ACE Network Subject Information Guide

Methods and Theory of Modern Optimisation

Semester 1, 2024

Administration and contact details

Host department	Mathematical Sciences, School of Science
Host institution	RMIT University
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Subject details

Handbook entry URL	**NA**
Subject homepage URL	**(will be on MoCaO website with password protection)**
Honours student hand-out URL	**On the MoCaO webpage**
Teaching period (start and end date):	**04/03/2024 – 07/06/2024**
Exam period (start and end date):	**10/06/2024 – 28/06/2024**
Contact hours per week:	2 hours lectures + 1 hour online consultation
ACE enrolment closure date:	**31st March 2024**
Lecture day(s) and time(s):	**Tuesdays and Thursdays (times TBA)**
Description of electronic access arrangements for students (for example, LMS)	**There will be a webpage hosted on the MoCaO website and a dropbox folder for access to files and for submission of assessment.**

Subject content

1. **Subject content description**

Nonlinear and convex optimisation has gained wide use in data mining and machine learning in recent years. This has spawned new research and renewed interest in the underlying mathematical foundations behind descent methods. The main tool of machine learning is stochastic gradient descent, which corresponds to a statistically unbiased approximation of a deterministic descent method and is used widely in regression and classification problems. Thus, a renewed interest in first order gradient-based convex optimisation has arisen out of the study of this and similar problems. The nonsmooth version arises in other problems found in Machine learning, such as the regularised risk minimisation with binary hinge loss. In the area of signal processing, similar nonsmooth problems are faced when recovering corrupted signals with sparse support (compressed sensing). Moreover, optimisation under uncertainty also draws its techniques from similar sources, with methods used to solve stochastic optimisation problems exploiting decomposition methods to create parallel programming approaches based on constrained convex optimisation, such as the so-called alternating direction method of multipliers (ADMM). Other feasibility methods based on projection-based methods also can be viewed as arising out of special cases of these general methods.

In the course we will take a modern view of the study of these and related problems, and the algorithms used to solve them. You will be introduced to the language of convex and nonsmooth optimisation, and shown how this powerful mathematical machinery allows one to analyse optimisation problems and develop algorithms for the solution of these problems. We will study the application of ADMM to various problem sets, including stochastic optimisation and feasibility problems. The ability to apply these techniques within the hyperspace of symmetric matrices allows the same methods to be applicable to a wider class of problems, and this aspect will also be explored.

Ultimately, we will look at the use of these techniques to some of the areas of application discussed above. Depending on the background of the student, scope is available in this course for projects involving theoretical work and/or more practical work involving the coding of methods.

- **Week-by-week topic overview**

Week 1: Convexity Preserving Operations, inner product spaces, symmetric matrices and Frobenius norm.
Week 2: Supports, Separation, relative interior and Subdifferentiability
Week 3: Fenchel Conjugate, Fenchel duality and Application in Optimization
Week 4: Lagrangian Duality and Penalty methods
Week 5: Introduction to Positive Semi-Definite problems.
Week 6: Simple subgradient methods, step size for methods and convergence

Week 7: Using subgradient methods in LPs and LR. Improvements when we have a gradient.
Week 8: The method of multipliers and alternating direction method of multipliers (ADMM).
Week 9: Using ADMM: Linear and Quadratic Programming Convex optimisation and Alternating Projections.
Week 10: Stochastic optimisation. Feasibility problems i.e. matrix completion
Week 11: Applications in Machine learning
Week 12: Inverse problems and Compressed sensing

2. **Assumed prerequisite knowledge and capabilities**

   Necessary: Any under-graduate courses in real analysis, basic vector calculus and linear algebra.

   Desirable: Familiarity with a computer programming language like Matlab, Julia, Python, etc.

3. **Learning outcomes and objectives**

   On completion of this course student should have gained:

   • Knowledge of advanced mathematical techniques in convex optimisation;
   • The ability to reformulate problems and identify and/or develop numerical algorithms for the solution of these problems;
   • Exposure to important classes of real world problems in areas like stochastic optimisation problems, machine learning and signal processing.

AQF specific Program Learning Outcomes and Learning Outcome Descriptors (if available):

AQF Program Learning Outcomes addressed in this subject	Associated AQF Learning Outcome Descriptors for this subject
Knowledge of advanced mathematical techniques in convex optimisation	K1 (knowledge)
The ability to reformulate problems and identify and/or develop numerical algorithms for the solution of these problems	S1 (Skills)
Exposure to important classes of real work problems in areas like machine learning and signal processing	S2 (Skills), A2 (Application of Skills)

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Learning Outcome Descriptors at AQF Level 8
Knowledge
K1: coherent and advanced knowledge of the underlying principles and concepts in one or more disciplines
K2: knowledge of research principles and methods
Skills
S1: cognitive skills to review, analyse, consolidate and synthesise knowledge to identify and provide solutions to complex problem with intellectual independence
S2: cognitive and technical skills to demonstrate a broad understanding of a body of knowledge and theoretical concepts with advanced understanding in some areas
S3: cognitive skills to exercise critical thinking and judgement in developing new understanding
S4: technical skills to design and use in a research project
S5: communication skills to present clear and coherent exposition of knowledge and ideas to a variety of audiences
Application of Knowledge and Skills
A1: with initiative and judgement in professional practice and/or scholarship
A2: to adapt knowledge and skills in diverse contexts
A3: with responsibility and accountability for own learning and practice and in collaboration with others within broad parameters
A4: to plan and execute project work and/or a piece of research and scholarship with some independence

4. Learning resources

Students will be provided with a full set of lecture notes that are typeset in Latex. Preferred computing platforms include Matlab or Python.

5. Assessment

Exam/assignment/classwork breakdown					
Exam/End of year project	40%	**Weekly Assignments**	60%		
Assignment due dates		**Weekly on weeks 2 to 12**		Click here to enter a date.	Click here to enter a date.
Approximate exam date				**Weeks 13 to 15**	

Institution honours program details

Weight of subject in total honours assessment at host department	12.5%
Thesis/subject split at host department	25%thesis / 75% course work
Honours grade ranges at host department	
H1	80 - 100%
H2a	75 - 79%
H2b	70 - 79 %
H3	60 - 79%