Arithmetic to Algebra through the Australian Curriculum

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What is Arithmetic?

Numbers

Addition

Subtraction

Multiplication

Division

Fractions

Decimals

Place Value

What is Algebra?

Letters

Variables

Equations

Formulas

Solving Unknowns

Substitution

Transposition

When does algebra start in the Australian Curriculum?

By the end of	
Foundation	students make connections between number names, numerals and quantities up to 10. Students count to and from 20 and order small collections.
Year 1	students describe number sequences resulting from skip counting by 2s, 5s and 10s. They identify representations of one half. Students count to and from 100 and locate numbers on a number line. They carry out simple additions and subtractions using counting strategies. They partition numbers using place value. They continue simple patterns involving numbers and objects.
Year 2	students recognise increasing and decreasing number sequences involving 2s, 3s and 5s. They represent multiplication and division by grouping into sets. Students identify the missing element in a number sequence. Students count to and from 1000. They perform simple addition and subtraction calculations using a range of strategies. They divide collections and shapes into halves, quarters and eighths. Students order shapes and objects using informal units.
Year 3	students recognise the connection between addition and subtraction and solve problems using efficient strategies for multiplication. They model and represent unit fractions. Students count to and from 10 000. They classify numbers as either odd or even. They recall addition and multiplication facts for single digit numbers. They continue number patterns involving addition and subtraction.
Year 4	students choose appropriate strategies for calculations involving multiplication and division. They recognise common equivalent fractions in familiar contexts and make connections between fraction and decimal notations up to two decimal places. They identify unknown quantities in number sentences. They describe number patterns resulting from multiplication. Students use the properties of odd and even numbers. They recall multiplication facts to 10 x 10 and related division facts. Students locate familiar fractions on a number line. They continue number sequences involving multiples of single digit numbers.
Year 5	students solve simple problems involving the four operations using a range of strategies. They check the reasonableness of answers using estimation and rounding. Students identify and describe factors and multiples. Students order decimals and unit fractions and locate them on number lines. They add and subtract fractions with the same denominator. Students continue patterns by adding and subtracting fractions and decimals. They find unknown quantities in number sentences.
Year 6	students recognise the properties of prime, composite, square and triangular numbers. They describe the use of integers in everyday contexts. They solve problems involving all four operations with whole numbers. Students connect fractions, decimals and percentages as different representations of the same number. They solve problems involving the addition and subtraction of related fractions. Students make connections between the powers of 10 and the multiplication and division of decimals. They describe rules used in sequences involving whole numbers, fractions and decimals. Students connect decimal representations to the metric system and choose appropriate units of measurement to perform a calculation. Students locate fractions and integers on a number line. They calculate a simple fraction of a quantity. They add, subtract and multiply decimals and divide decimals where the result is rational. Students calculate common percentage discounts on sale items. They write correct number sentences using brackets and order of operations.
Year 7	students solve problems involving the comparison, addition and subtraction of integers. They make the connections between whole numbers and index notation and the relationship between perfect squares and square roots. They solve problems involving percentages and all four operations with fractions and decimals. Students represent numbers using variables. They connect the laws and properties for numbers to algebra. They interpret simple linear representations and model authentic information. Students use fractions, decimals and percentages, and their equivalences. They express one quantity as a fraction or percentage of another. Students solve simple linear equations and evaluate algebraic expressions after numerical substitution.
Year 8	students solve everyday problems involving rates, ratios and percentages. They recognise index laws and apply them to whole numbers. They describe rational and irrational numbers. They make connections between expanding and factorising algebraic expressions. Students use efficient mental and written strategies to carry out the four operations with integers. They simplify a variety of algebraic expressions. They solve linear equations and graph linear relationships on the Cartesian plane.
Year 9	Students apply the index laws to numbers and express numbers in scientific notation. They expand binomial expressions. They find the distance between two points on the Cartesian plane and the gradient and midpoint of a line segment. They sketch linear and non-linear relations.
Year 10	students solve problems involving linear equations and inequalities. They make the connections between algebraic and graphical representations of relations. Students expand binomial expressions and factorise monic quadratic expressions. They find unknown values after substitution into formulas. They perform the four operations with simple algebraic fractions. Students solve simple quadratic equations and pairs of simultaneous equations.

Is it this simple?

What about:

Order of operations

Recognising patterns

Describing rules

Working with unknowns

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Algebra-Arithmetic connection

- Important arithmetic ideas for algebra: Laying the foundations in the upper primary years

- Very relevant to all of primary and junior secondary as well

Get It Together Activity

Number Shapes 1

$$\triangle + \star = 3$$

Number Shapes 1

$$\star + \star = \triangle$$

Number Shapes 1

$$\triangle + \square = 6$$

Number Shapes 1

$$\square = 4$$

Get It Together Activity

Number Shapes 2

☆ + △ = 9

Number Shapes 2

☆ − □ = 2

Number Shapes 2

□ + △ = ☆

Number Shapes 2

□ + 1 = 6

Get It Together Activity

Number Shapes 3

$$\square + \square + \triangle = \star$$

Number Shapes 3

$$\triangle + 1 + \triangle = \square$$

Number Shapes 3

$$\square + \star + \triangle = 11$$

Number Shapes 3

$$\star - \square = 4 \times \triangle$$

Get It Together Activity

Number Shapes 4

$$3\star + 1 = \square$$

Number Shapes 4

$$\square - 2\triangle = 2$$

Number Shapes 4

$$\frac{\triangle - 1}{\star} = \frac{\square}{\square}$$

Number Shapes 4

$$2\star + \triangle = \square$$

Algebra

Algebra is a language

- a way of saying and communicating.

Algebra is more succinct than English

- making it easier to manipulate

- but also more open to incomprehension

Algebra

Algebra is a powerful means of communicating abstract and complex ideas.

It has its own rules which must be learnt and practised.

It is an ideal way to see and express general statements.

Algebra

Consider the consecutive numbers

10, 11, 12

Multiply the 1st and 3rd numbers

10 x 12 =

Now, square the middle number

11 x 11 =

Where would we place this in the curriculum?

Year 4

Algebra

What is the difference between the answers?

$10 \times 12 = 120$

$11 \times 11 = 121$

Difference is 1

Extending thinking: Does this always work for consecutive whole numbers?

Algebra

We can use algebra…

Let x, $x + 1$, $x + 2$ represent three consecutive whole numbers

Algebra

I want to show that the difference between the square of the middle number and the the product of the first and third, is one.

I want

$$(x + 1)^2 - x\,(x + 2) = 1$$

For our number example:

$$(10+1)^2 - (10)\,(10 + 2) = 1$$

Algebra

It does not take much manipulation to see that the result *will* always be true

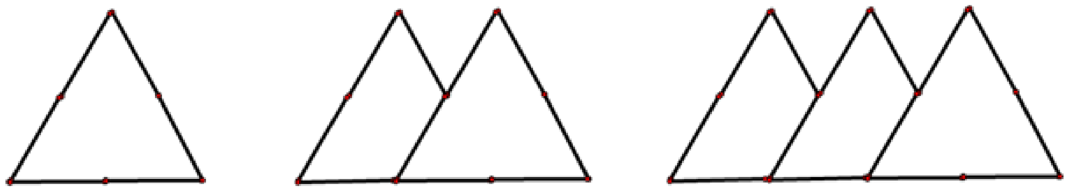
$$(x + 1)^2 - x\,(x + 2) = 1$$

Why stop here?

