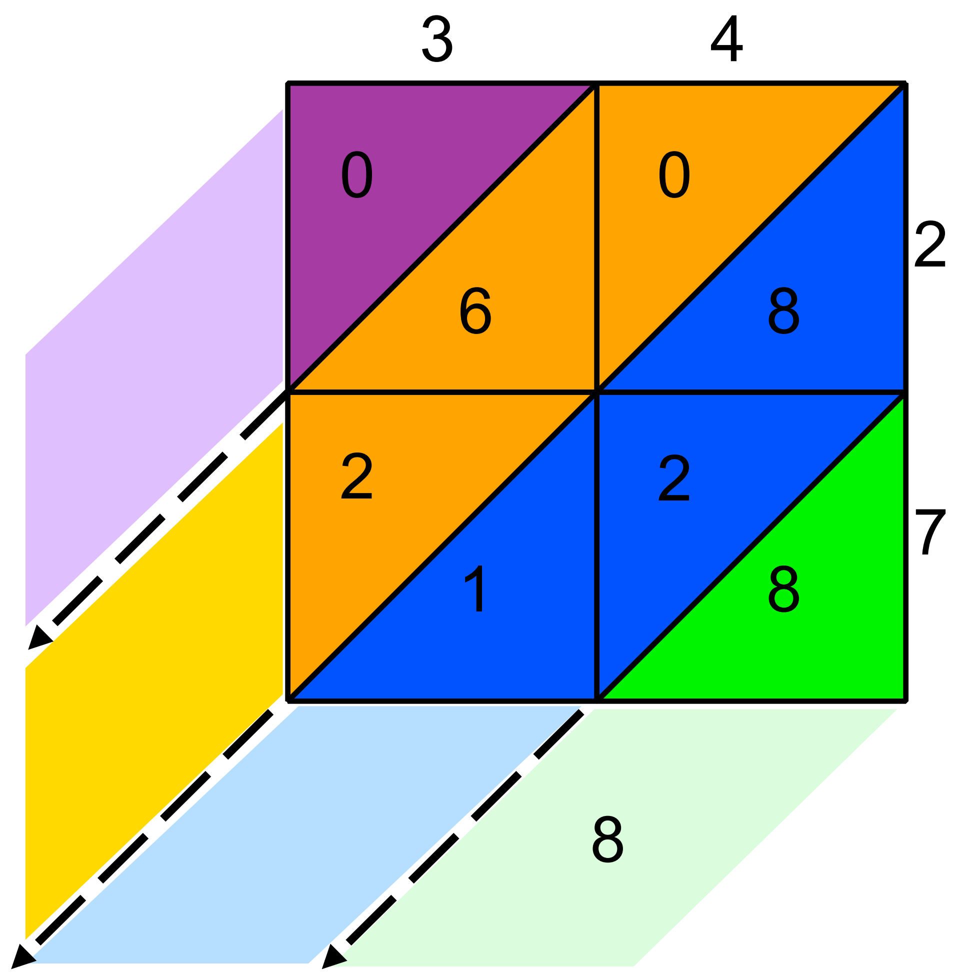
Lattice Multiplication



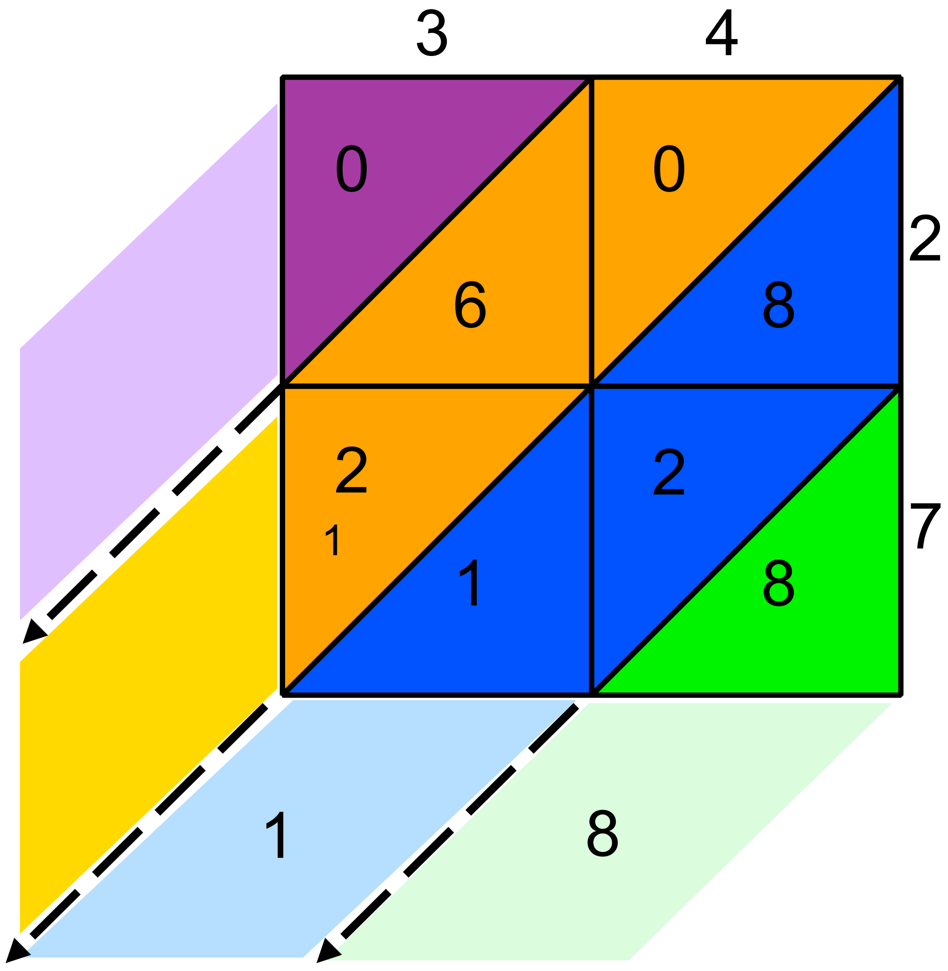
Lattice Multiplication



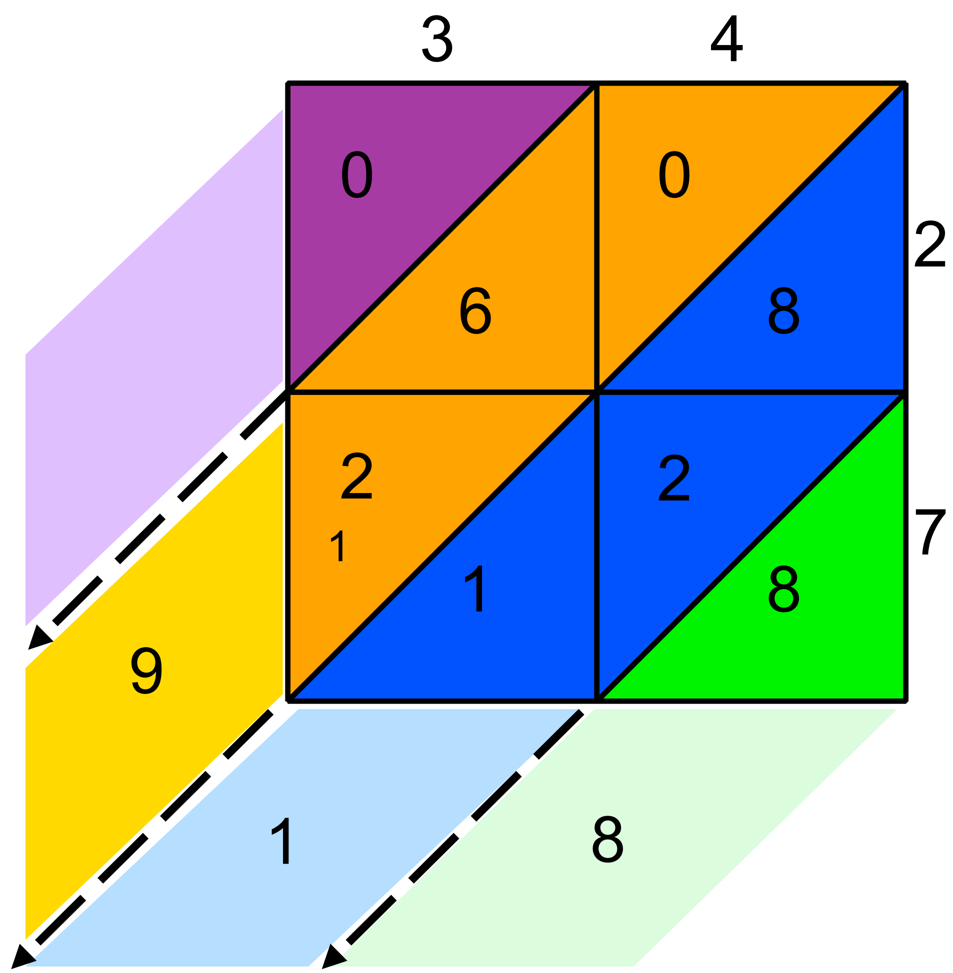
Lattice Multiplication



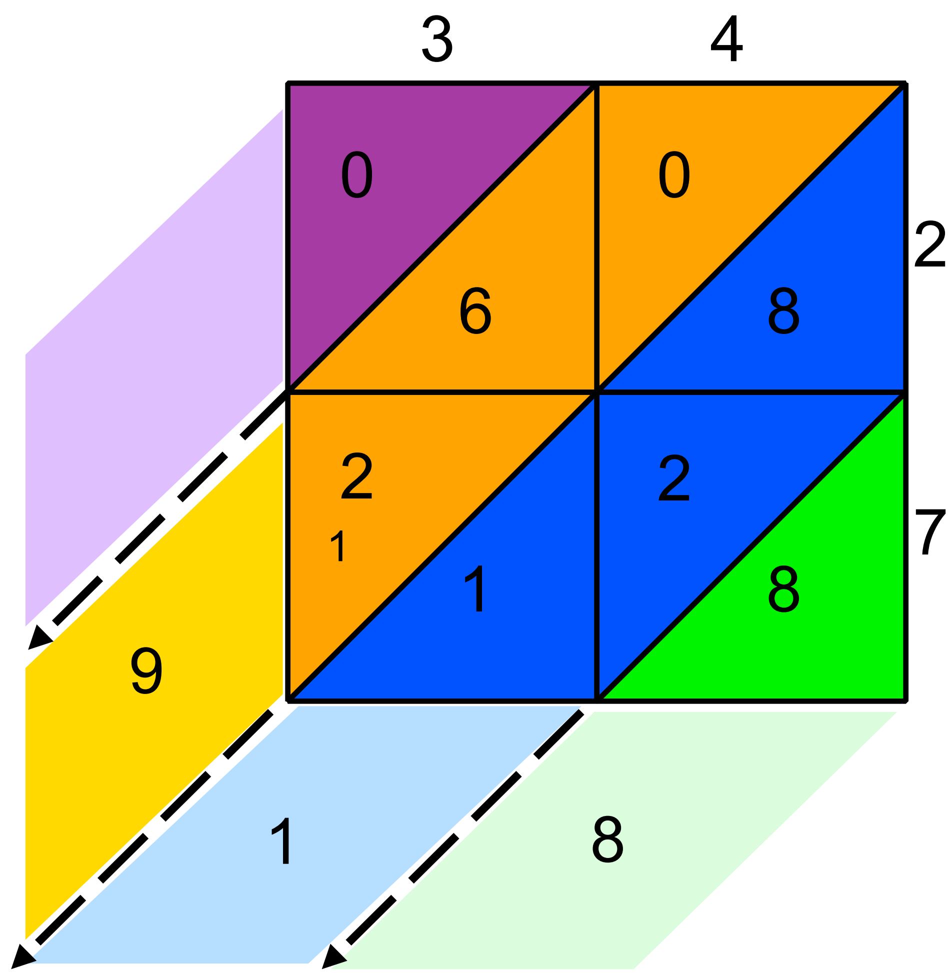
Lattice Multiplication



Lattice Multiplication



Lattice Multiplication

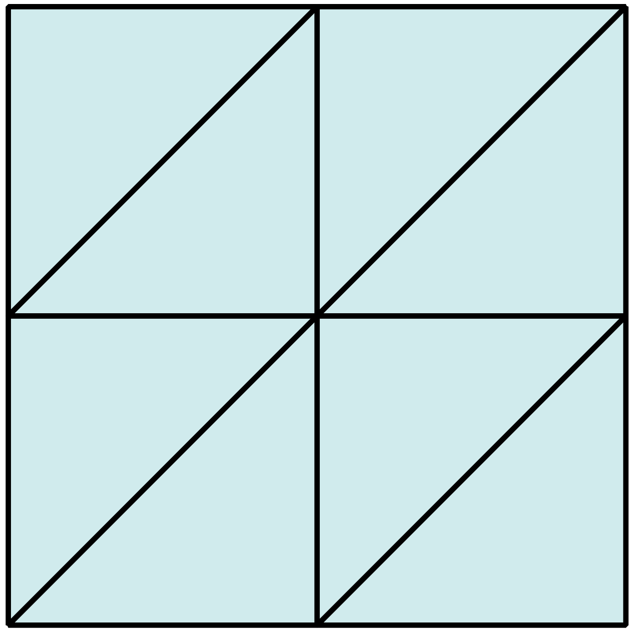


34 x 27 = 918

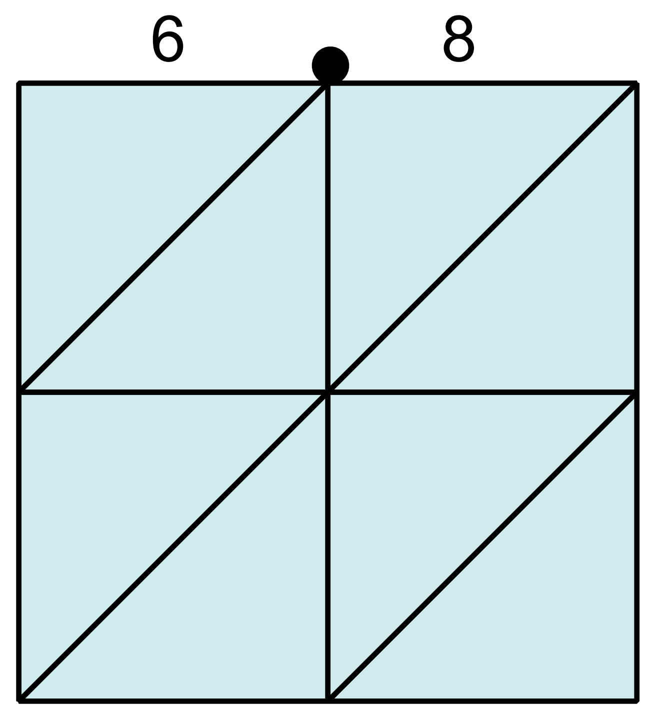
Lattice Multiplication

6.8×9.3

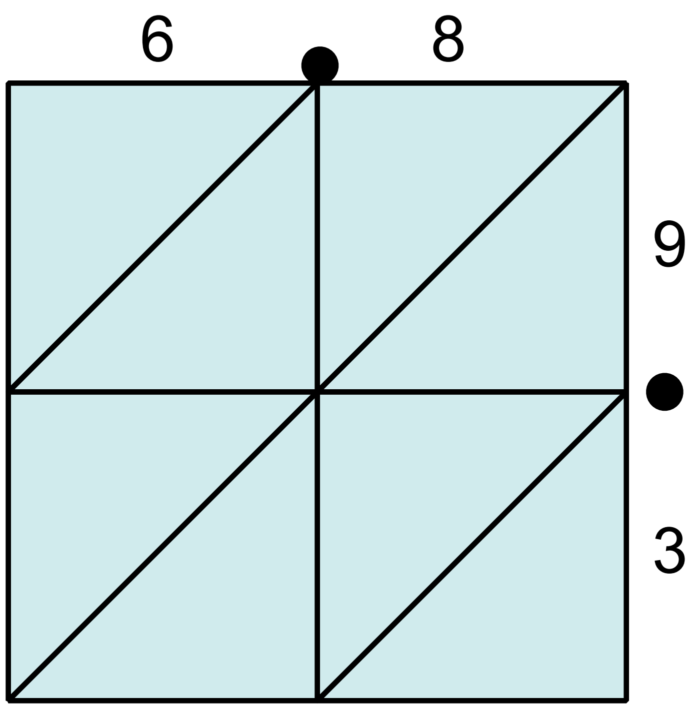
Lattice Multiplication



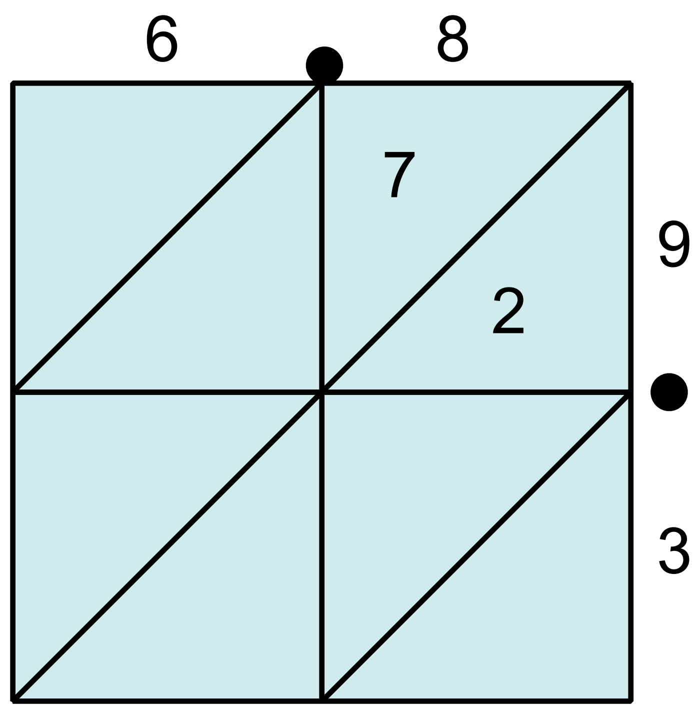
Lattice Multiplication



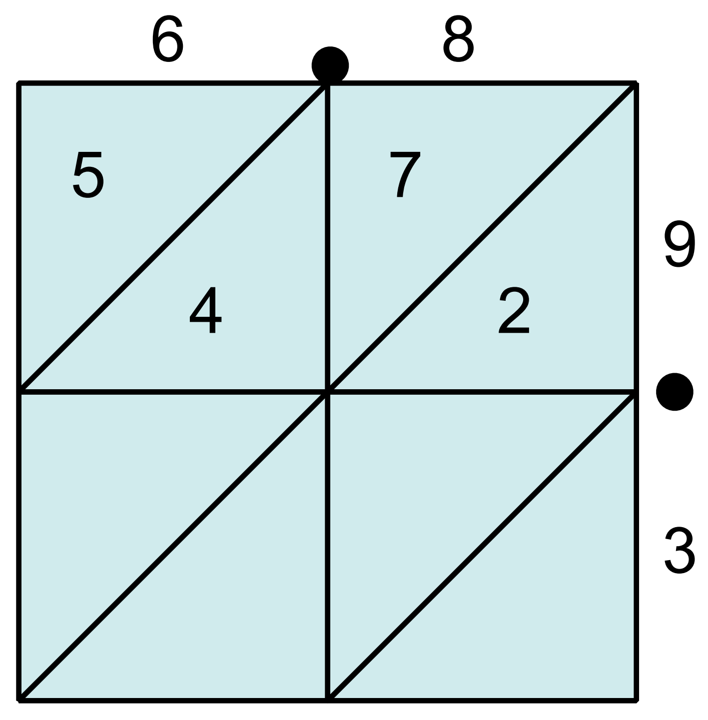
Lattice Multiplication



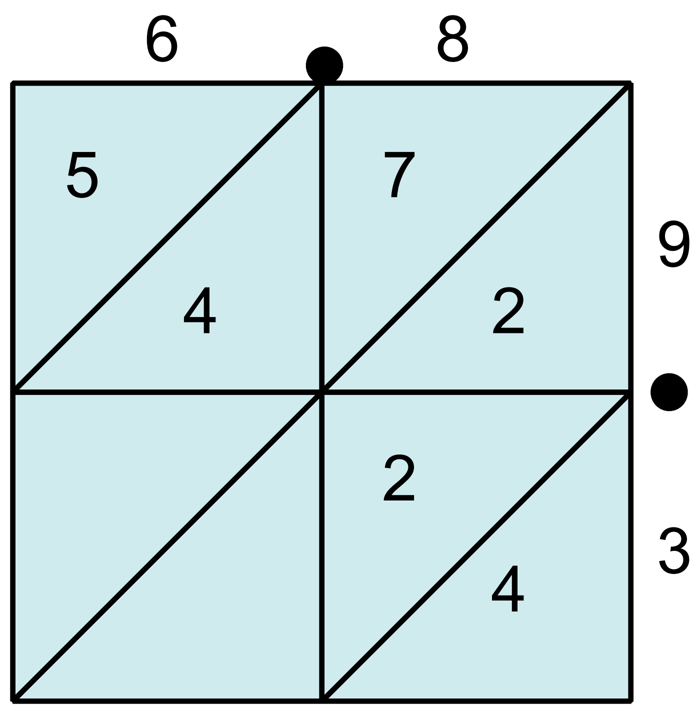
Lattice Multiplication



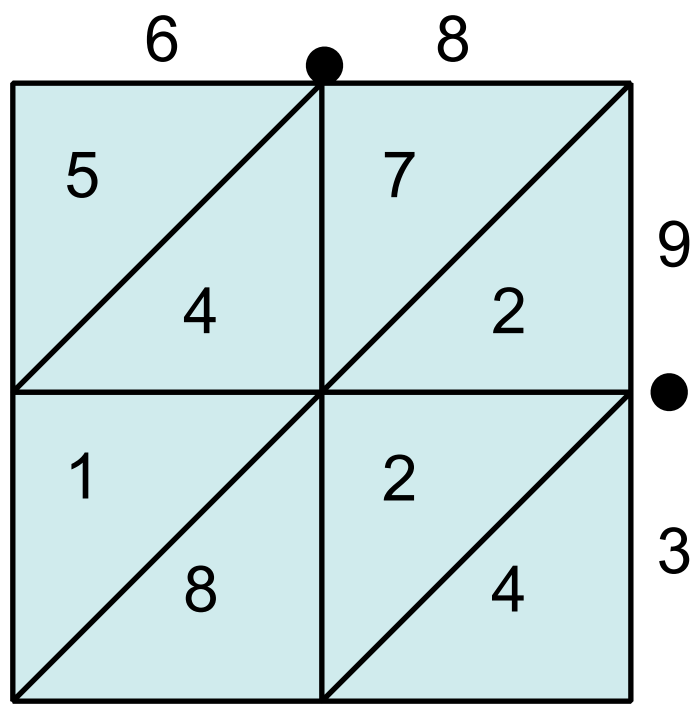
Lattice Multiplication



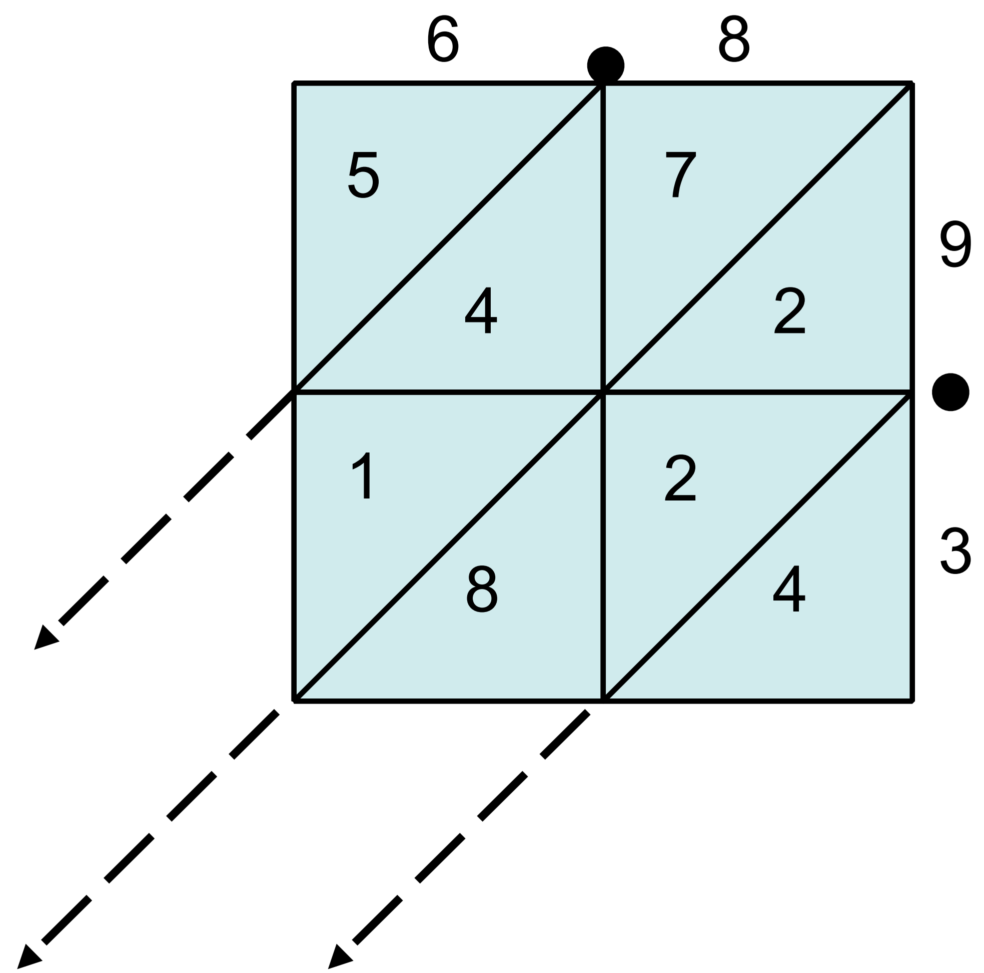
Lattice Multiplication



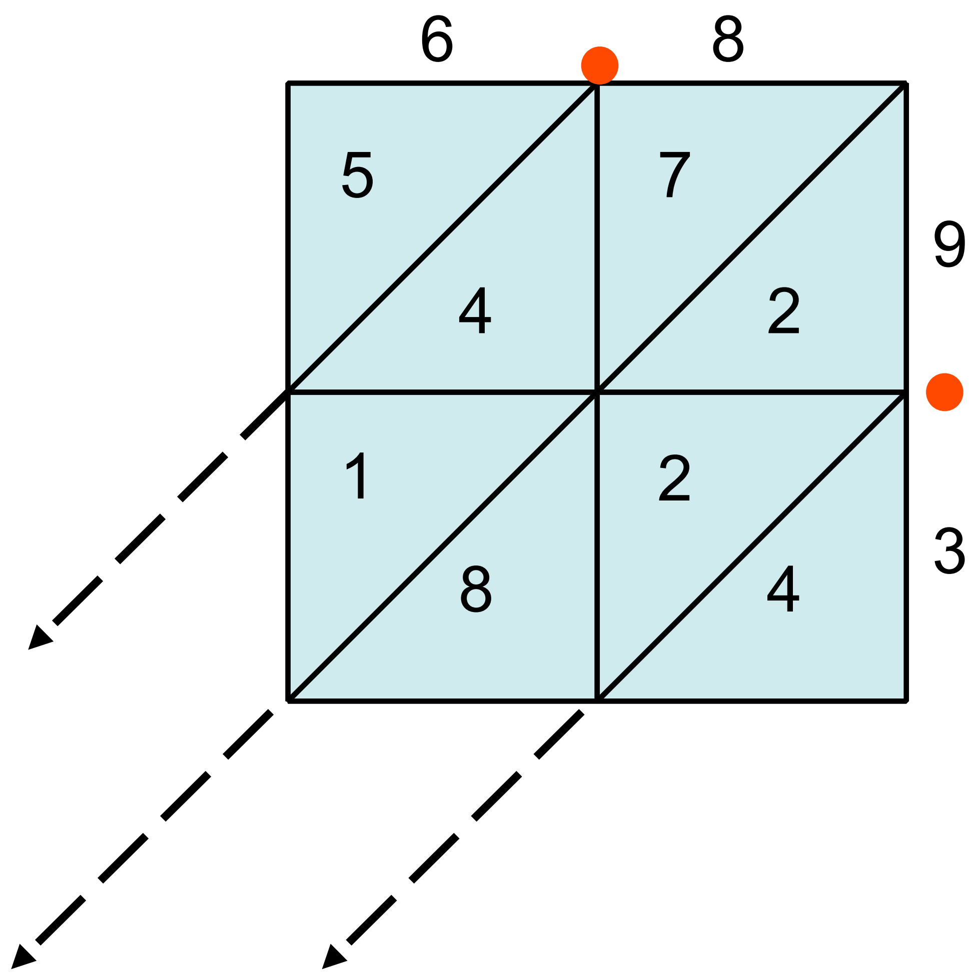
Lattice Multiplication



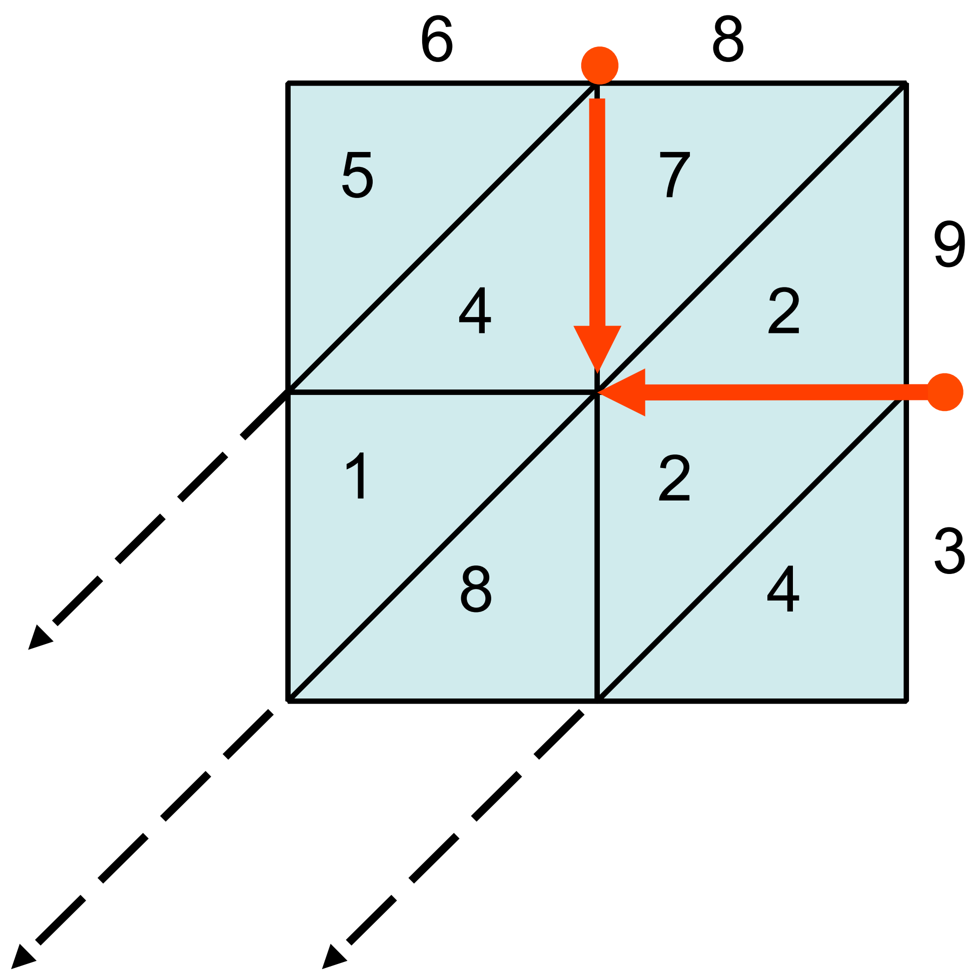