Why not test consecutive even numbers, or consecutive odd numbers…

Mountains Activity

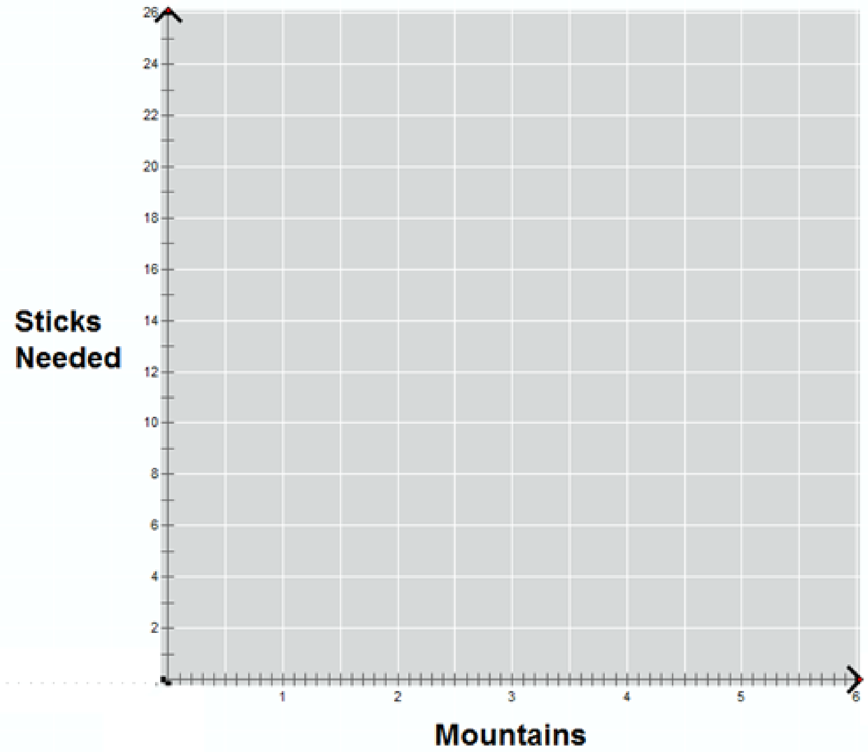
Starting with the Diagram



Use your sticks to work out how many you need to make each of the mountain ranges given in the table

How many Mountains	1	2	3	4	5
How many sticks					

Plot points for each of the pairs of values in the table.



Describe how to create the pattern in words:

Now write an equation for the pattern:

An Example Activity

6	**Number and Place Value**	**Identifies and describes properties of prime, composite, square and triangular numbers**	[**ACMNA122**]()

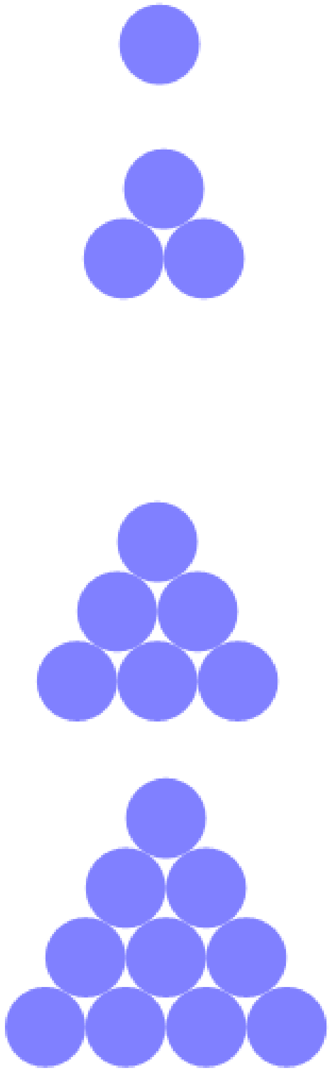
Triangle Numbers

1

3

6

10

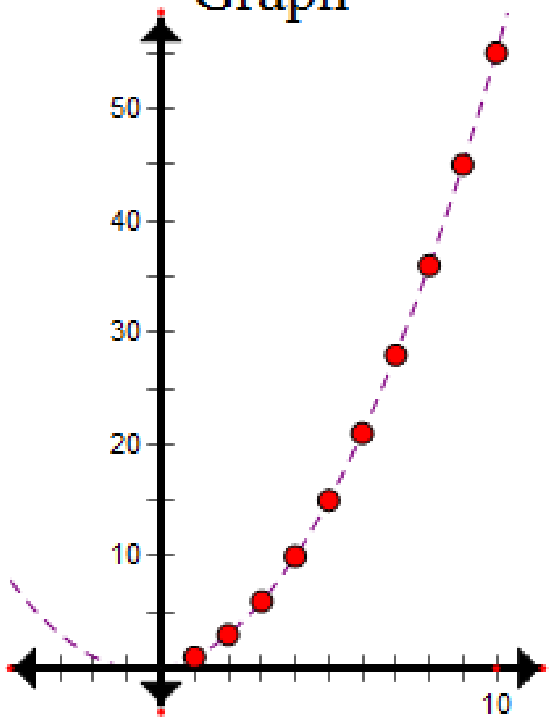


Number of Rows	Number of Counters in Bottom Row

Formula

$$\frac{n\,(n+1)}{2}$$

Graph



BUT WAIT … THERE'S MORE!

Algebra

Generalisation: The Essence of Algebra

Algebra

Has had some bad press

- Algebra is hard
- When will I ever use it?

- Where does algebra have applications in my life?

Algebra

Perhaps one reason for the *attitude* people have to algebra

is that it has been taught without the links to arithmetic being made explicit.

Algebra

Students need to develop an overall framework

to help them make sense of how

the various parts fit together

and

the purpose of them.

Algebra

Why teach it?

As well as simply being part of a well rounded mathematical education, Algebra supports

-problem solving

-logical deduction

-abstraction

-seeing and expressing generalisation

Addition

Any order principle of addition

13 + 25 + 45 +27

- Why is it important to think about arithmetic in this way?
- Does it apply only to addition?

Any order principle of addition

$$3x + 4y + 7x + 11y$$

Any order principle of multiplication

$$25 \times 7 \times 4 \times 3$$

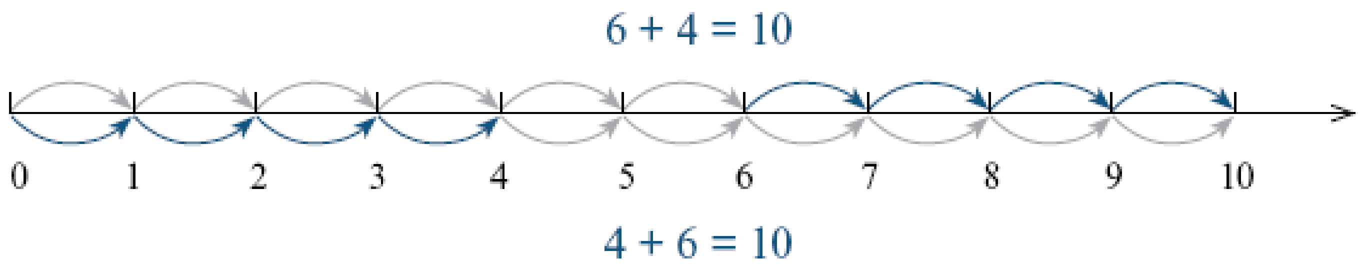
Any order principle of multiplication

$3xy^2 \times 4x2y^3$

Examples

Using the number line

The order in which we perform addition does not matter. For example, 6 + 4 = 4 + 6. This can be shown on the number line.



This property is called the **commutative law for addition.**

We can produce a long list of arithmetic statements such as

$$4 + 3 = 3 + 4$$

$$2 + 6 = 6 + 2$$

$$8 + 3 = 3 + 8$$

Each is an example of the commutative property of addition.

One algebraic statement defines the commutative law

$$a + b = b + a$$

where *a* and *b* are whole numbers.

Importance of building blocks and sequencing

- The teaching of number and algebra are inextricably linked.

- We cannot expect an improvement in student's learning of algebra until we succeed in building their understanding of arithmetic, i.e. knowledge of number and number operations, and mental computation techniques.

Do this calculation in your head

123456789 × 52 - 123456789 × 42

And the answer is:

‘A strong grounding in high school mathematics through Algebra correlates powerfully with access to college, graduation from college, and earning in the top quartile of income from employment. The value of such preparation promises to be even greater in the future’.

‘ Claims based on Piaget’s highly influential theory, and related theories of “developmental appropriateness” that children of particular ages cannot learn certain content because they are “too young,” “not in the appropriate stage,” or “not ready” have consistently been shown to be wrong. Nor are claims justified that children cannot learn particular ideas because their brains are insufficiently developed, even if they possess the prerequisite knowledge for learning the ideas’.

Exemplifying relational thinking: 1

Able to relate terms in a numerical expression across both sides of the equal sign without calculation

Example: Grade 5 student #19 completed the number sentence 23 + 15 = 26 + ☐ by saying, "Comparing 23 and 26, since 26 is three more, so 15 has to become three less."

Exemplifying relational thinking: 2

Able to show relationships between pairs of numbers or between groups of numbers which make the relationship true.

Example: Grade 5 student #38 analyzed the number sentence 43 + □ = 48 + 76 by placing the two pairs of numbers directly under each other:

$$\begin{array}{ccc} 43 & + & \square \quad = \\ 48 \uparrow_{-5} & + & 76 \uparrow_{+5} \end{array}$$

Exemplifying relational thinking: 3

Able to explain in words why a given numerical expression is true by stating the mathematical rules connecting all terms.

Example: Grade 5 student #21 solved like this:

-1

73 + 49 = 72 + □

The student said: ***"the box has to contain an opposite number (to –1) that produces zero."***

Exemplifying relational thinking: 4

Able to express *specific* conditions or a *specific* rule which make a numerical expression true.

**Example: Grade 5 student #37 analyzed the number sentence 23 + 15 = 26 + ☐ in the following specific way:
"Because I have to make 23 + 15 equal to a 'three larger' number and a 'three smaller' number, I get 12. Left side and right side are balanced."**

Exemplifying relational thinking: 5

Able to generate other number sentences which exhibit the same or similar specific relationship or rule as the original sentence.

Example: Grade 5 student #15 solved
746 – 262 + ☐ = 747 by placing 263 in the box.
The student explained that "the result of the calculation on the left-hand side has to give a number on the right-hand side which is one more".

Exemplifying relational thinking: 6

Able to re-express a given number sentence in a form which allows one to show the truth of the original expression.

Examples:
Grade 7 student #29 re-expressed
39 – 15 = 41 – ☐ as
39 – 15 = 41 – (15 + 2)
Grade 8 student #10 re-expressed
99 – ☐ = 90 – 59 as
(90 + 9) – ☐ = 90 – 59
(90 + 9) – (59 + 9) = 90 – 59

Exemplifying relational thinking: 6 con't

Able to re-express a given number sentence in a form which allows one to show the truth of the original expression.

Grade 7 student #32 transformed
39 – 15 = 41 – ☐ to
39 + (–15) = 41 + (–☐)
and then applied the procedures that are used to deal with addition sentences: "*an increase in the first number on each side has to be balanced by a (corresponding) decrease in the second number on each side*. *In this way (–15) becomes (–17).*"

Features of relational thinking

What defines relational thinking?

the focus is on the sentence, viewed as a whole

the equals symbol stands for equivalence or balance

relational thinking depends on being able to refrain from calculation (i.e. keep the sentence open)

comparing pairs of known numbers (either side of the equals sign) to find the missing value.

the strategies depend on the nature of the numbers and the operations involved

Arithmetic thinking

Is appropriate and needed where relations between numbers are not evident

simplifies an expression through calculation so that an answer can be obtained

cannot be used to deal successfully with expressions involving literal symbols

some students choose to use it in some contexts

for other students it is their only strategy

Teaching implications

Introducing young children to relational thinking is not easy when teachers' vision has for so long been restricted to arithmetic as calculation.

In the primary school, this means attending to the structure of arithmetic operations.

Without these experiences, many students fail to understand these structures that are necessary for a successful transition to algebra.

Calculator "Logic"

There are two ways that calculators work:

4 Function – Immediate Action or Arithmetic Logic

And

Scientific – Algebraic logic

Give specific attention to the “equals sign” and “difference”

Step away from treating the equality sign as simply denoting a result of a computation.

Consider the possibility of more than one term on the right side of the equal sign. Ask students to give many possible meanings to a sentence such as 7 = …….

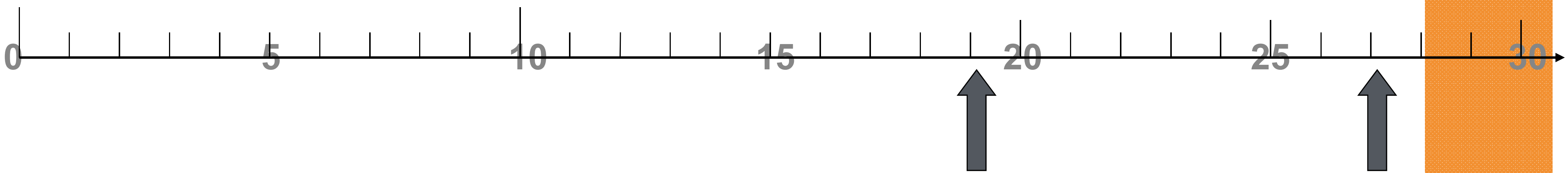
Introduce students to considering the equality sign as meaning “is the same as” or “has the same value as”.

Use “difference” rather than “subtraction”. Represent this difference on a number line

The Importance of Subtraction as Difference

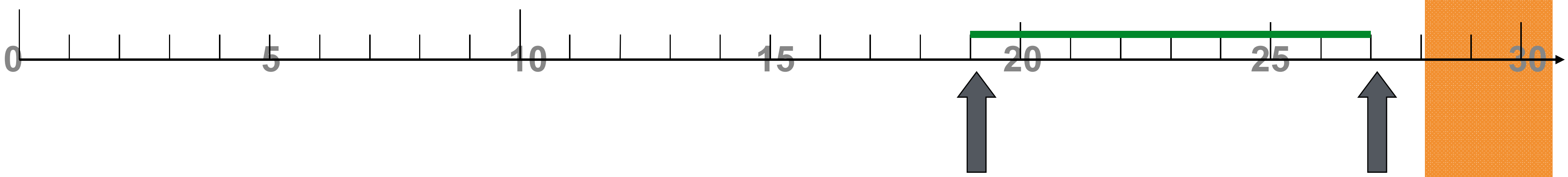
$$27 - 19 = \square - \square$$

What is the difference between 27 and 19?



$$27 - 19 = \square - \square$$

Here is the difference between 27 and 19



The blue bar has a fixed length (here it is 8 units)

$$27 - 19 = \boxed{26} - \boxed{}$$

The difference between 27 and 19 is the same as the difference between 26 and what number?

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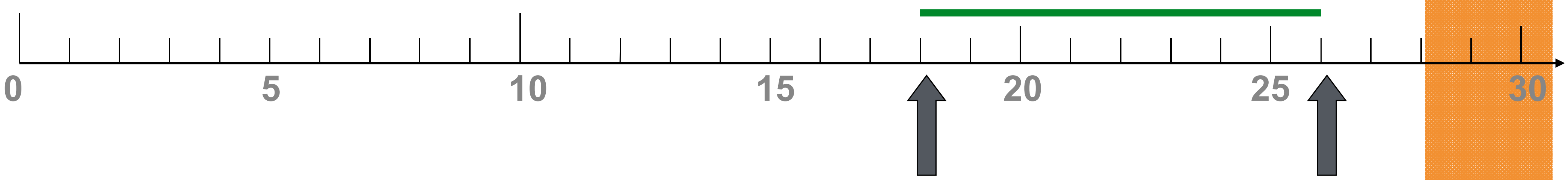
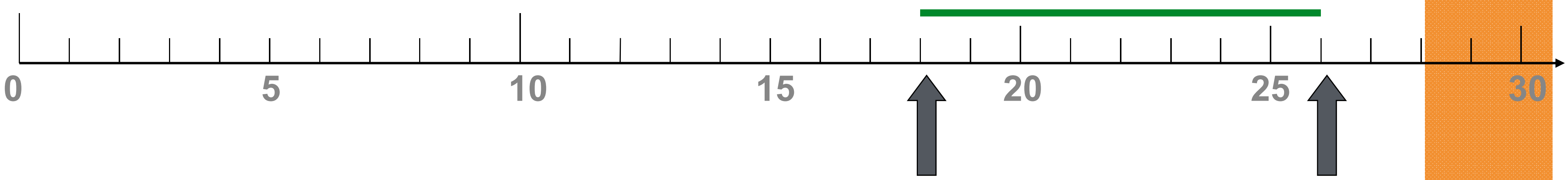


The blue bar has moved one unit to the left

$$27 - 19 = \boxed{26} - \boxed{18}$$

The difference between 27 and 19 is the same as the difference between 26 and what number?



Both numbers have decreased by 1 to keep the difference the same

$$27 - 19 = \boxed{} - \boxed{9}$$

The difference between 27 and 19 is the same as the difference between what number and 9?