Lattice Multiplication



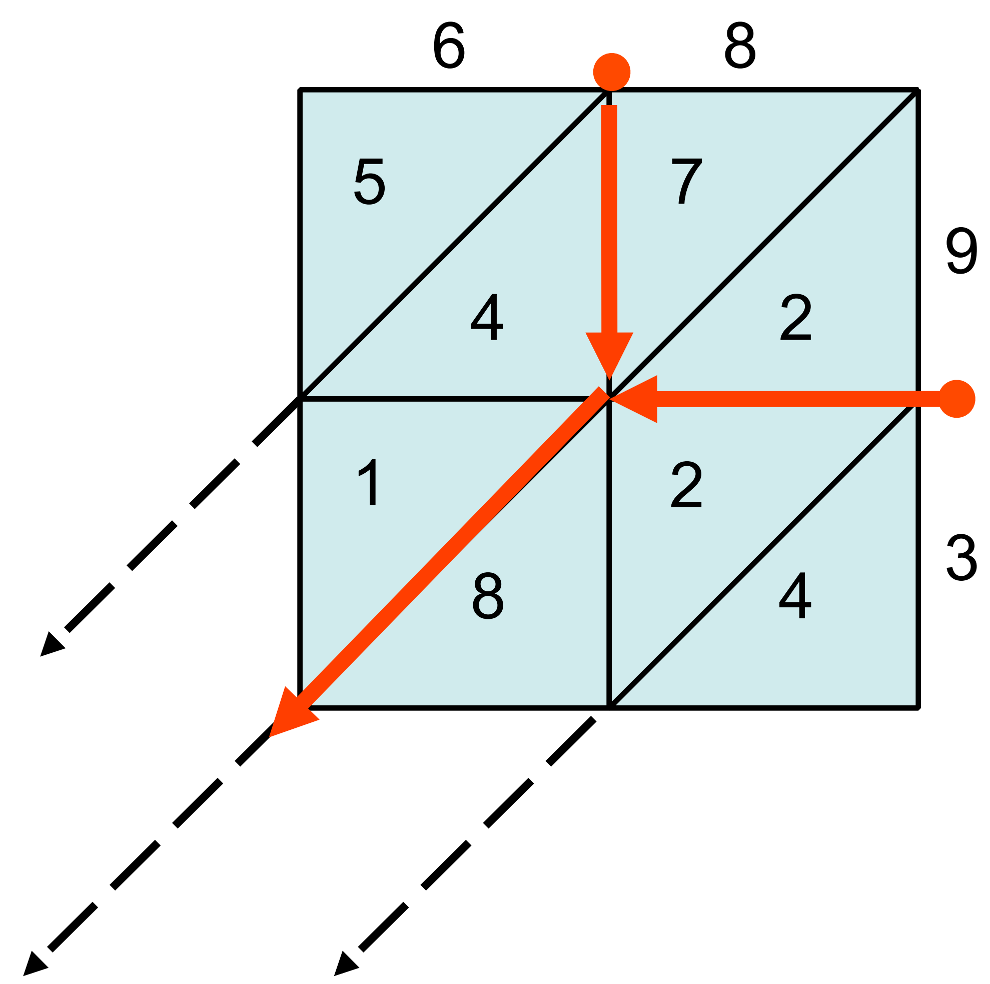
Lattice Multiplication



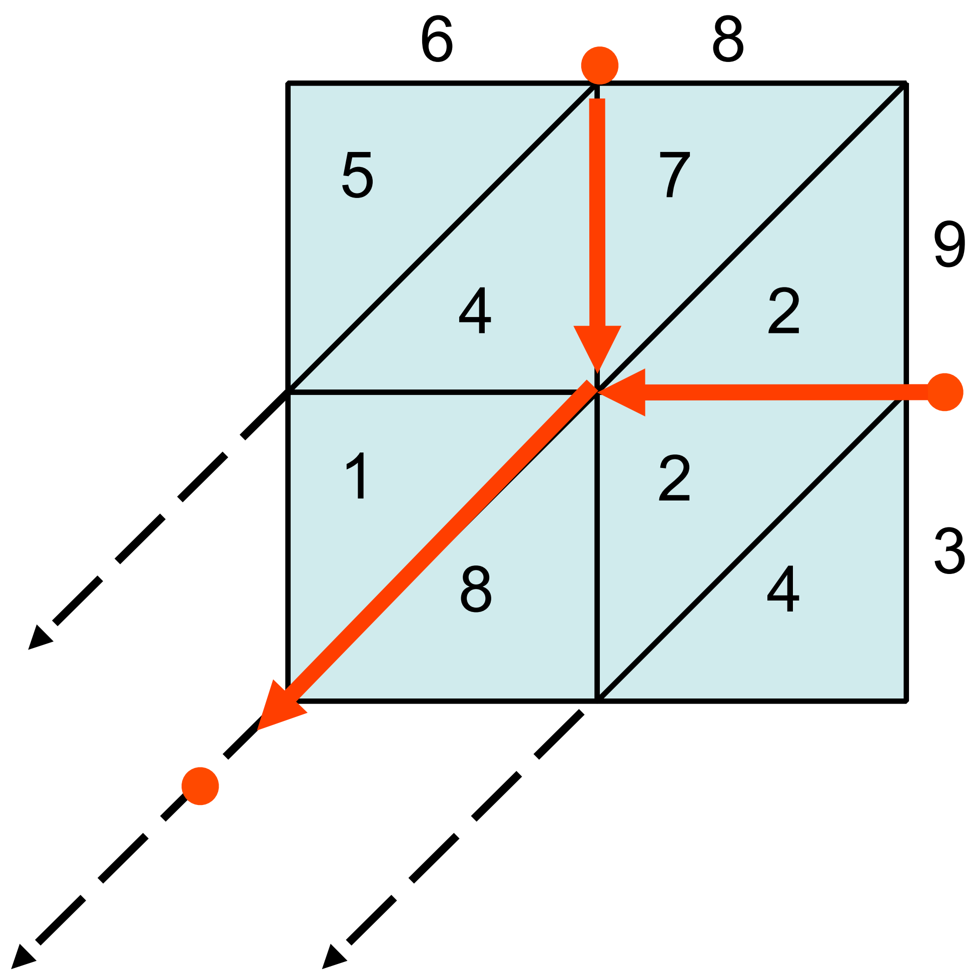
Lattice Multiplication



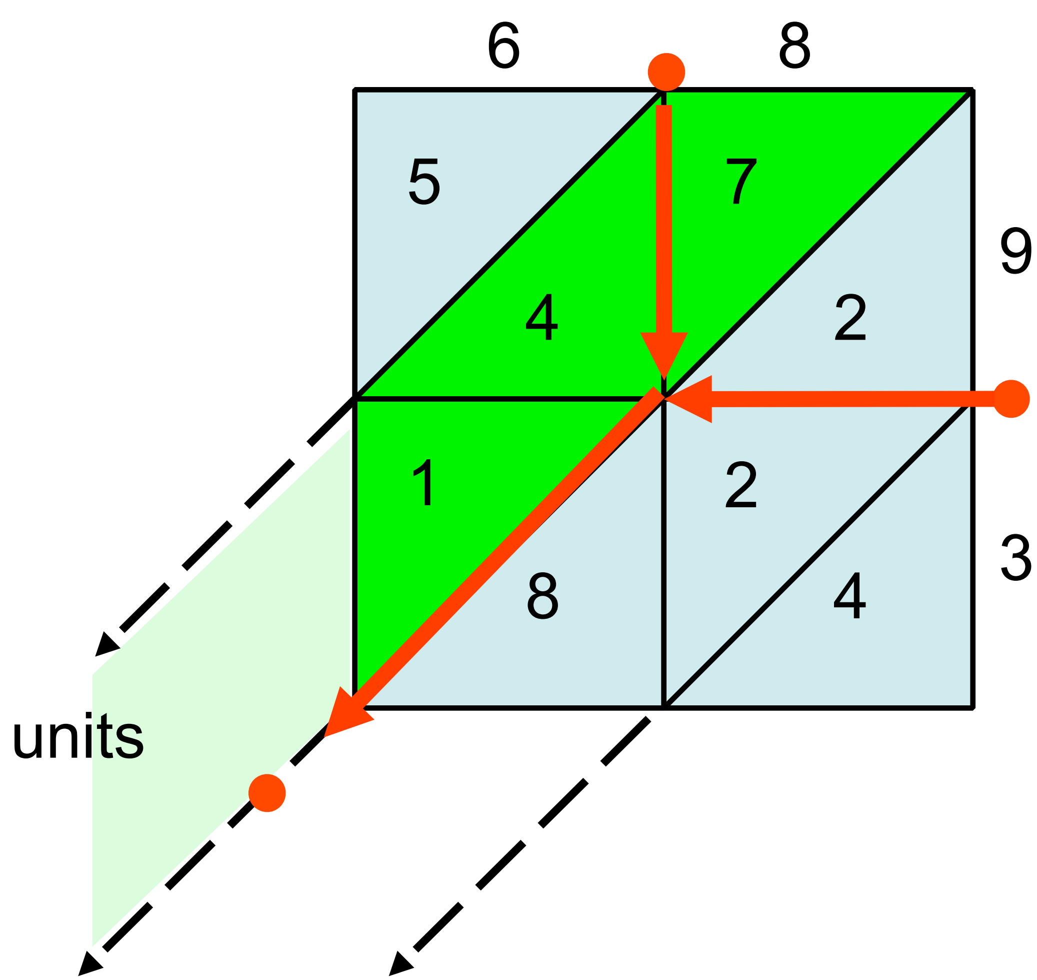
Lattice Multiplication



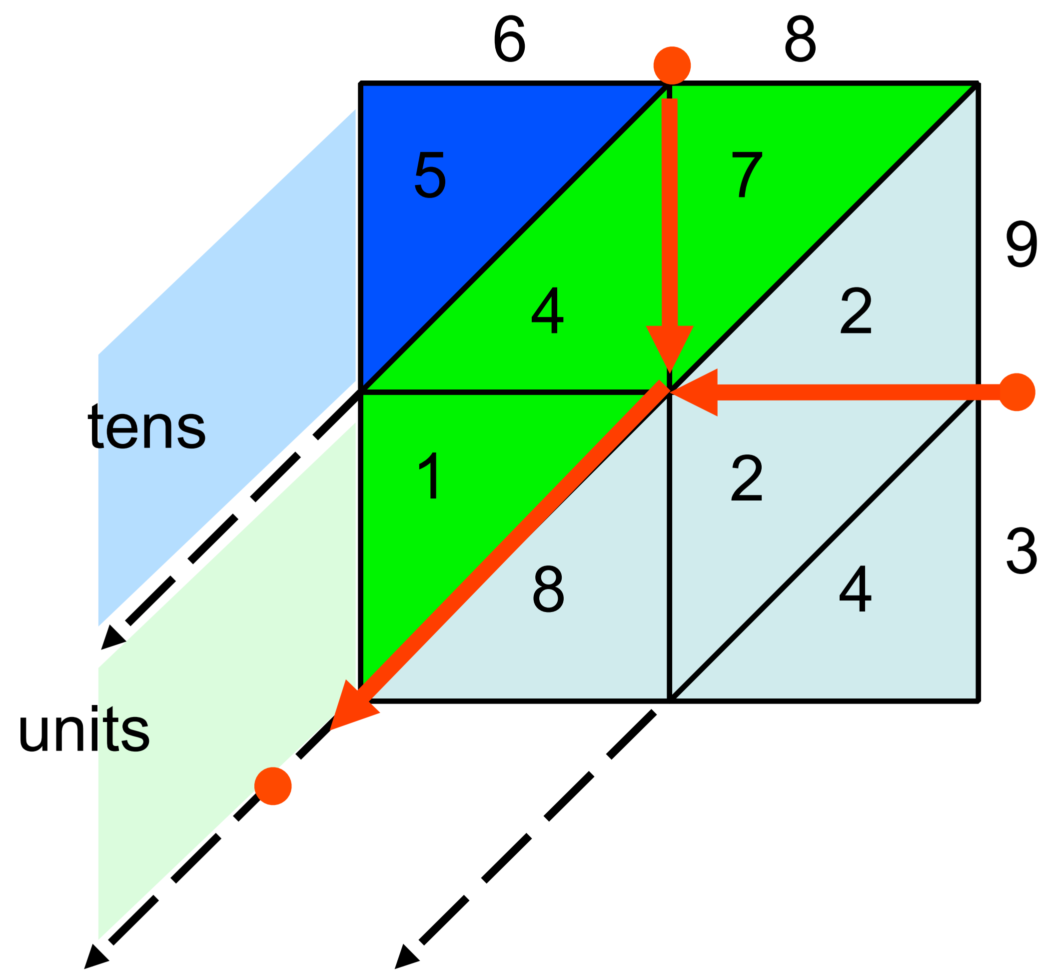
Lattice Multiplication



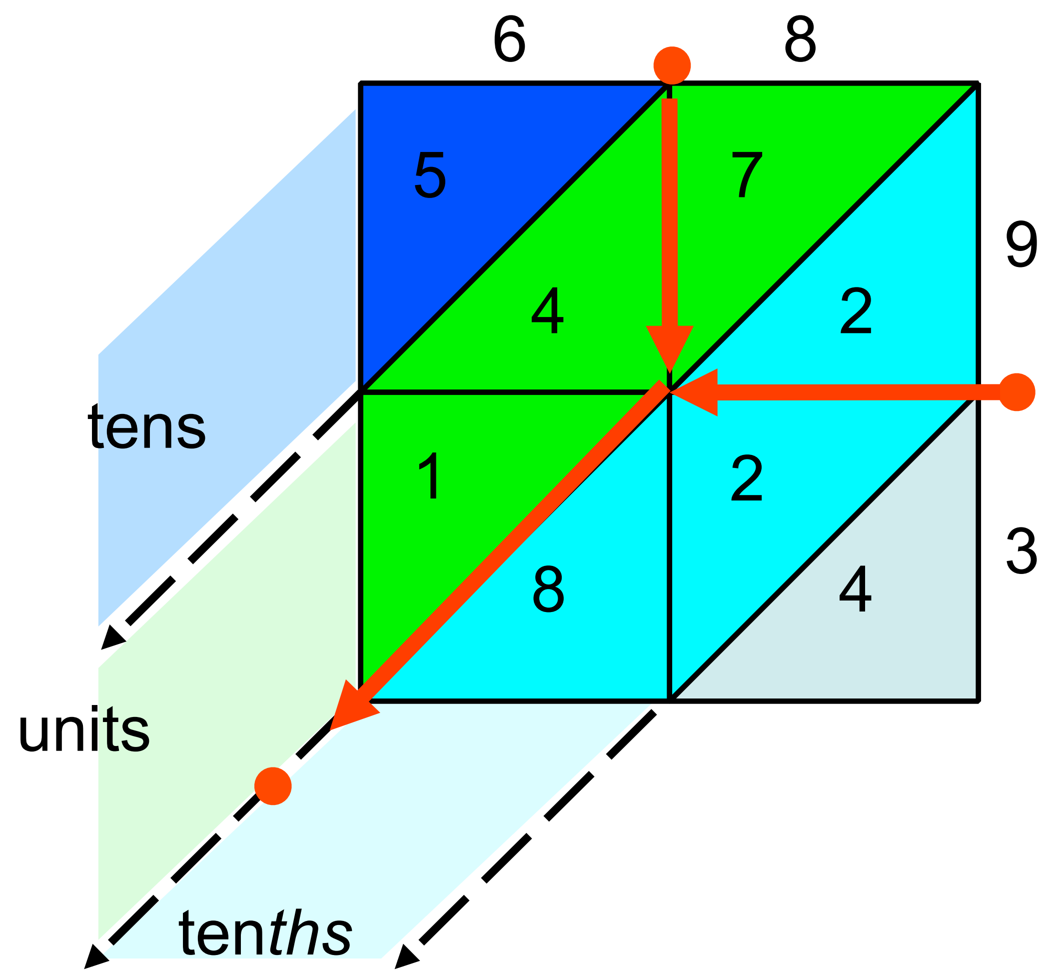
Lattice Multiplication



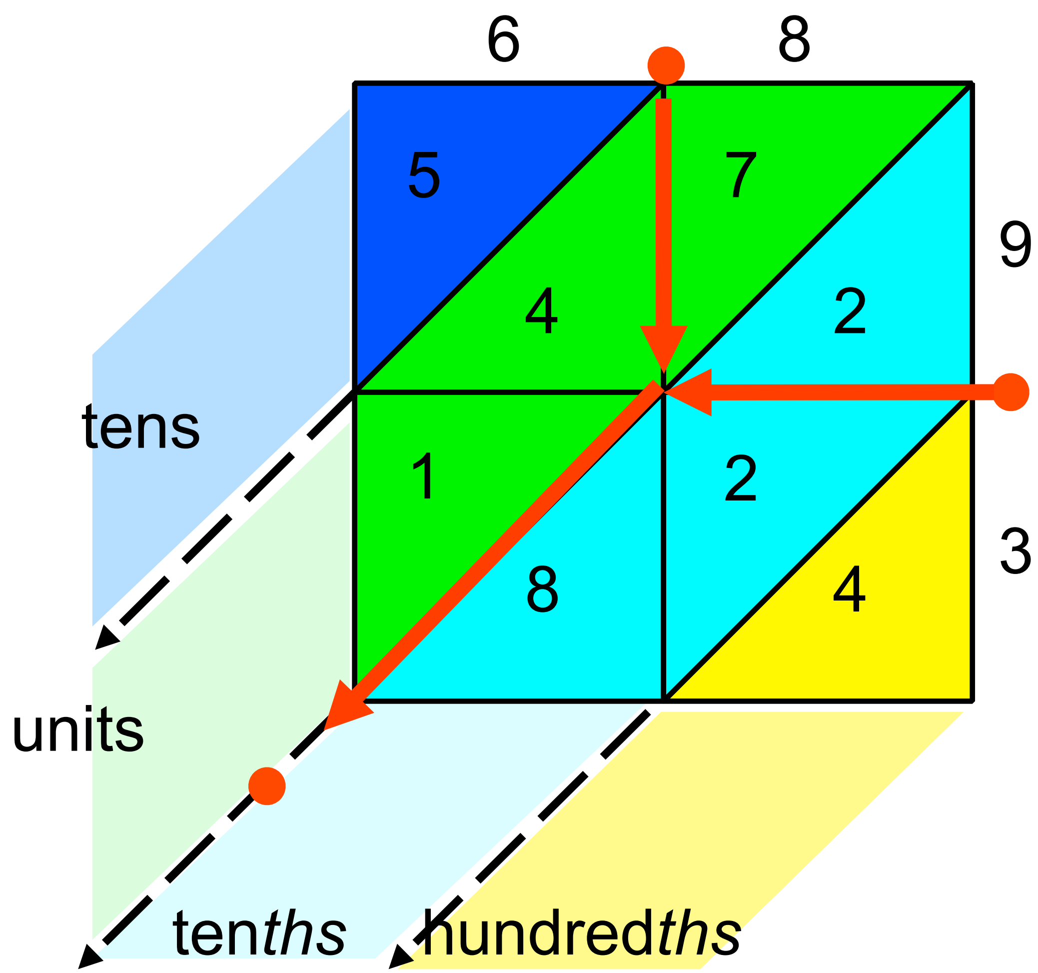
Lattice Multiplication



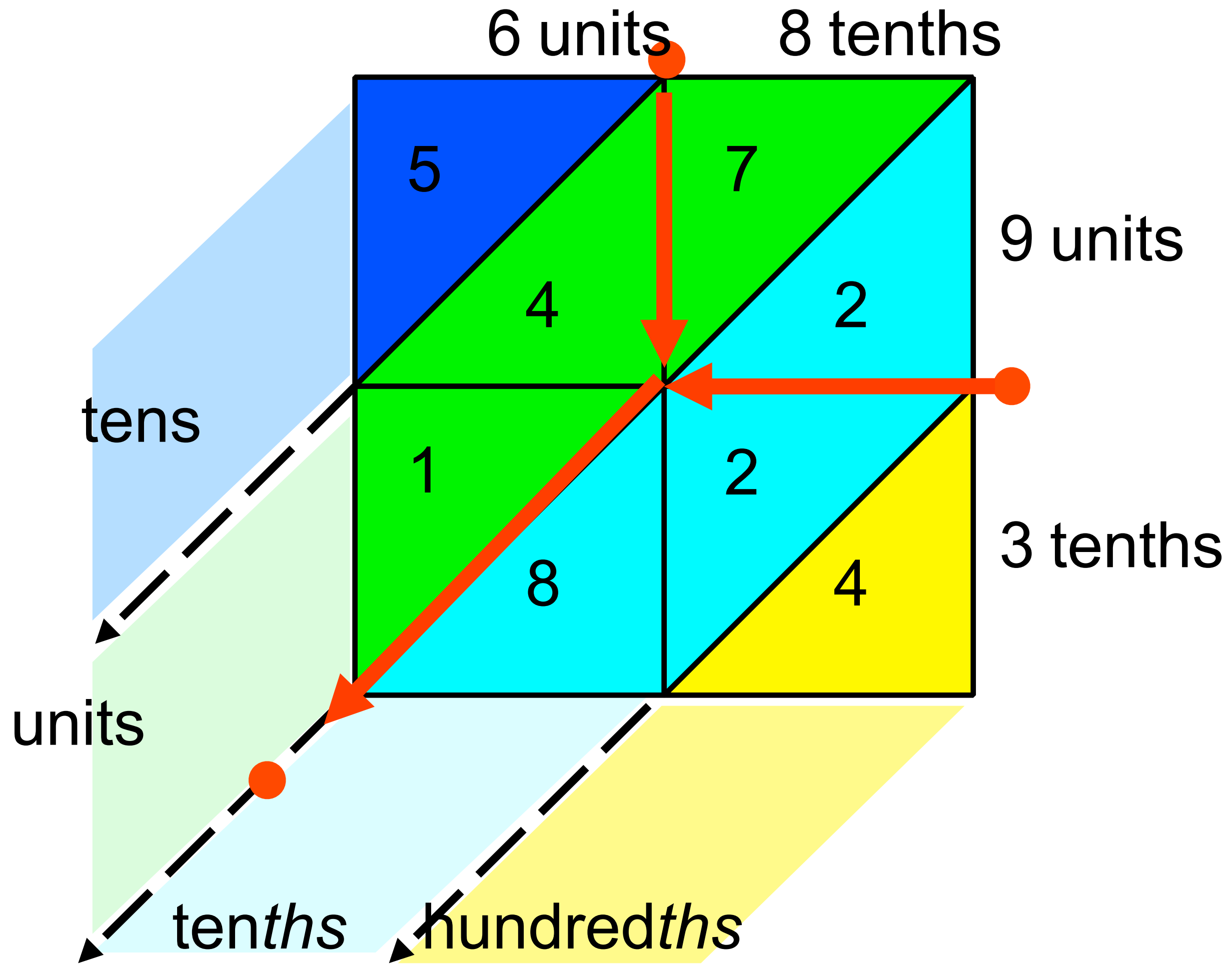
Lattice Multiplication



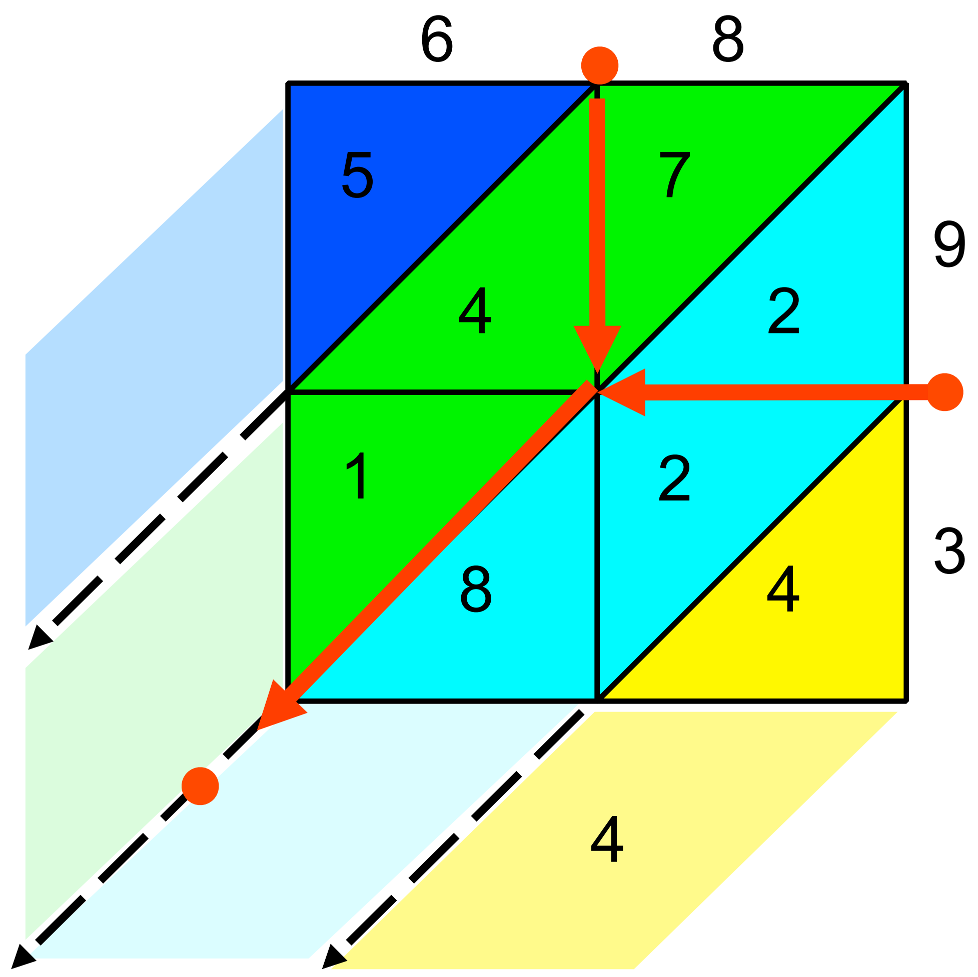
Lattice Multiplication



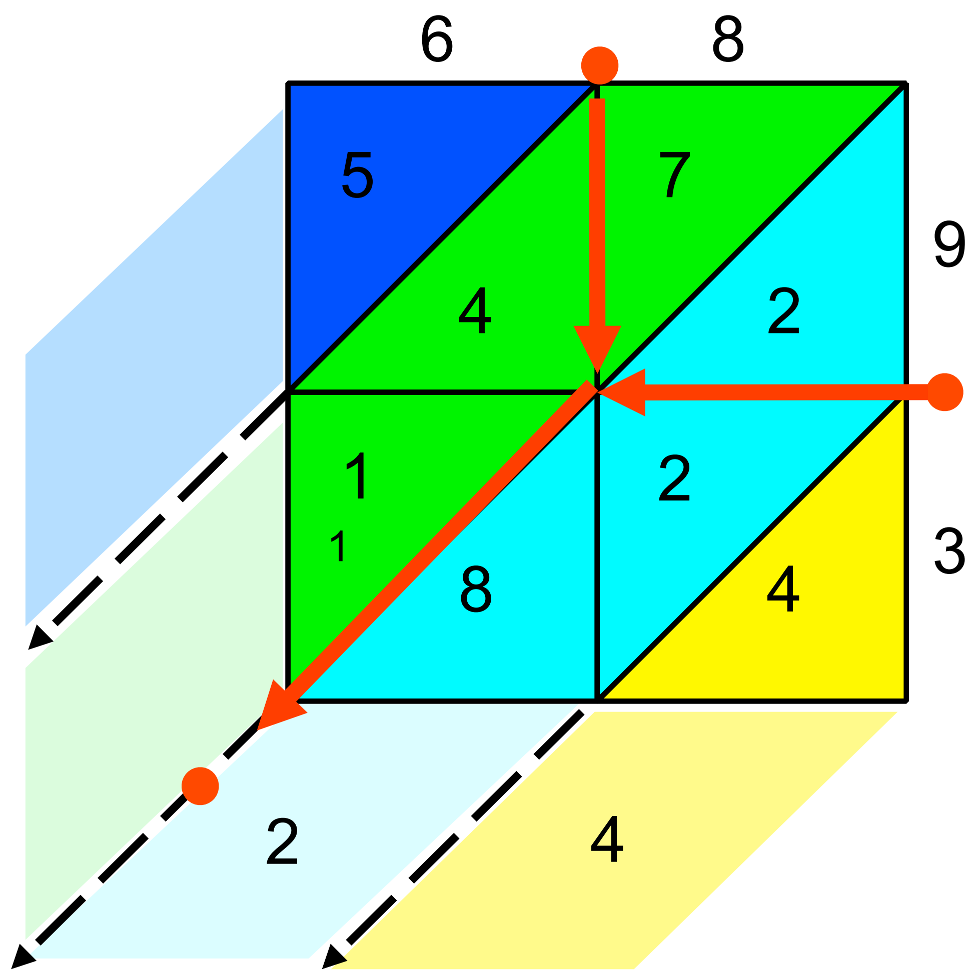
Lattice Multiplication



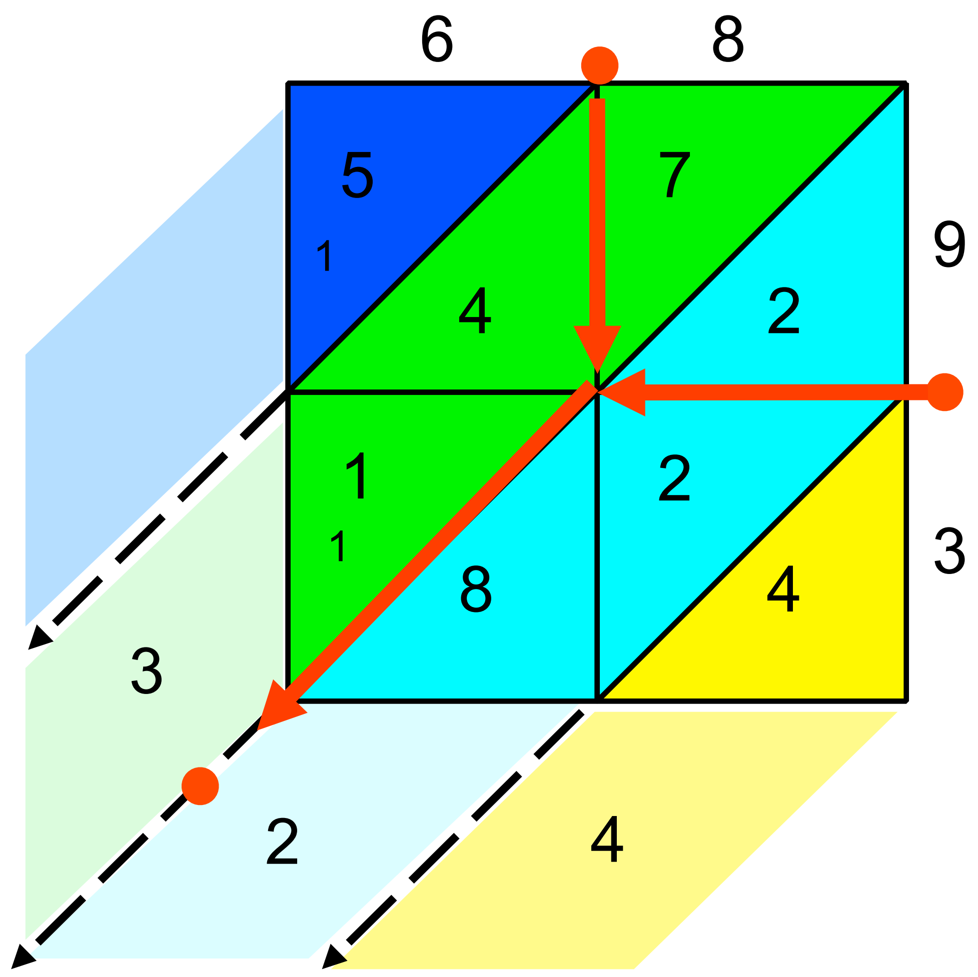
Lattice Multiplication



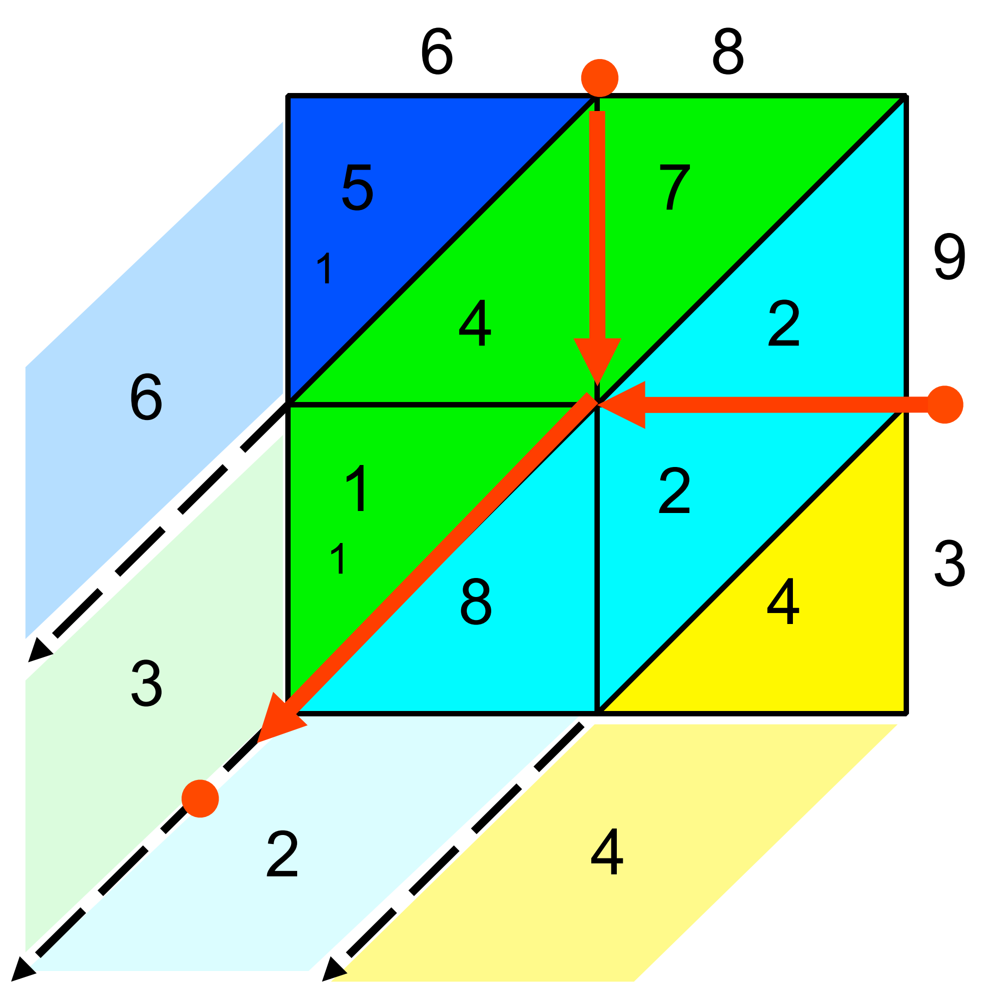
Lattice Multiplication



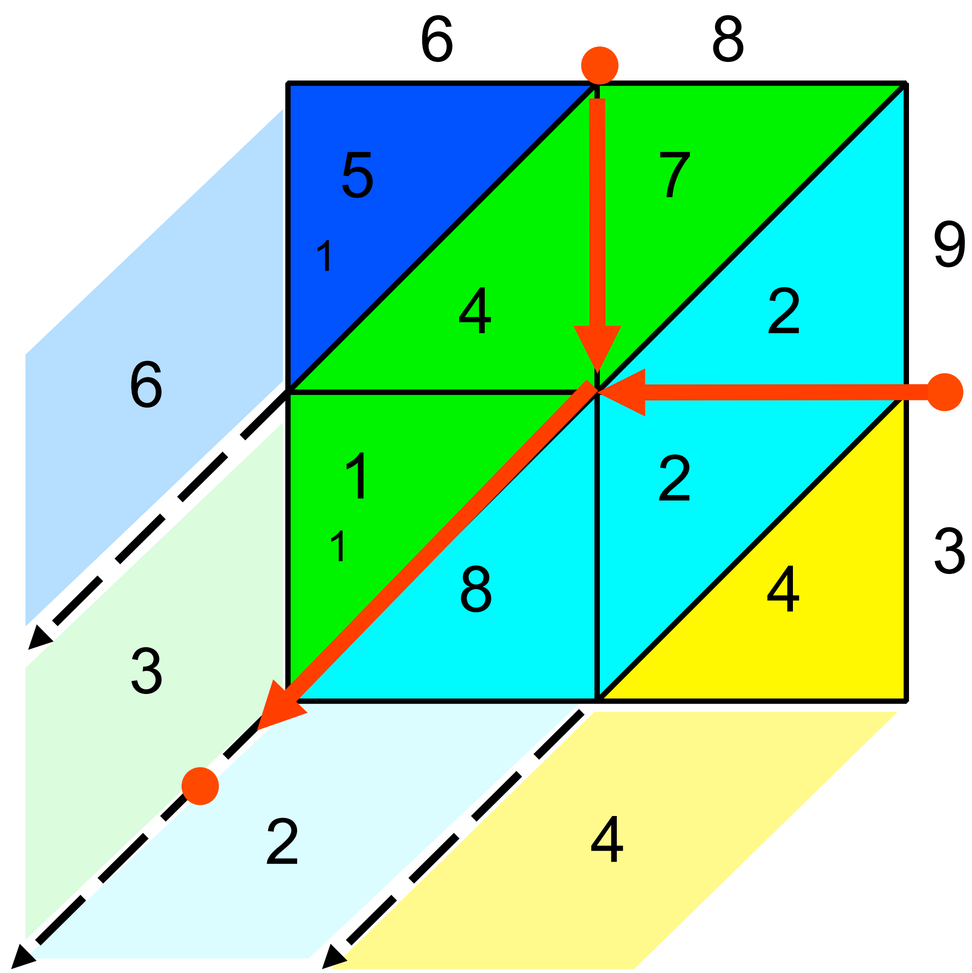
Lattice Multiplication



Lattice Multiplication



Lattice Multiplication



6.8 x 9.3 = 63.24

Egyptian multiplication

- Only need the 2 times table and doubling