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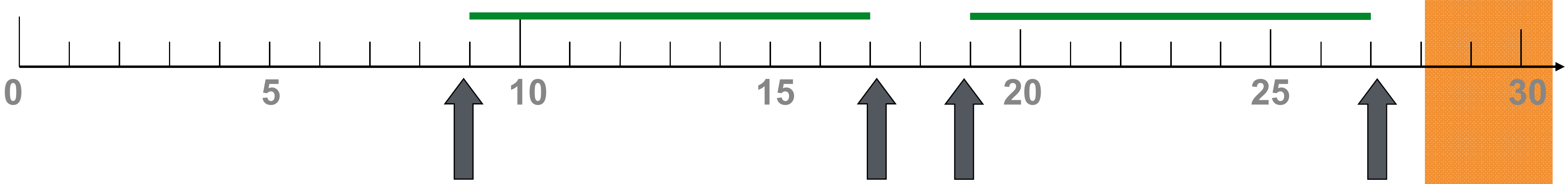
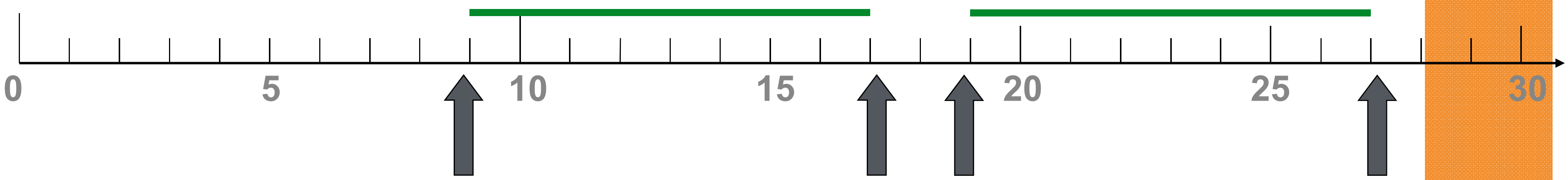
The difference between 27 and 19 is the same as the difference between what number and 9?



The blue bar has moved ten units to the left - so the missing number is 17

$27 - 19 = \boxed{17} - \boxed{9}$

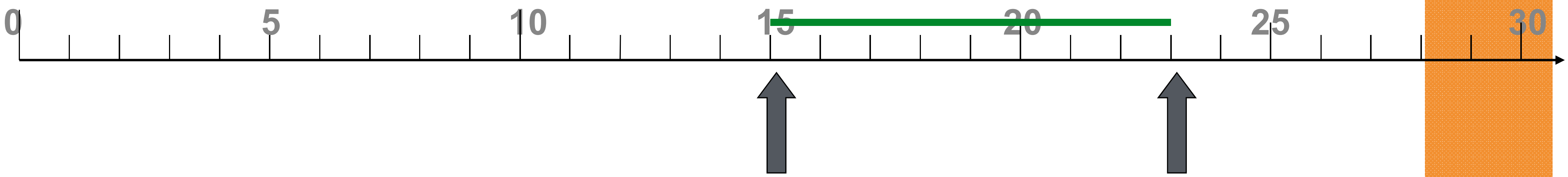
The difference between 27 and 19 is the same as the difference between what number and 9?



Both numbers have decreased by 10 units to keep the difference the same

$$27 - 19 = \square - \square$$

The difference between 27 and 19 is the same as the difference shown below. What are the numbers?



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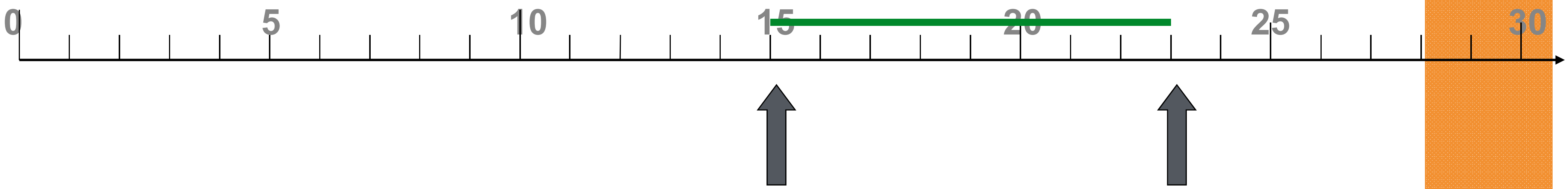
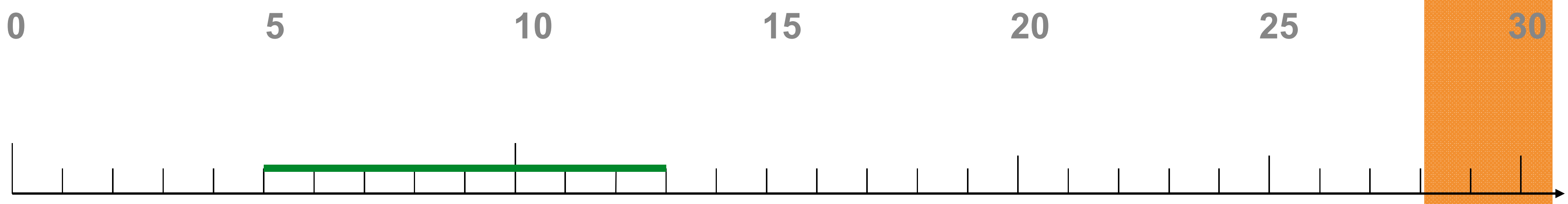


The missing numbers are 23 and 15 because the blue bar has moved 4 units to the left

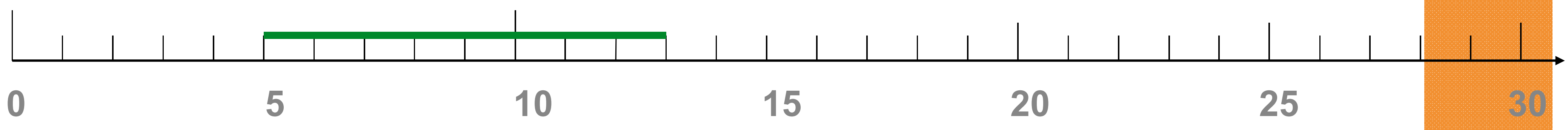
$$27 - 19 = \boxed{13} - \boxed{}$$

The difference between 27 and 19 is the same as the difference between 13 and what number?



$$27 - 19 = \boxed{13} - \boxed{}$$

The difference between 27 and 19 is the same as the difference between 13 and what number?



The first number has decreased by 14, so the second number must also decrease by 14 to keep the difference the same

Tidy Up Making Connections Slides

Making connections

Let students discuss how different thinking applies to:

97 + 39

+3 – 3

100 + 36

Making connections

Let students discuss how different thinking applies to:

$$97 + 39$$

$$\color{red}{+3} \qquad \color{red}{-3}$$

$$100 + 36$$

$$97 - 39$$

$$\color{red}{+1} \qquad \color{red}{+1}$$

$$98 - 40$$

Making connections

Let students discuss how different thinking applies to:

97 + 39

+3 **– 3**

100 + 36

97 – 39

+1 **+1**

98 – 40

It applies to working with fractions and decimals as well:

3.7 + 4.45

+0.3 **–0.3**

Making connections

Let students discuss how different thinking applies to:

97 + 39

+3 – 3

100 + 36

97 – 39

+1 +1

98 – 40

It applies to working with fractions and decimals as well:

3.7 + 4.45

+0.3 –0.3

4.0 + 4.15

What about 5.31 – 3.8?

Making connections to multiplication and division

Students also need to explore how relational thinking applies to multiplication and division (next lecture):

96×25

$\div\, 4 \qquad \times\, 4$

24×100

$192 \div 25$

$\times\, 4 \qquad \times\, 4$

$768 \div 100$

Relational thinking and calculation

For all four operations, there are payoffs:

For addition:

■ + 99

■ + 399

■ + 9.9

■ + 49

Relational thinking and calculation

For all four operations, there are payoffs:

For subtraction:

- $\blacksquare - 99$
- $\blacksquare - 9.9$
- $\blacksquare - 49$
- $\blacksquare - 1\frac{2}{3}$

Relational thinking and calculation

For all four operations, there are payoffs:

For multiplication:

$\blacksquare \times 25$

$\blacksquare \times 125$

$\blacksquare \times 2.5$

$\blacksquare \times 0.25$

Relational thinking and calculation

For all four operations, there are payoffs:

For division:

- $\div$ 25
- $\div$ 2.5
- $\div$ 0.5
- $\div$ 0.25

Relational thinking

Is a powerful way of drawing attention to some fundamental **structures** of arithmetic

Two key ideas are:

equivalence of expressions, and

compensation, including knowing the **direction** in which compensation takes place

These ideas also provide a key foundation for **algebraic thinking**

How would you calculate?

$$\begin{array}{r} 3000 \\ -\underline{1563} \end{array}$$

Can relational thinking help us to find alternatives?

$$\begin{array}{r} 3000 \\ -\underline{1563} \end{array}$$

Some students find algorithms for subtraction (e.g. method of decomposition) difficult and time consuming

Given 3000 – 1563

Increase both numbers by 37 gives

3037 – 1600

This can be calculated more easily!! 1437

(Why did we choose 37? Because it makes the second number 1600 and easier to subtract)

We can also change the first number:

Given 3000 – 1563

Decrease both numbers by 1: 2999 – 1562

$$\begin{array}{r} 2999 \\ -\ \underline{1562} \\ 1437 \end{array}$$

This can be calculated more easily!! 1437

- Is a powerful way of drawing attention to some fundamental structures of arithmetic

- Two key ideas are:

  –equivalence of expressions, and

  –compensation, including knowing the direction in which compensation takes place

- These ideas also provide a foundation for algebraic thinking

Summarising what we found + -

We can keep the **sum** or the **difference** the same by making particular adjustments to each number using the operations of **addition** and **subtraction**.

- To keep the **sum** the same, one number is increased by a certain amount and the other number is decreased by that same amount.
- To keep the **difference** the same, both numbers are increased (or decreased) by the same amount.

In each case, the amount of increase or decrease can be **any type of number** (whole, fraction, decimal)

In each case, the type of increase or decrease is **"additive"** (which means involves only addition or subtraction)

Summarising what we found x ÷

We can keep the **product** or the **quotient** the same by making particular adjustments to each number using the operations of **multiplication** and **division**.

- To keep the **product** the same, one number is increased by a certain amount and the other number is decreased by that same amount.
- To keep the **quotient** the same, both numbers are increased (or decreased) by the same amount.

In each case, the amount of increase or decrease can be **any type of number** (whole, fraction, decimal)

In each case, the type of increase or decrease is **"multiplicative"** (which means involves only multiplication or division)

Fact Families and contexts for operations

- Students should be able to obtain related facts from a single number fact:
  - E.g., from $6 \times 5 = 30$ we also know

    $5 \times 6 = 30$

    $30 \div 6 = 5$

    $30 \div 5 = 6$
- They should also have experience of relating a worded situation to a numerical expression
  - E.g., the number sentence $6 \times 5 = 30$ could arise from "there are 6 tables, and each table has 5 people sitting at it, so there are 30 people all together"

Inverse Operations

"Siblings"

- Understand the relationship between addition/subtraction and multiplication/division and even square/square root

- Equivalence of

$$6 \times 4 = 24$$
$$\Updownarrow$$
$$24 \div 4 = 6$$

$$12 + 8 = 20$$
$$\Updownarrow$$
$$20 - 8 = 12$$

$$7^2 = 49$$
$$\Updownarrow$$
$$\sqrt{49} = 7$$

Role of the Equals Sign

- As teachers you need to set the example.
- Only write the equals sign between quantities that are equal
- Don't use the equals sign for "run on calculations"
  - E.g., if solving "a boy has five marbles and then a friend gives him ten more, and then he loses two" then do NOT write

$$5 + 10 = 15 - 2 = 13$$

Number Properties

- Need to know the multiples

- Need to know how to factorise numbers (and then rearrange the factors)

  e.g., $24 = 2 \times 2 \times 2 \times 3$
  $= 8 \times 3 = 6 \times 4 = 12 \times 2$

- Useful to know the square numbers:
  1, 4, 9, 16, 25, 36, ...

Algebra

Students need to be familiar with

- arithmetic
- patterns in arithmetic
- relationships between numbers and operations

Algebra

While arithmetic is still being consolidated, care should be taken not to *over do* algebra.

For example,

20% of an amount is $6, what is 100%?

Play with

Algebra

The tendency might be to use algebra

$$20\% \text{ of } x = 6$$

$$\frac{x}{5} = 6$$

$$x = 30$$

Algebra

I prefer the unitary method

10% is $3

100% is $30

Number

Number is an abstract construct of the human mind.

Like anything abstract it is difficult to grasp – literally!

In order to use the concept of numbers effectively we need to internalise their properties, they need to become second nature to us.

Number line

Use the number line to illustrate

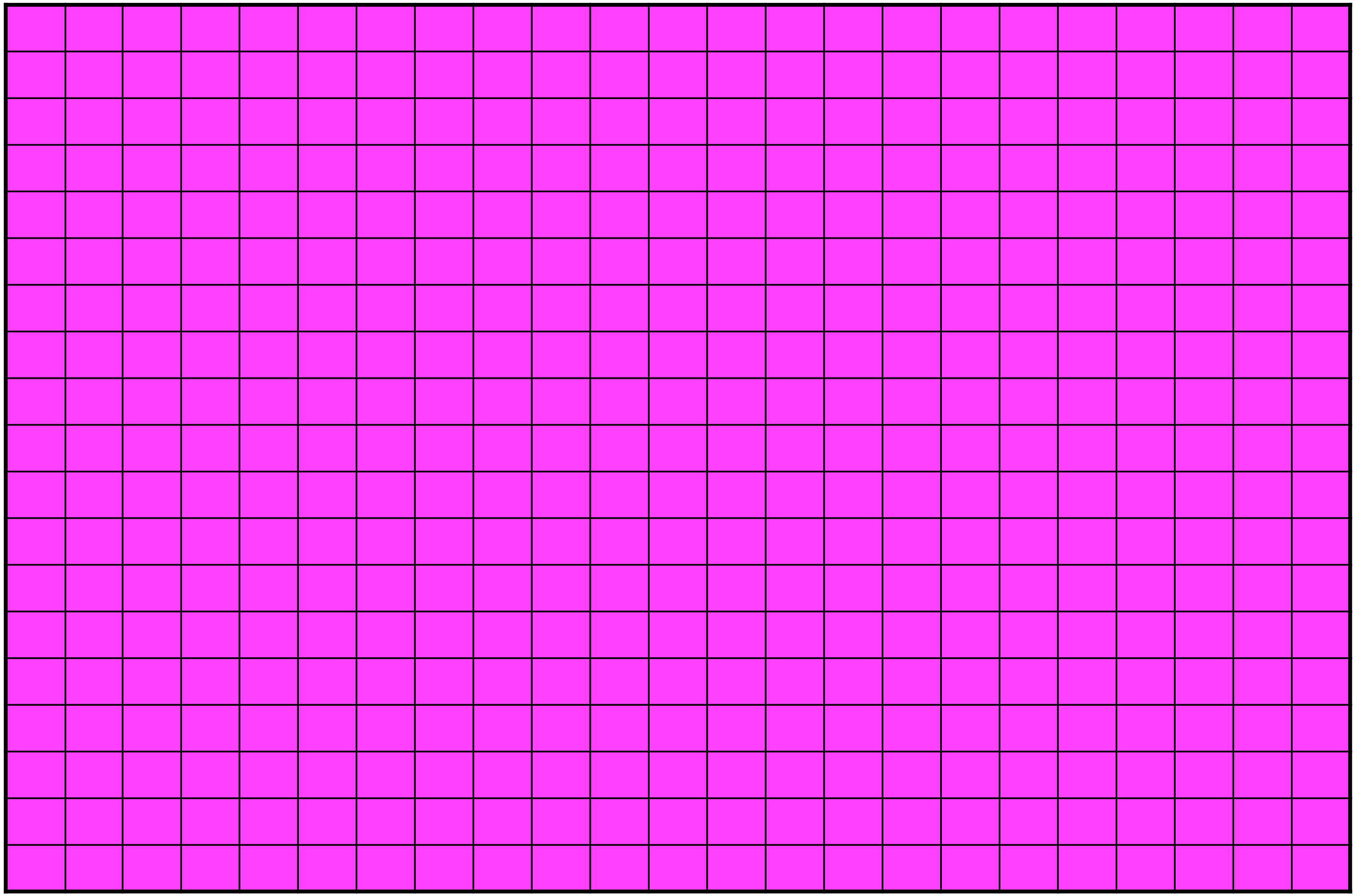
- Order
- Addition
- Subtraction
- Multiplication
- Division

Step through introduction to number lines - do not assume students have used them before.