- Still used in rural Ethiopia, Russia and some Arab countries

With thanks to Roger Blackman
Penleigh and Essendon Grammar School, Victoria

Egyptian multiplication

21 x 22

1.	22

Egyptian multiplication

21 x 22

Generate doubles

Egyptian multiplication

21 x 22

1.	22
2.	44

Egyptian multiplication

21 x 22

1.	22
2.	44
4.	88

Egyptian multiplication

21 x 22

1.	22
2.	44
4.	88
8.	176

Egyptian multiplication

21 x 22

1.	22
2.	44
4.	88
8.	176
16	352

Egyptian multiplication

21 x 22

1.	22
2.	44
4.	88
8.	176
16.	352

Notice the powers of 2!

Stop doubling before you get to a power of 2 larger than 21.

Egyptian multiplication

21 x 22

1	22
2	44
4	88
8	176
16.	352

21 = 16 + 4 + 1

So the powers of two needed to compile 21 are

$21 = 2^4 + 2^2 + 2^0$

Egyptian multiplication

21 x 22

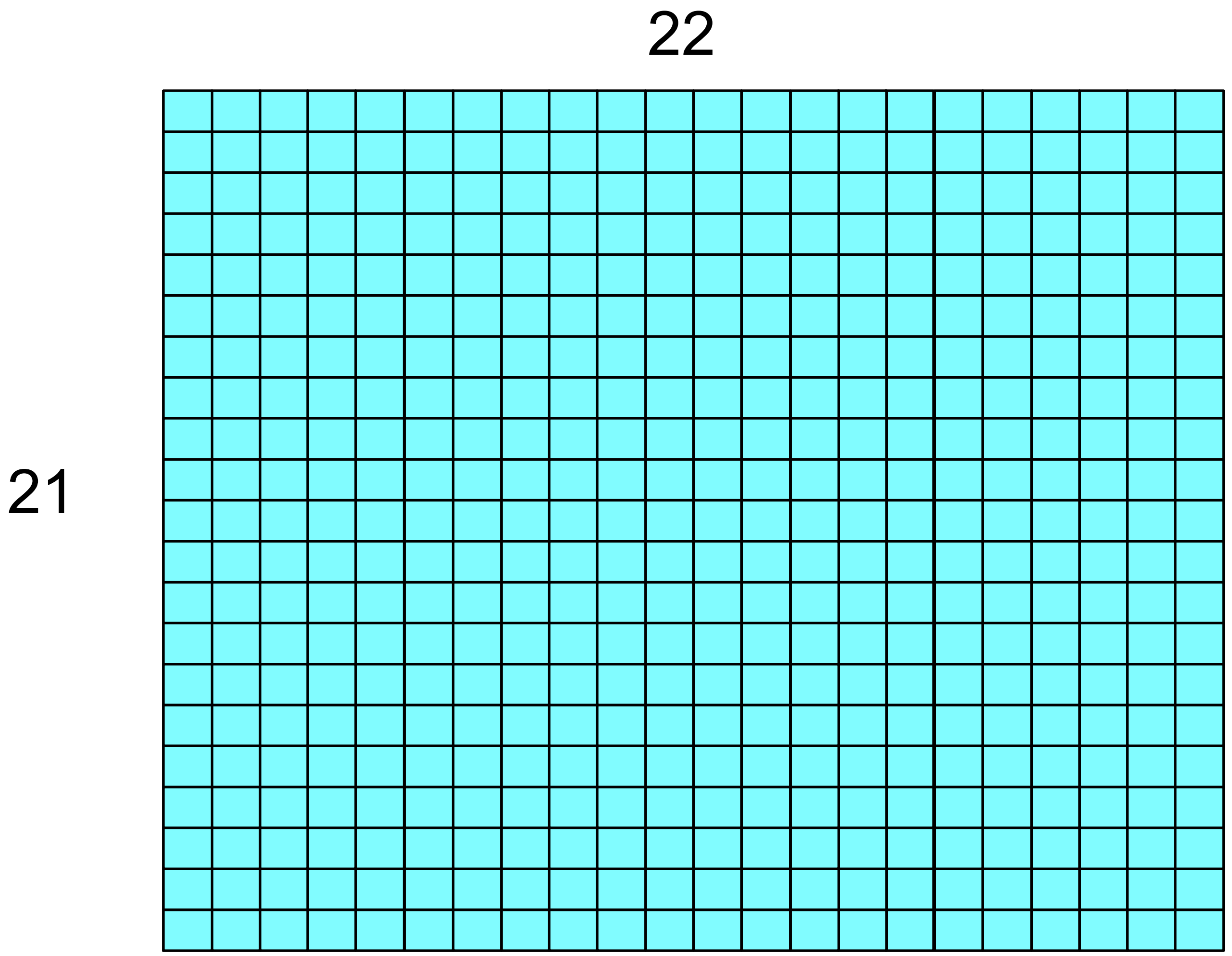
1	22
2	44
4	88
8	176
16.	352
462	

Add the right hand numbers that match the powers of 2 used.