Number lines are used later to illustrate similar properties for fractions, decimals and integers.

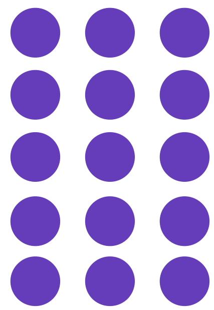
Multiplication - More than repeated addition

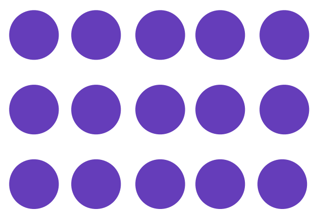
More than repeated addition

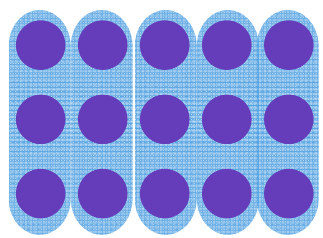


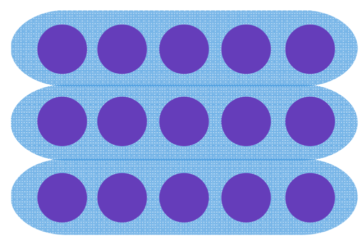
Multiplication- arrays

15



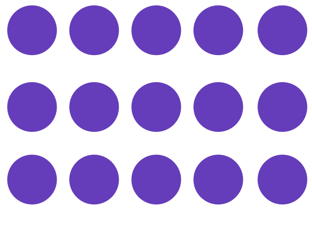


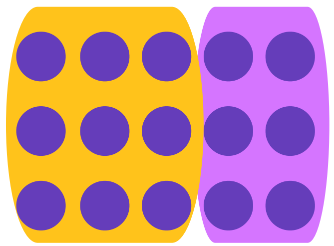


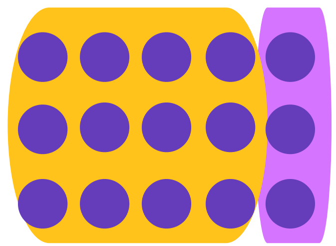


Multiplication- arrays

15

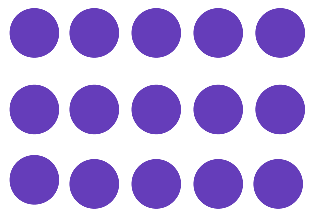






Multiplication- arrays

15



The factors of 15 are
3 and 5

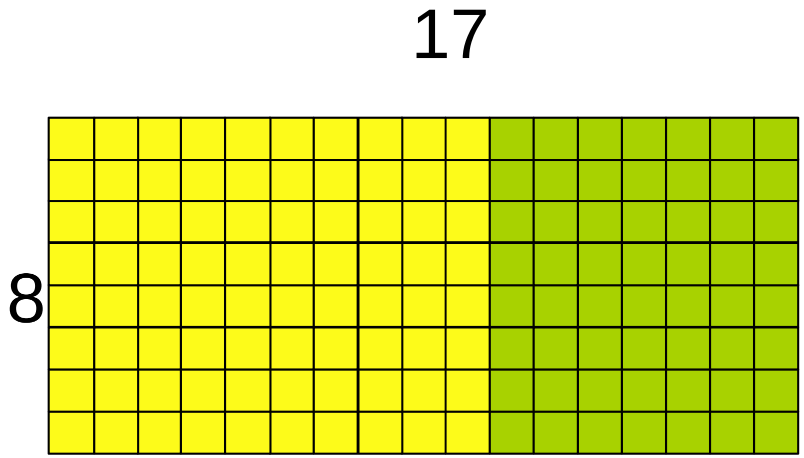
1 and 15

Multiplication

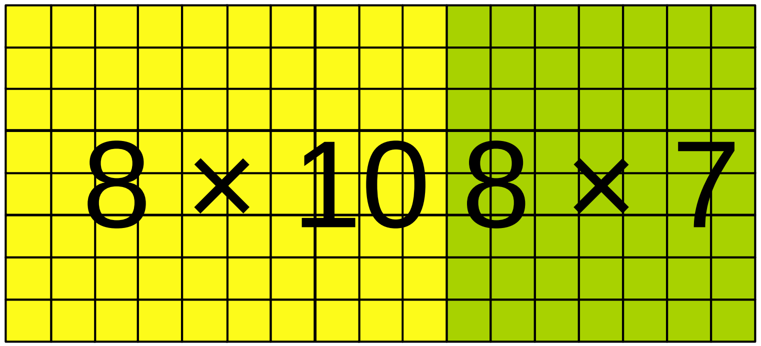
Linking arrays and areas with the multiplication algorithm.

For example, 8×17

Multiplication



Multiplication



$$8 \times 17 = 8 \times 10 + 8 \times 7$$
$$= 80 + 56$$
$$= 136$$

Multiplication

$$\begin{array}{r} {}^{5}1\ 7 \\ \times \quad\quad 8 \\ \hline 1\ 3\ 6 \\ \hline \end{array}$$

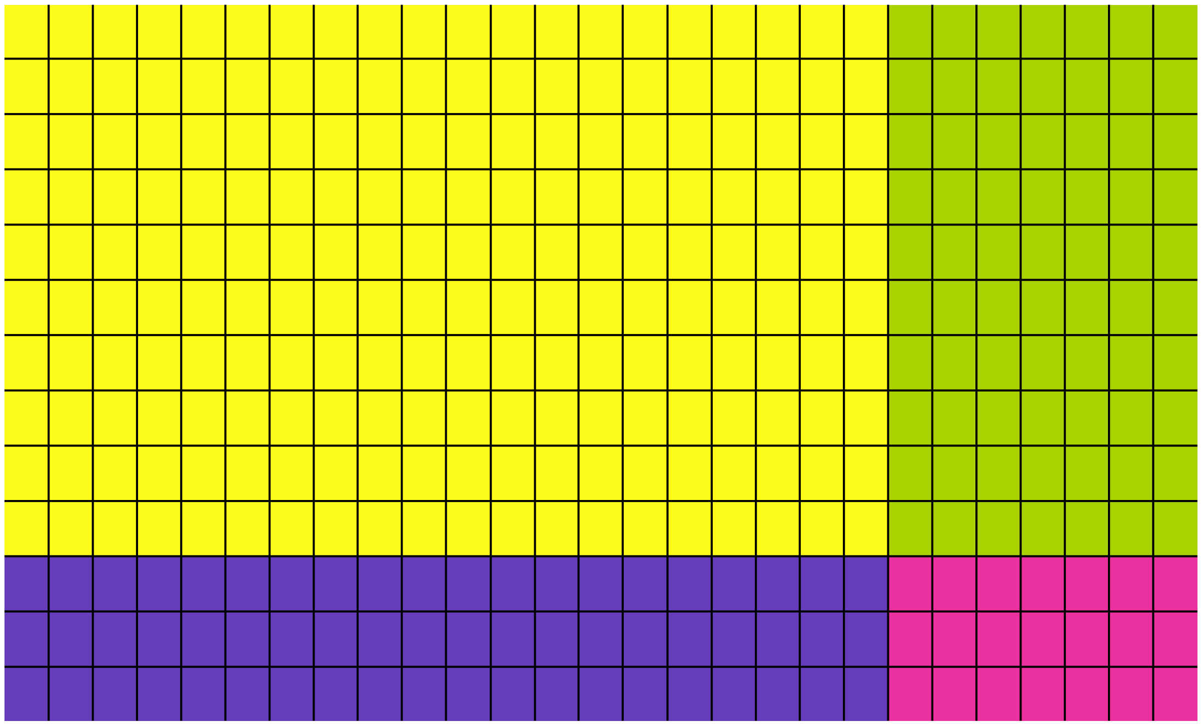
Multiplication

Linking arrays and areas with the long multiplication algorithm.

For example, 27 × 13

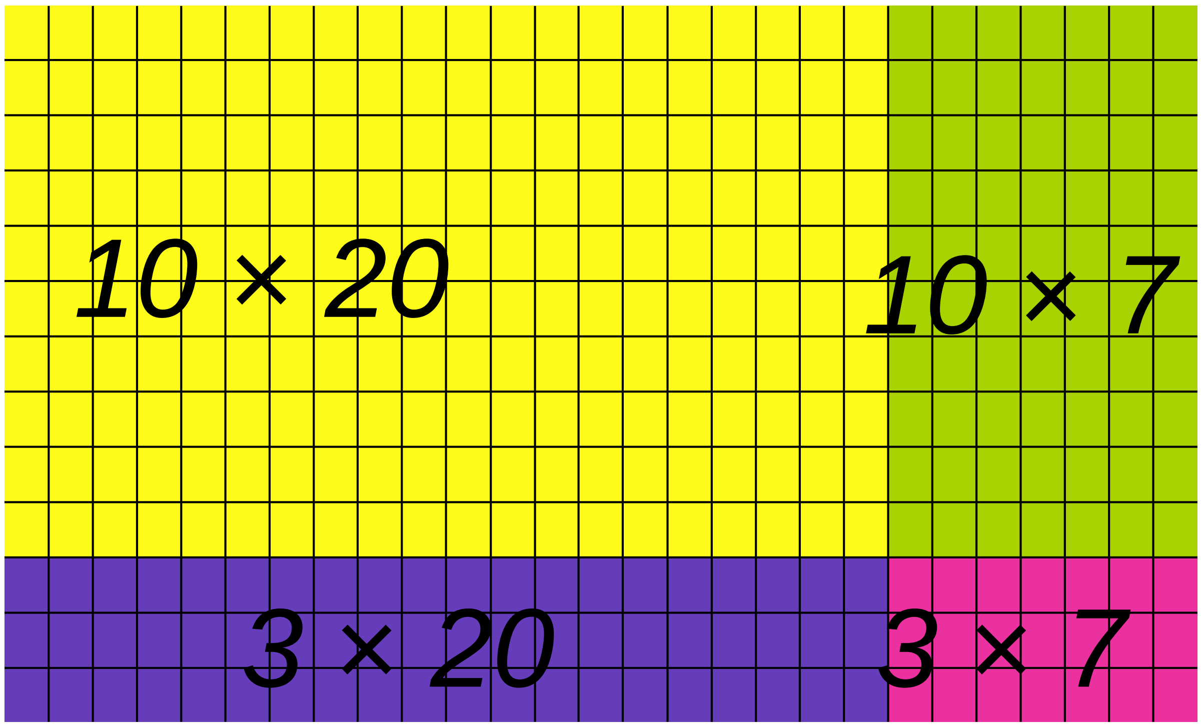
Multiplication

Draw an array



Multiplication

Highlight the chunks



Multiplication

10 times 7 is 70

10 times 20 is 200

Algorithm

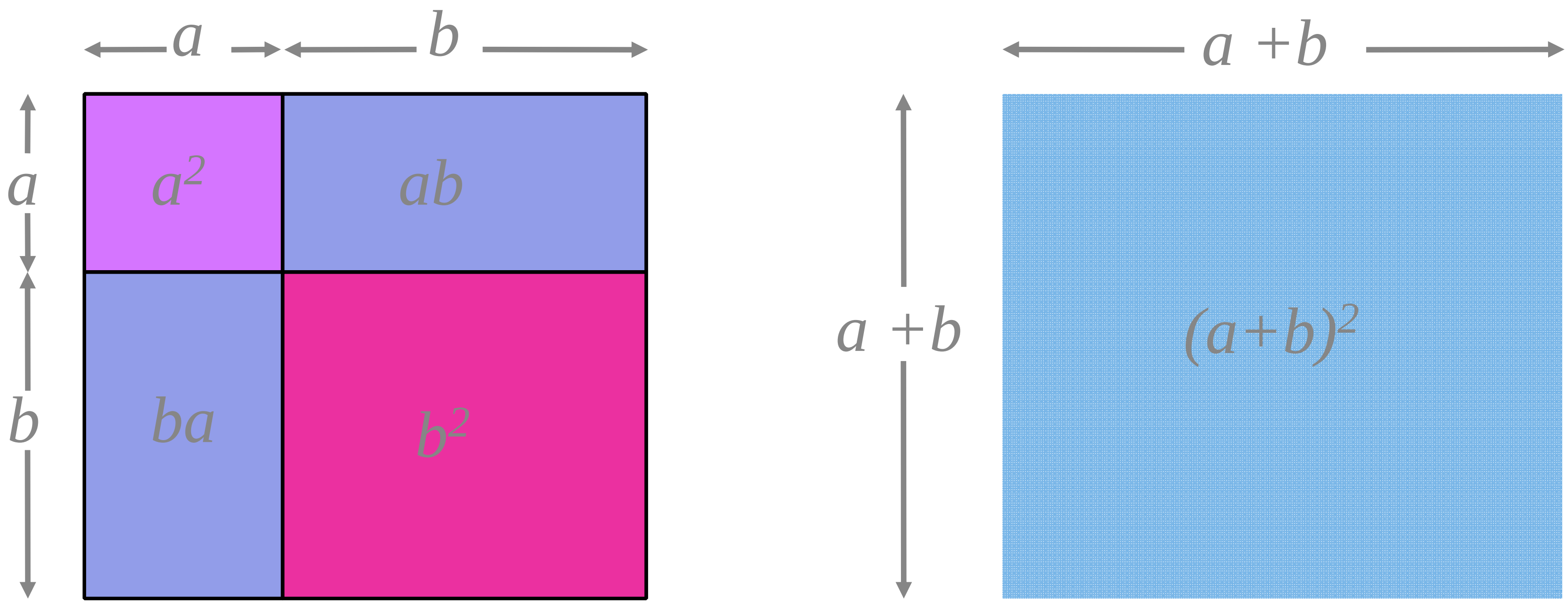
$$\begin{array}{r} {}^{2}27 \\ \times \quad \underline{13} \\ 81 \\ \underline{270} \\ \underline{351} \end{array}$$

3 times 7 is 21

3 times 20 is 60 plus 20 is 80

- Area model and multiplication are fundamentally linked. It can be used in the introduction of algebra but must be based on a sound understanding of area and multiplication with numbers.

Algebra - Area



$$(a+b)^2 = a^2 + 2ab + b^2$$

Mental arithmetic - Multiplication and division

It is all about using and understanding the most efficient methods.

Handy to have good numbers for students to use these strategies on.

Order of Operations

▪

Number Shapes

Concept Areas

Algebra. Solving simultaneous equations using manipulatives, this time in symbolic number sentences.

For Each Group:

- The number shapes mat from the facing page.
- Manipulatives for the students to place on the mat to represent the solution. The manipulatives should be relatively small, so they can fit. The biggest number in any solution is ten.

Description

These problems are abstract, using shapes—squares, triangles, and stars—to represent numbers. Students can represent proposed solutions by placing beans on the mat, and then easily change their minds by moving the beans around. Ideally, each student becomes responsible for seeing to it that his or her equation is satisfied. When all the students agree, the problem is solved.

While the first two problems are accessible to students in early elementary school, the last is definitely a problem for middle school and beyond.

Other Comments

These problems help further students' understanding of variables.

In *Kids With Stuff*, students solved simultaneous equations that arise out of situations. This family is more abstract, so in that sense, harder. Yet students traditionally have more trouble with "word problems" than with traditional "technique" problems. So which of these sets should you do first?

Perhaps you should mix them. Word problems make you represent a concrete situation in the language of mathematics—they are problems in abstraction. When students learn techniques without a context—divorced totally both from situations and the meaning of the numbers—they can exhibit what may be the most damning set of skills: mastery of technique without the ability to abstract. They may do well in computation, but they can't think.