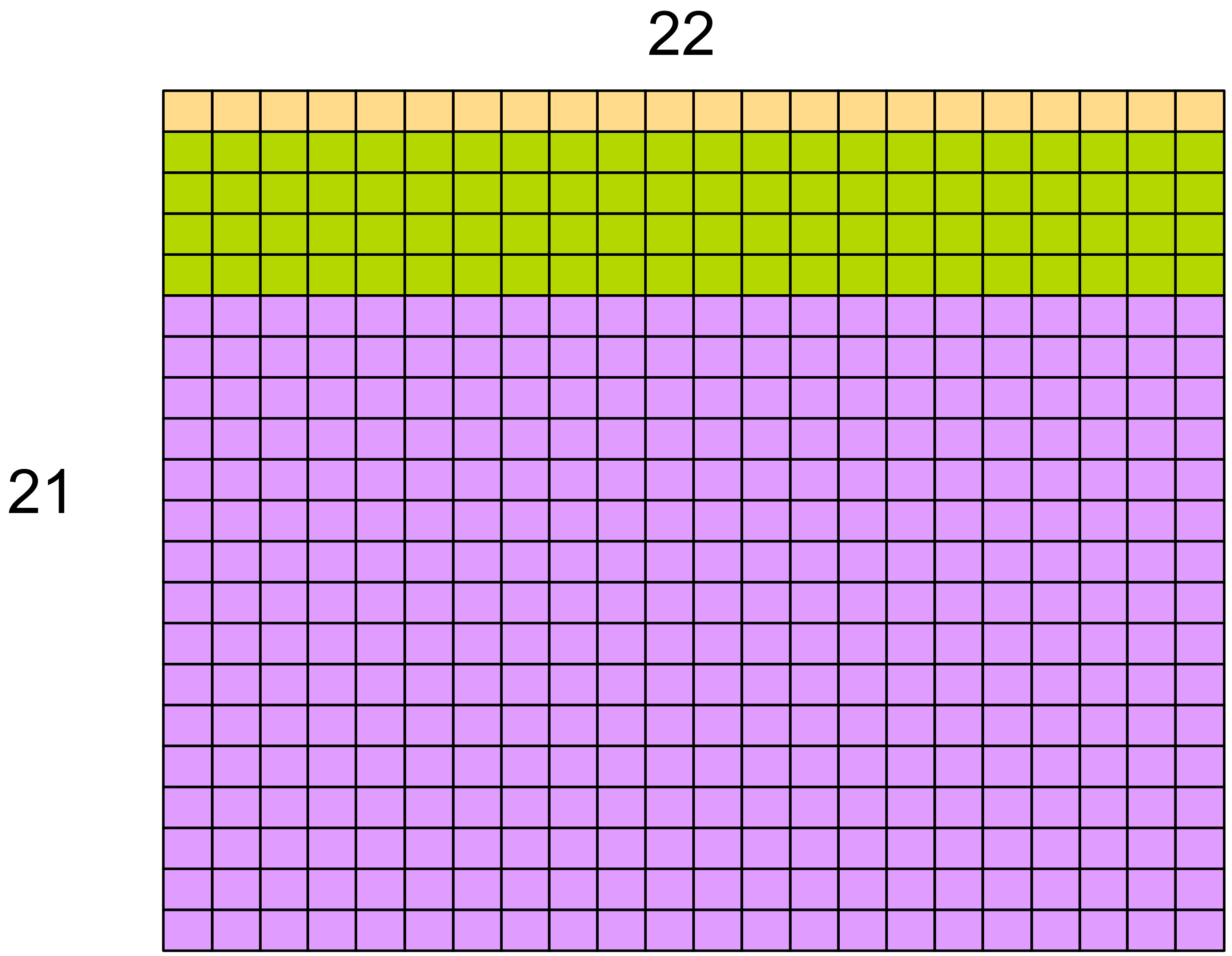
Egyptian multiplication

21 x 22 = 462

Egyptian multiplication



Egyptian multiplication



Russian Peasant method for multiplication

One of these methods for multiplying is often called the Russian peasant algorithm.

You don't need multiplication facts to use the Russian peasant algorithm;

you only need to double numbers, cut them in half, and add them up.

Russian Peasant method for multiplication

Here are the rules:

- Write each number at the head of a column.
- Double the number in the first column, and halve the number in the second column.
- If the number in the second column is odd, divide it by two and drop the remainder.
- If the number in the second column is even, cross out that entire row.
- Keep doubling, halving, and crossing out until the number in the second column is 1.
- Add up the remaining numbers in the first column. The total is the product of your original numbers.