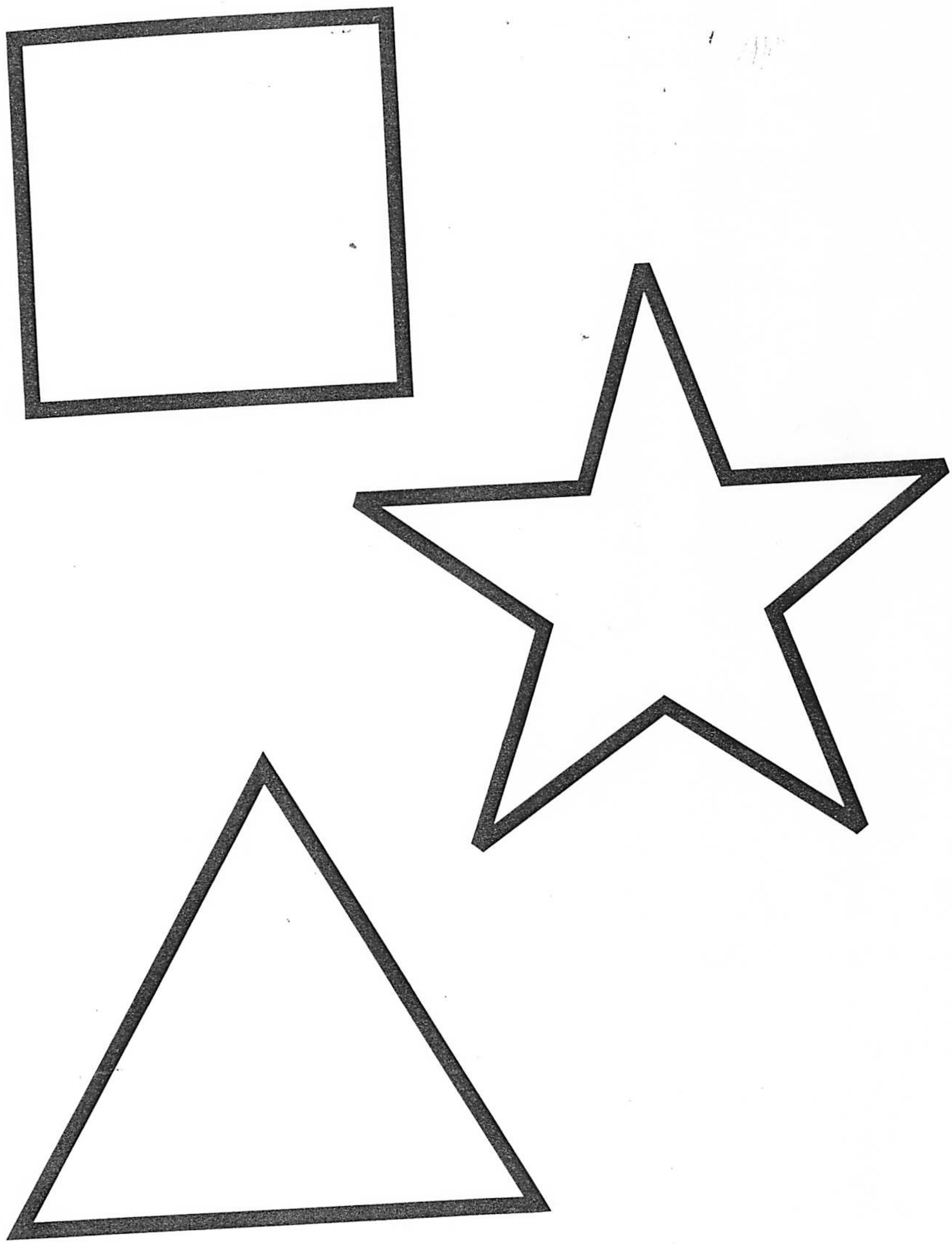
Note Well

There are only three regular clues and one optional for each of these. (There are only three shapes, so you can get a solution with three clues.) Some teachers let students use the extra, blank clues to make up their own.

Possible Debriefing Questions

Which did you find easier—these problems or the ones in *Kids With Stuff*? Why?

How did your group use the manipulatives?



from *Get It Together* • EQUALS, Lawrence Hall of Science

Number Shapes 1

△ + ☆ = 3

Number Shapes 1

☆ + ☆ = △

Number Shapes 1

△ + □ = 6

Number Shapes 1

□ = 4

Number Shapes 1

Number Shapes 1

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Number Shapes 2

$☆ + \triangle = 9$

Number Shapes 2

$☆ - \square = 2$

Number Shapes 2

$\square + \triangle = ☆$

Number Shapes 2

$\square + 1 = 6$

Number Shapes 2

Number Shapes 2

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Number Shapes 3

$$\square + \square + \triangle = \star$$

Number Shapes 3

$$\triangle + 1 + \triangle = \square$$

Number Shapes 3

$$\square + \star + \triangle = 11$$

Number Shapes 3

$$\star - \square = 4 \times \triangle$$

Number Shapes 3

Number Shapes 3

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Number Shapes 4

$3\star + 1 = \square$

Number Shapes 4

$\square - 2\triangle = 2$

Number Shapes 4

$$\frac{\triangle - 1}{\star} = \frac{\square}{\square}$$

Number Shapes 4

$2\star + \triangle = \square$

Number Shapes 4

Number Shapes 4

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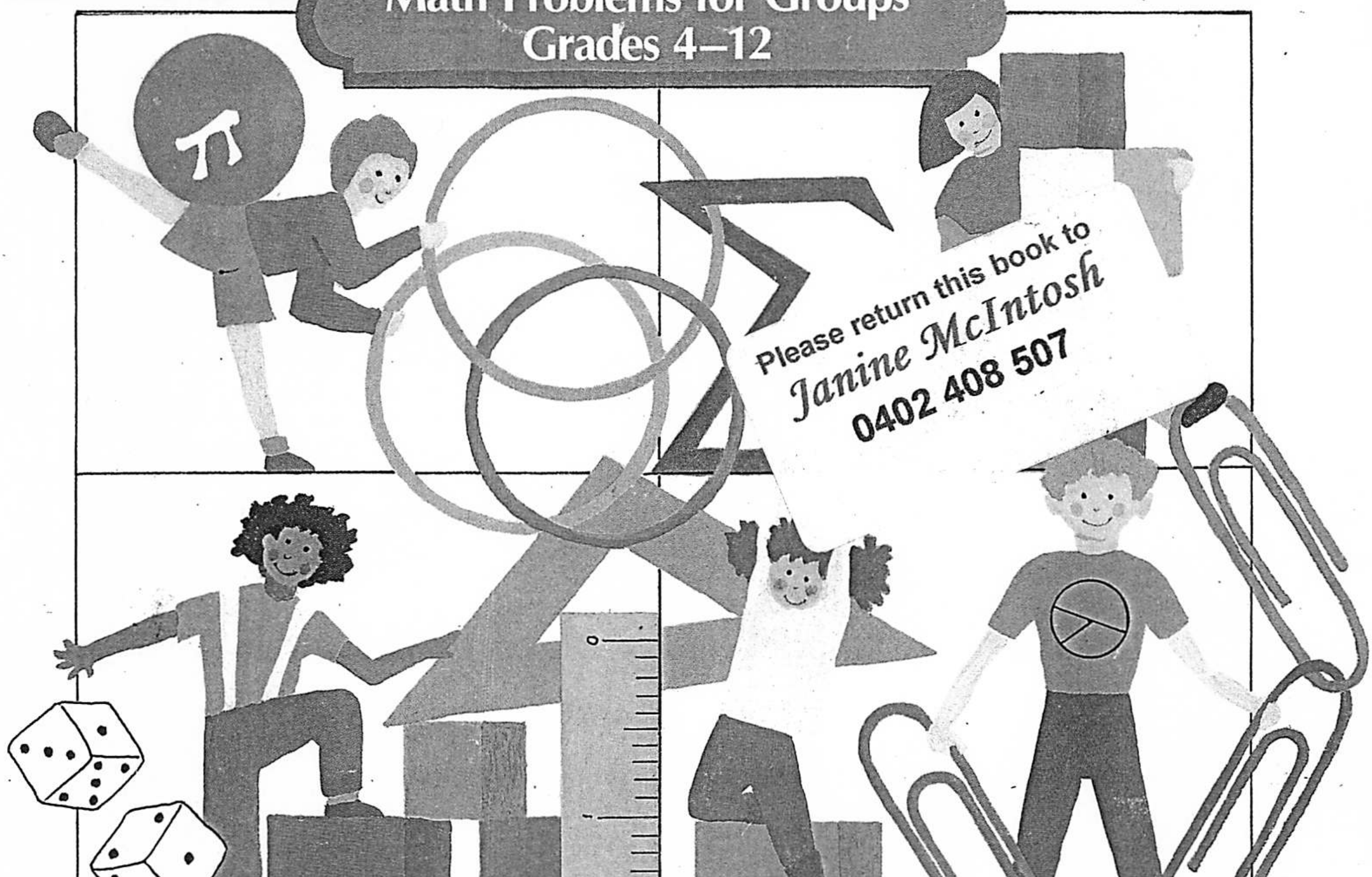
GET IT TOGETHER

Math Problems for Groups
Grades 4–12



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