Russian Peasant method for multiplication

Keep doubling, halving, and crossing out until the number in the second column is 1.

If the number in the rhs is even cross out the whole line.

~~57~~	~~86~~
114	43
228	21
~~456~~	~~10~~
912	5
~~1824~~	~~2~~
3648	1

Adding up the lhs gives 4902

57 x 86 = 4902

Division

Models for division

- Arrays
- Number line

Other division ideas to consider

- Long division
  - more natural
  - explains process to a greater degree
  - Use as a demonstration of an idea rather than an assessable skill
- Short division

Division

Division is the inverse of multiplication

$$\text{number} \; \underset{\div 3}{\overset{\times 3}{\rightleftarrows}} \; 3 \times \text{number}$$

Direct problems on division can be phrased as indirect problems in terms of multiplication.

Division

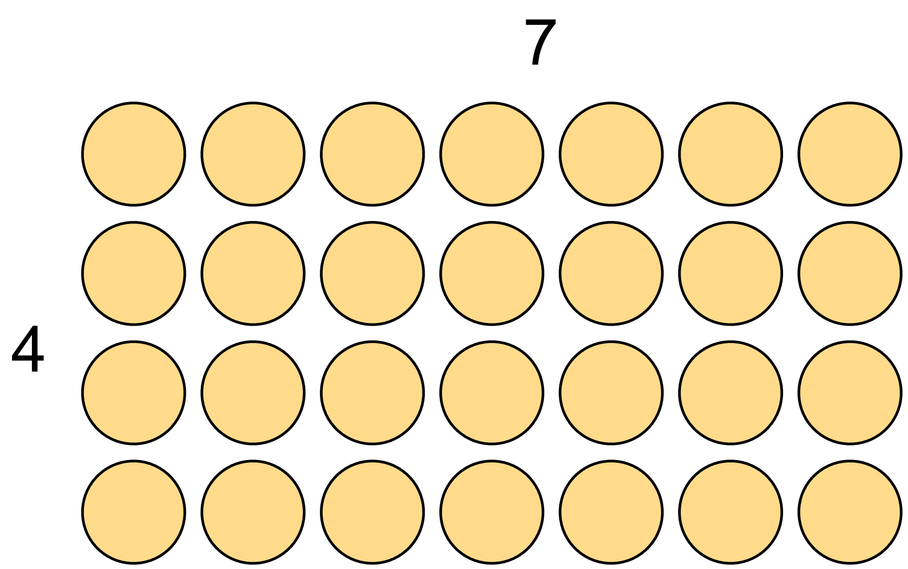
The direct problem $8 \div 2 = ?$
can be interpreted as

either $2 \times ? = 8$ or $? \times 2 = 8$.

As with addition, the only difference between these
expressions is the way in which they are modelled.

Division - arrays

Arrays can be used to model division.



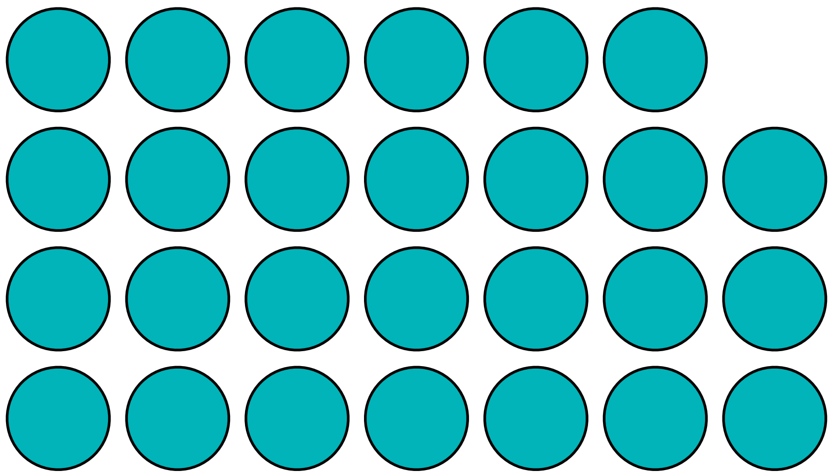
$$4 \cdot 7 = 28$$

$$28 \div 7 = 4$$

$$28 \div 4 = 7$$

Division - arrays

What about 27 ÷ 4?



The closest array is $6 \cdot 4$.
We write $27 \div 4 = 6 \cdot 4 + 3$ (p = qx + r)

Division

Multiplication by 0 is not invertible

$$\text{number} \underset{?!?!?!?!}{\overset{\text{x } 0}{\rightleftarrows}} 0$$

This means division by 0 is **not defined**.

Division

Division is often harder for students than multiplication because

1. Whole numbers not closed under division.

Eg $7 \div 3$ is not a whole number

We sometimes do our best and write

$$7 \div 3 = 2 \text{ r } 1$$

dividend

divisor

quotient

remainder

to mean $7 = (2 \times 3) + 1$.

Division

2. Division is not commutative.

eg $4 \div 2 \neq 2 \div 4$

3. Division is not associative.

eg $8 \div (4 \div 2) \neq (8 \div 4) \div 2$

4. Division is not distributive over addition

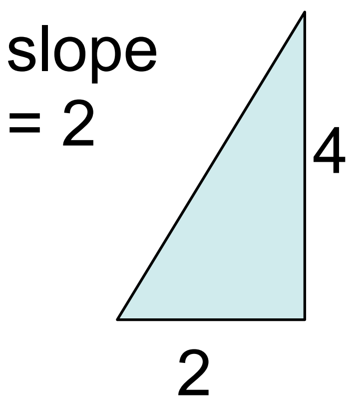
eg $8 \div (2 + 2) \neq (8 \div 2) + (8 \div 2)$

Division

Geometry of Division: Slopes

4 ÷ 2 is the slope of an incline which rises
4 units for every 2 units run horizontally.

Thought of as “rise over run”.



Not great for calculations but generalizes well to fractions.

Long Division

Long division

- –more natural
- –explains process to a greater degree
- –Use as a demonstration of an idea rather than an assessable skill

Long Division

We could use the basic layout of the algorithm to perform this calculation.

```
        1 3 8
     ┌────────
5    )  6 9 3
        5 0 0      100 x 5 = 500
       ──────
        1 9 3
        1 0 0       20 x 5 = 100
       ──────
          9 3
          5 0       10 x 5 =  50
       ──────
          4 3
          4 0        8 x 5 = 40
       ──────
            3
```

So
693 ÷ 5
= 138 r 3

Note: people often suppress the 0's.

Long Division

What facts do we need to know to use the algorithm?

The algorithm will not help us calculate a division where the dividend is less than ten times the size of the divisor.

To use the algorithm, we need to know the product of the divisor by 0,1,2,3,4,5,6,7,8,9.
Why?

The division algorithm is the only standard algorithm where we start on the left. At each step we subtract the highest single digit multiple of the divisor times the corresponding power of 10. At the next stage we must be subtracting a single digit multiple of a lower power of 10 because if we had 10 times it we would have identified it in a previous stage.

Division

Short division

$$\begin{array}{r|l} & 1\ \ 7\ 6 \\ \hline 3 & 5^{2}3^{2}4 \end{array}$$

$534 \div 3 \quad = (500 + 30 + 4) \div 3$

$= (500 \div 3) + (30 \div 3) + (4 \div 3)$

$= 100 + (230 \div 3) + (4 \div 3)$

$= 100 + (210 \div 3) + (24 \div 3)$

$= 100 + 70 + 6$

$= 176$

Divisibility tests

- A number is divisible by 2 if it is even.
- A number is divisible by 3 if the sum of its digits is divisible by three.
- A number is divisible by 4 if its last two digits are divisible by 4.
- A number is divisible by 5 if it ends in 0 or 5.
- A number is divisible by 6 if it is even and divisible by 3.
- NO EASY TEST FOR NUMBER 7 ! Use short division
- A number is divisible by 8 if the last three digits are divisible by 4.
- A number is divisible by 9 if the sum of its digits is divisible by 9.
- A number is divisible by 10 if it ends in zero.

Divisibility tests

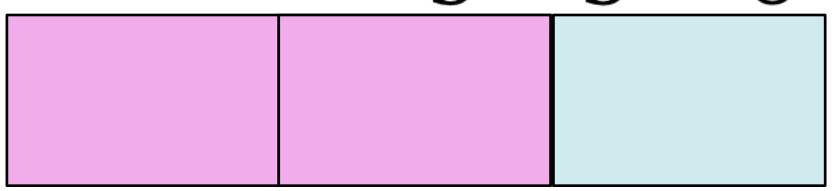
It is fairly easy to show why divisibility by 3 and by 9 works.

876 = 800 + 70 + 6

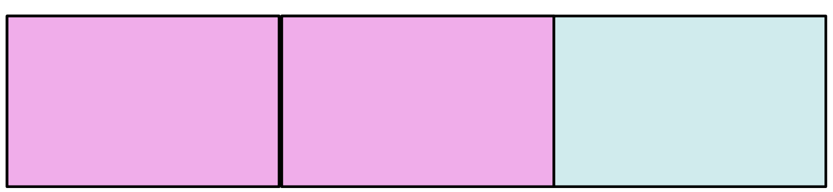
= (8 x 99 + 8) + (7 x 9 + 7) + 6

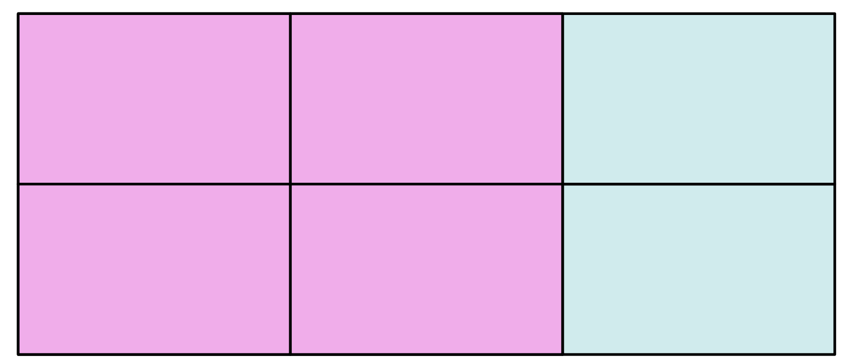
Fractions - some misconceptions

A child says $\frac{2}{3} + \frac{2}{3} = \frac{4}{6}$ and draws this picture



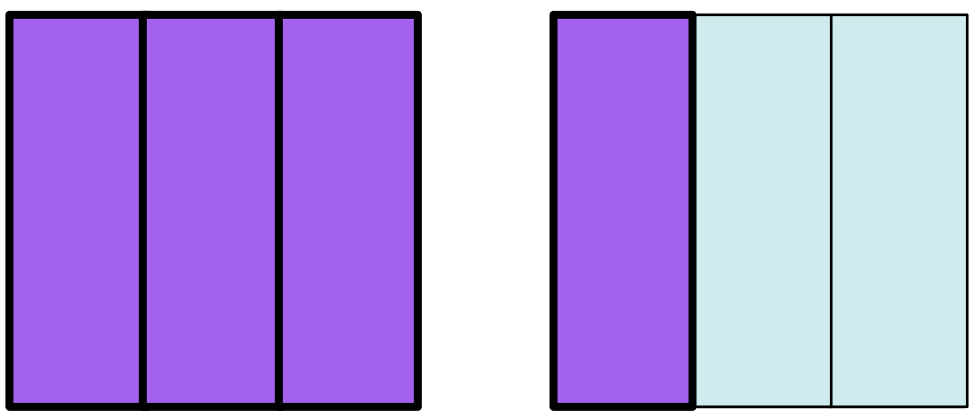
Add them together





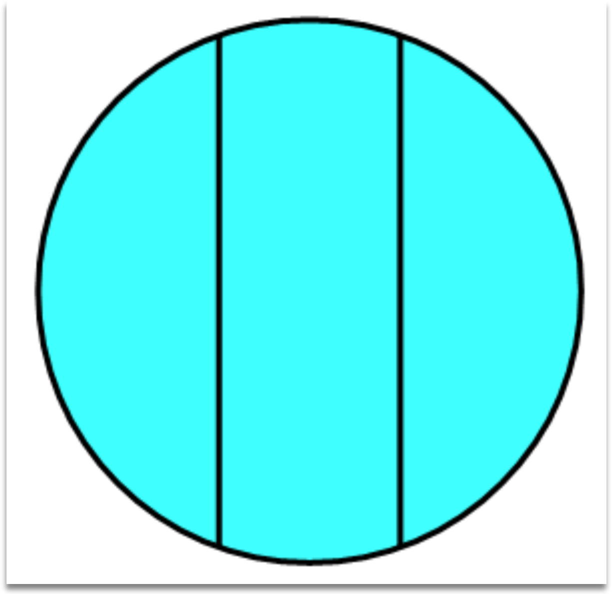
Fractions - some misconceptions

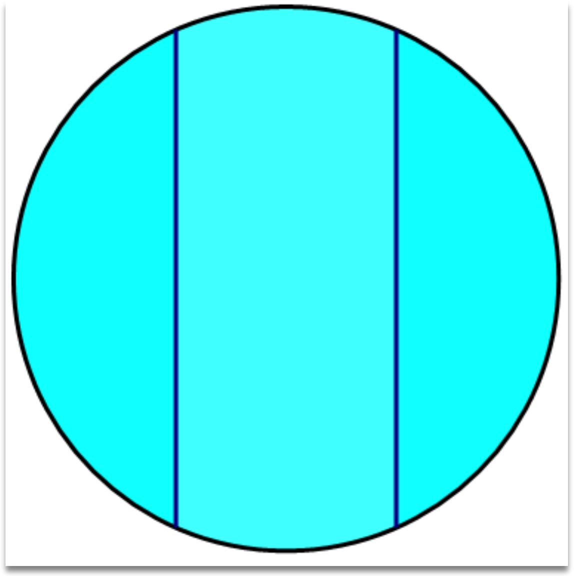
What fraction does this picture represent?

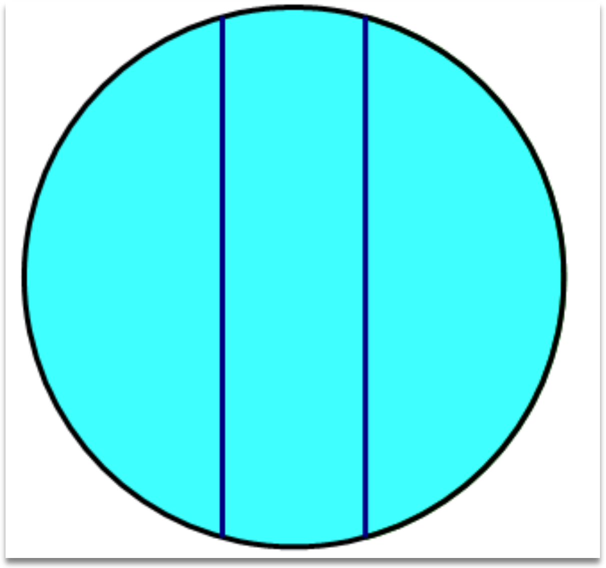


Fractions - some misconceptions

Which circle is divided into thirds by area?







Equal Widths

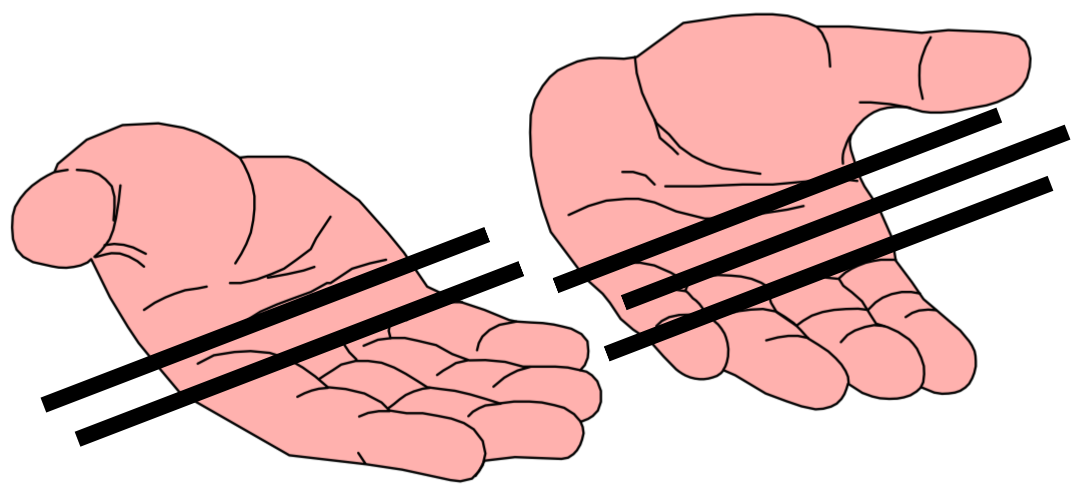
Centre is Half of the circle

Each is $\frac{1}{3}$ total area

Fractions - some misconceptions

Fractions - early strategies

Stick in hands

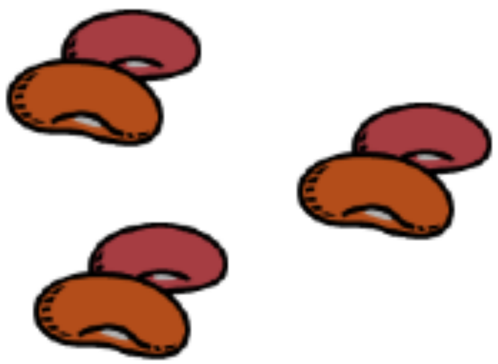


- Small children should be exposed to puzzles and to cutting things up to develop a sense of ‘part/whole’ relationships

Fractions - early strategies

Magic beans

(Lima beans sprayed gold on one side)



- Take a handful.
- Throw them.
- Talk about the number of gold out of the total number of beans.
- Link to ‘numerator’ and ‘denominator’.

Fractions - the number line

Introducing the number line

- Mark in zero and one other reference point
- Convention of negative numbers to the left, zero in the middle and positive numbers to the right
- Move towards children drawing their own

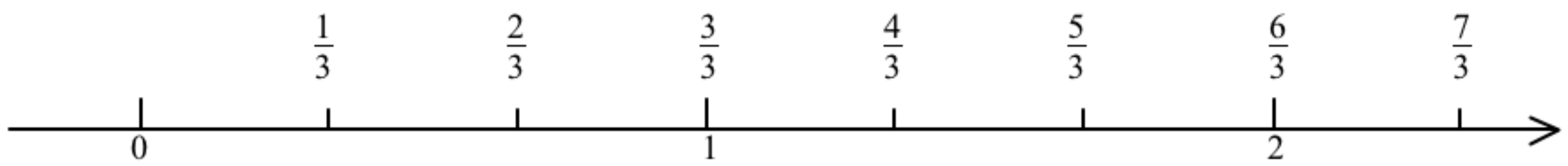
Use

- Masking tape on the floor
- String across the room
- Chalk in the playground
- Magnetized numbers on a blackboard or whiteboard
- Cash-register rolls
- Number ladder

Fractions - the number line

- Step through the introduction of the number line very slowly
- Do not assume this has been done before
- Remind the children all the time, where is the one?

Fractions - the number line



To represent thirds on a number line

- Draw a line segment
- Mark in whole numbers   0, 1, 2 etc
- Divide the segment into three equal lengths
- Label each marker one third, two thirds etc

Watch for confusion...

- Defining fractions on number line

- A fraction is both
  - A point **on** the number line

    AND
  - The distance from 0, a length

Fractions - folding paper

Folding paper helps develop the vocabulary needed

Connects to the number line

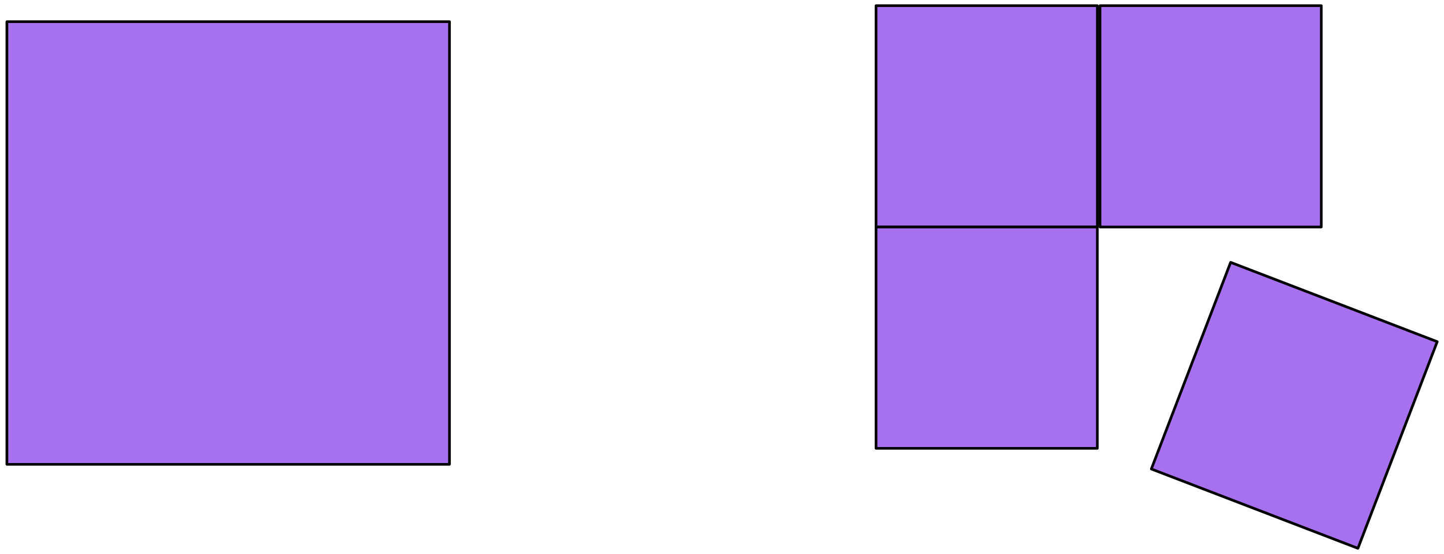
Start by folding paper strips.
 Streamers are cheap and easy to use.

Begin with
 halving quartering eighthing
Then move on to
 thirding sixthing twelfthing
 fifthing tenthing

Fractions - folding paper

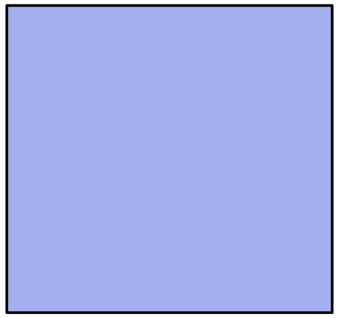
Make posters using kindergarten squares

- Show understanding of cutting the whole
- Begin to introduce the idea of equivalence



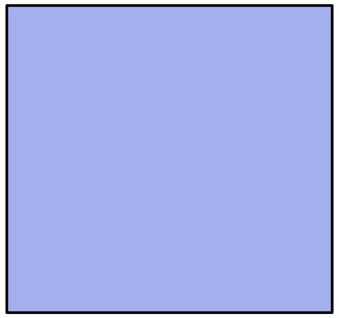
Fractions - the unit square

The unit square is a square with each side of length 1 unit.



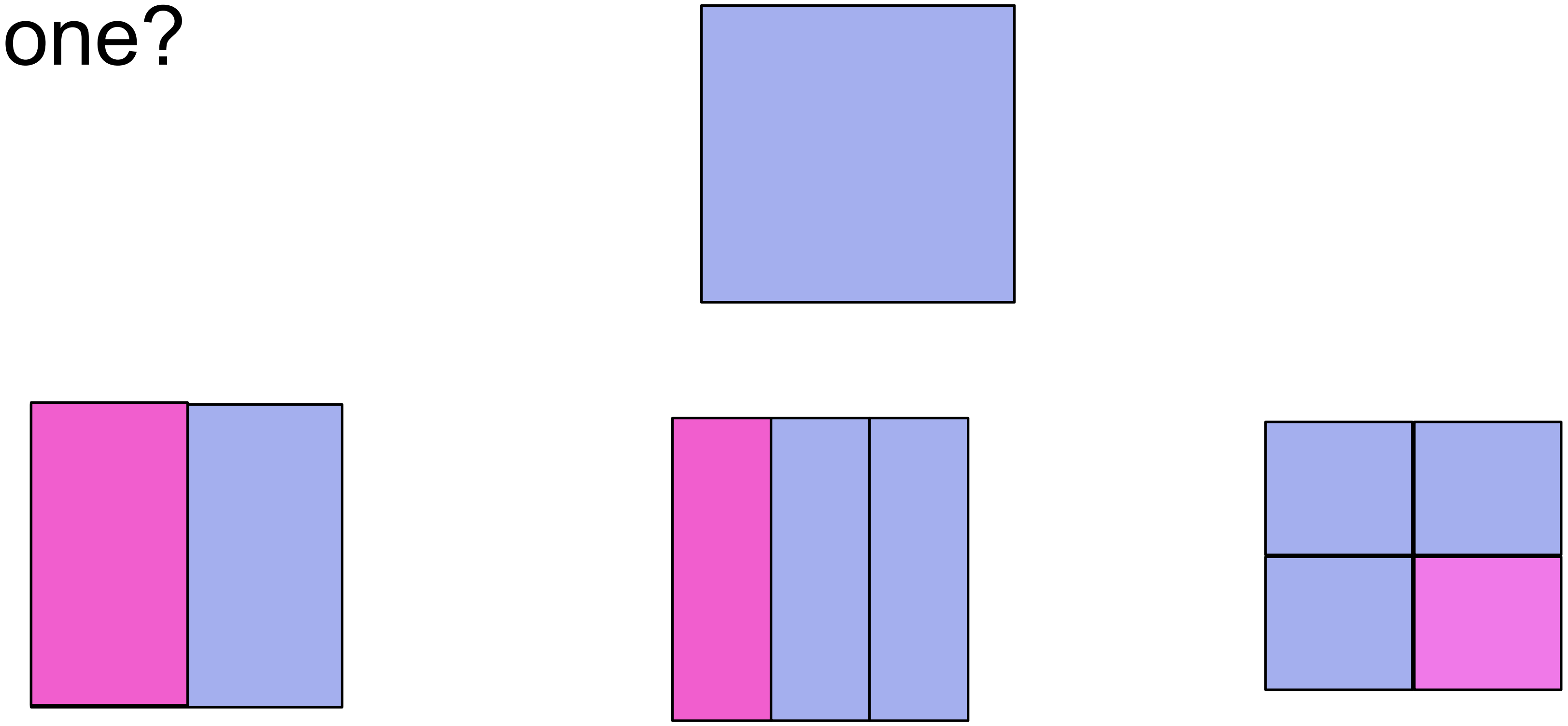
Fractions - the unit square

Remind the children all the time, what is the one?



Fractions - the unit square

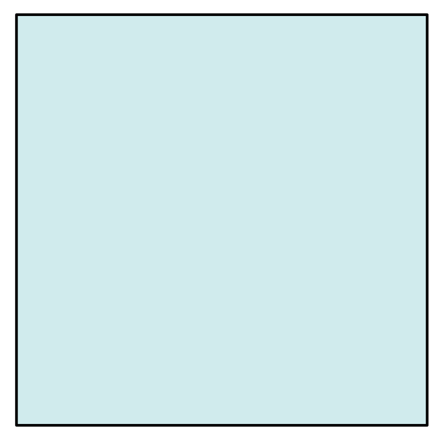
Remind the children all the time, what is the one?



Fractions - multiplication

Based on a unit square:

- Side length = 1
- Area = 1

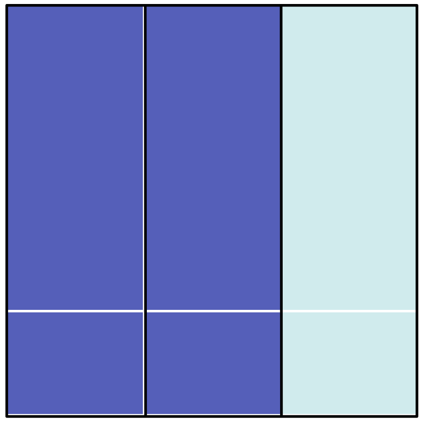


Fractions - multiplication

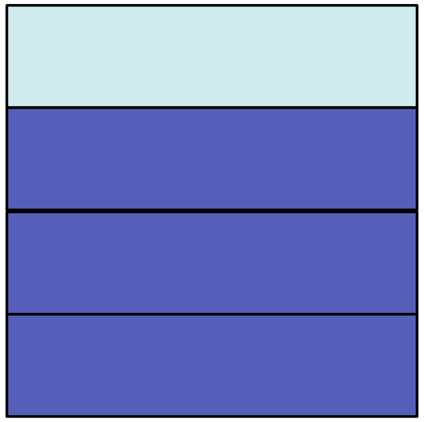
Shade parts of the unit square

- Denominator gives number of parts
- Numerator tells us “How many parts to take”

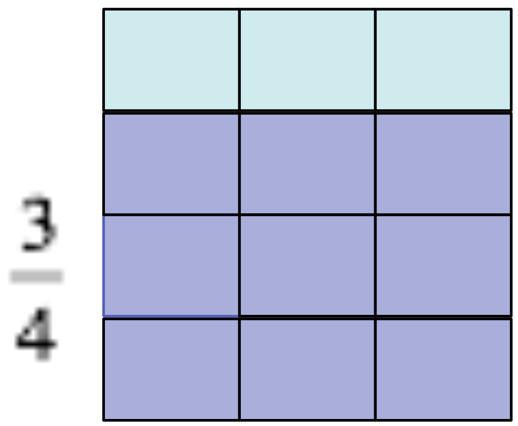
$\frac{2}{3}$



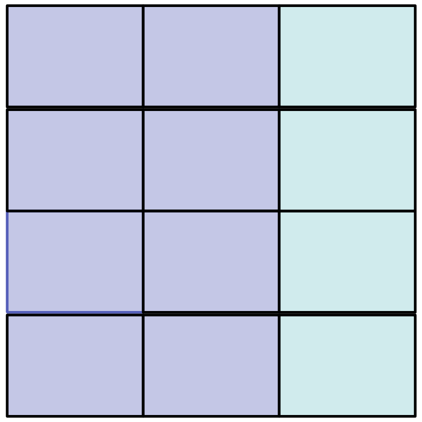
$\frac{3}{4}$



Fractions - multiplication



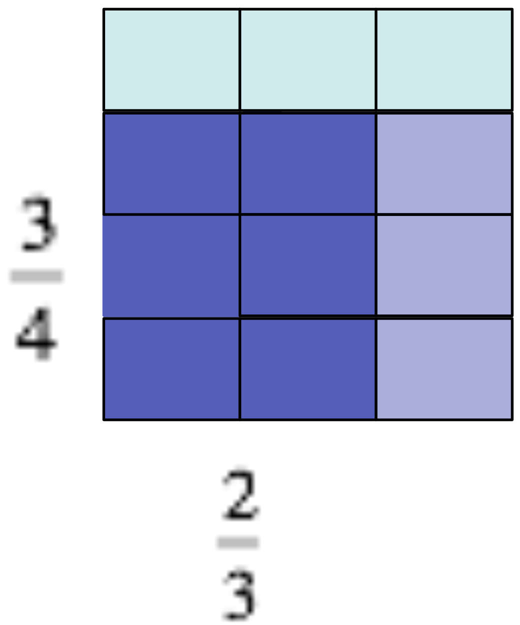
Fractions - multiplication



$$\frac{2}{3}$$

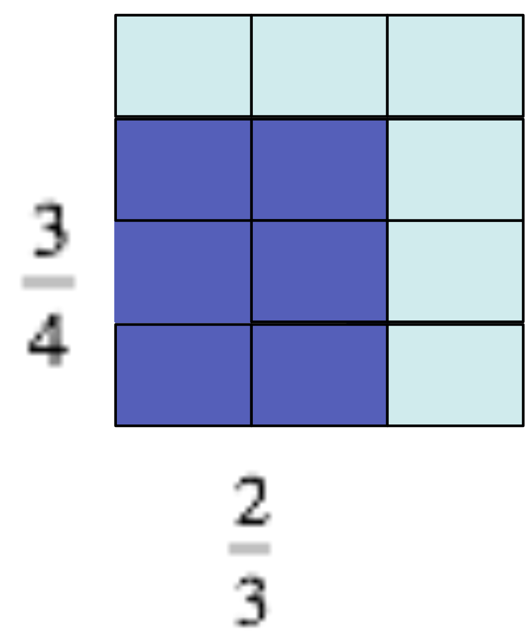
Fractions - multiplication

$\frac{2}{3}$ of $\frac{3}{4}$



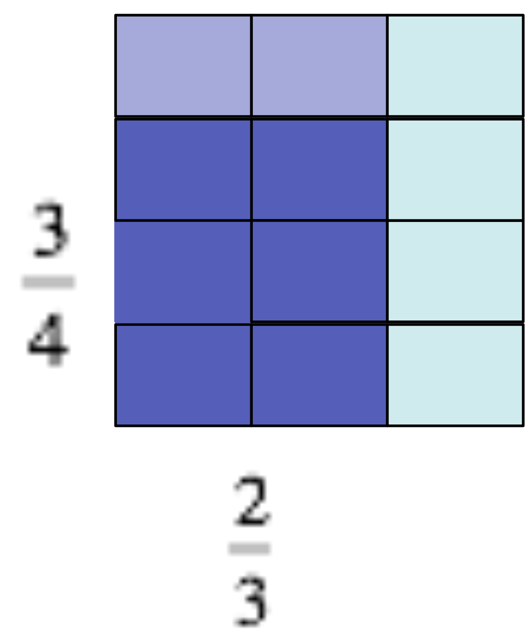
Fractions - multiplication

$$\frac{2}{3} \cdot \frac{3}{4}$$



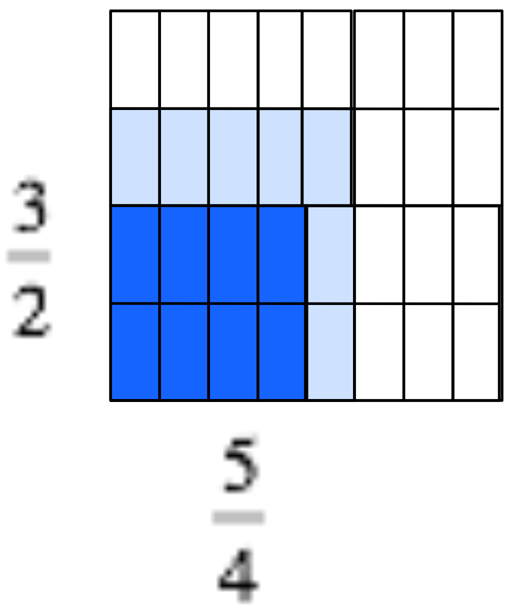
Fractions - multiplication

$\frac{3}{4}$ of $\frac{2}{3}$



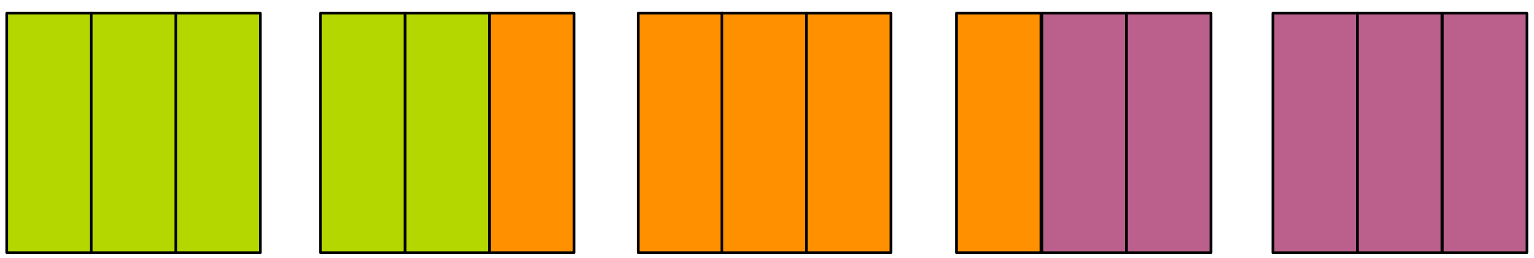
Fractions - multiplication

$$\frac{5}{4} \cdot \frac{3}{2}$$



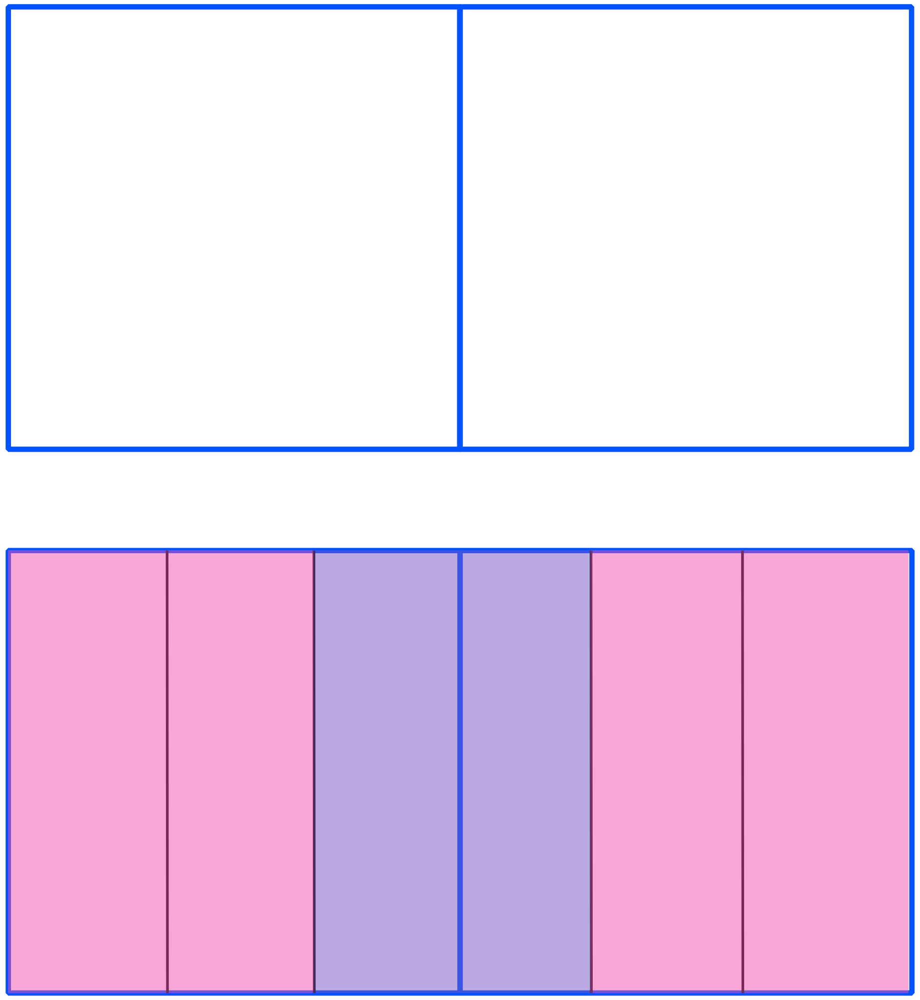
Fractions - division

5 ÷ 3



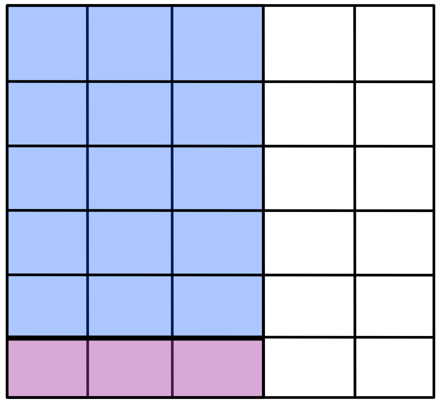
Fractions - division

2 ÷ 3



Fractions - division

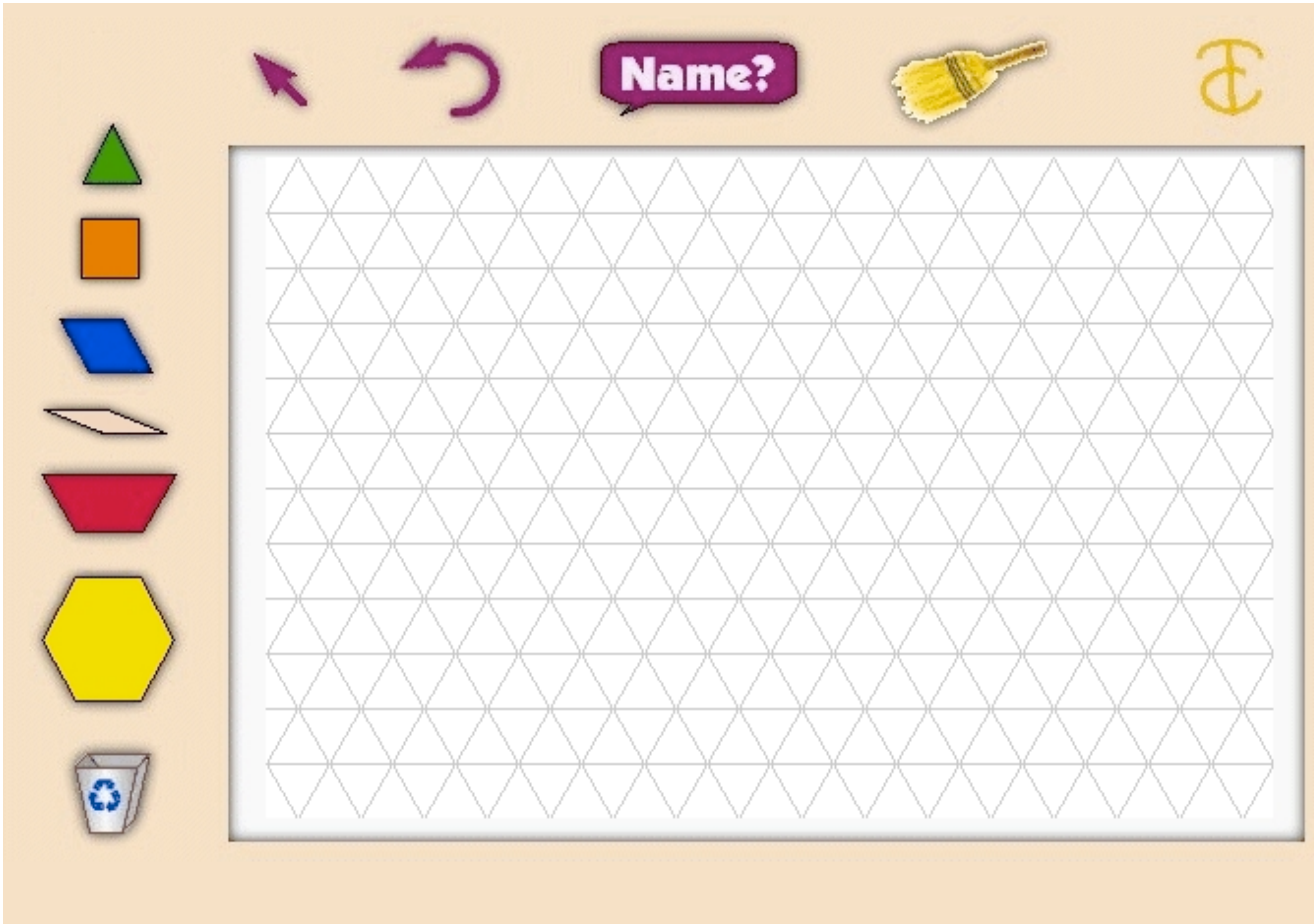
$\frac{3}{5} \div 6$



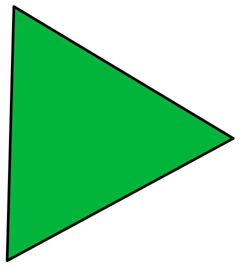
Fractions - pattern blocks

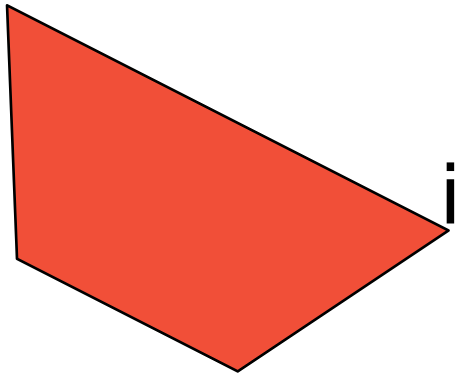
A different way of using manipulatives

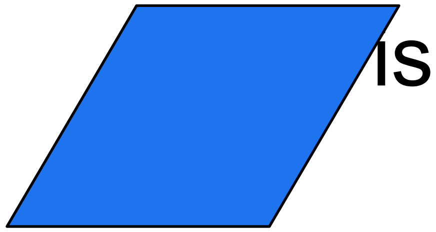
- Pattern blocks can be used to represent fractions
- The issue here is to make sure we always know what the one is
- Use after unit square and number line are firmly understood
- There is a nice java applet on the web
- http://www.arcytech.org/java/patterns/patterns_j.shtml


Name?

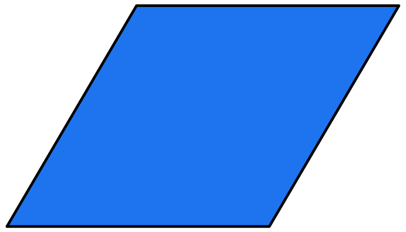
Fractions - pattern blocks

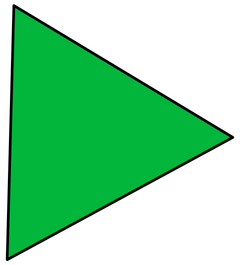
If  is 1,

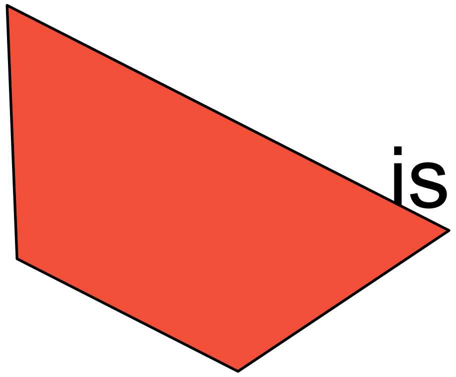
then  is 3

and  is 2.

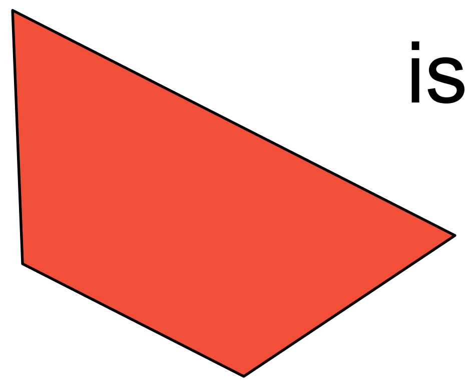
Fractions - pattern blocks

If  is 1,

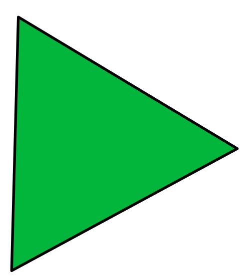
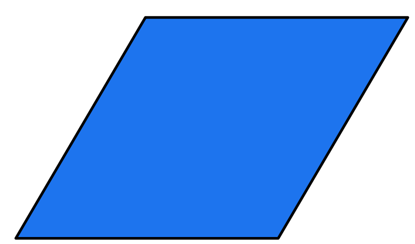
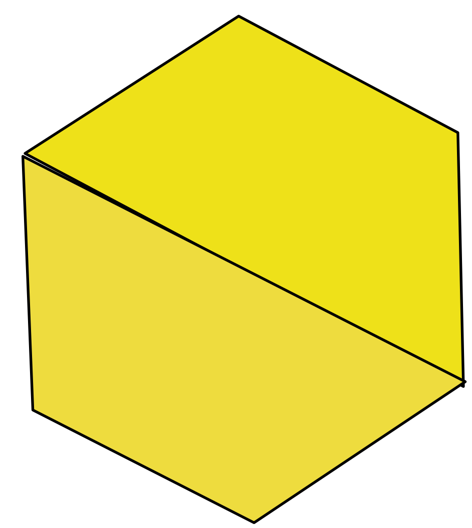
then  is $\frac{1}{2}$

and  is 1 . $\frac{1}{2}$

Fractions - pattern blocks

If  is 1

Then what value do each of these have?

Fractions - using dominoes

Making connections

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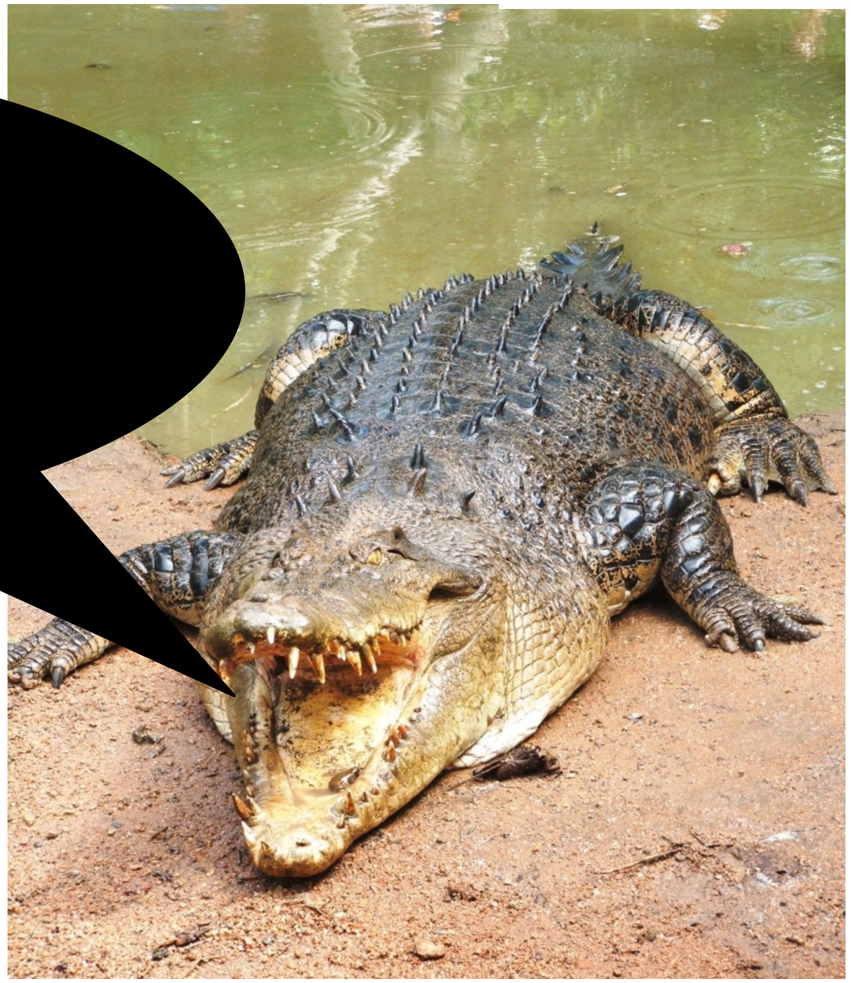
MATHEMATICS

Where's the
Maths?



Making connections

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We thought…
We said…
We wrote…
We saw…
We heard…
We know…
We drew…
We said…
We asked…
We felt…
We liked…
We learnt…
We didn't like…

We found out...
We already knew…
We remembered…
We used equipment…
We need to find out…
It was interesting when…
The tricky bit was…
We didn't know that…
It was cool when…
Congratulations to…
We discovered…
Our group worked well when…
A new word we learnt was…
The strategy we used was…
The important thing to remember is…

Never say
anything a
child can say!