



## ACE Network Subject Information Guide

### Algebraic Number Theory

Semester 2, 2023

#### Administration and contact details

Host department	School of Information and Physical Sciences
Host institution	University of Newcastle
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#### Subject details

Handbook entry URL	TBC
Subject homepage URL	TBC
Honours student hand-out URL	TBC
Teaching period (start and end date):	17 July – 21 October 2023
Exam period (start and end date):	30 October – 10 November 2023
Contact hours per week:	2
ACE enrolment closure date:	17 July 2023
Lecture day(s) and time(s):	TBC
Description of electronic access arrangements for students (for example, LMS)	Access will be arranged for the relevant Canvas page.

## Subject content

### 1. Subject content description

Number theory is the study of the integers. However, in order to study questions about the integers, one is often forced to study more general sets numbers. For example, in order to determine which prime numbers can be written as the sum of two squares,  $p = x^2 + y^2$ , one really needs to consider numbers of the form  $x + iy$ , which are irrational but algebraic.

Algebraic Number Theory is thus the study of algebraic numbers (i.e. solutions to polynomial equations with integer coefficients). These are elements in number fields, i.e. finite extensions of the field of rational numbers, for example  $\mathbb{Q}(i)$ , the Gaussian numbers.

Each such number field contains a subring of algebraic integers (e.g.  $\mathbb{Z}[i]$ , the Gaussian integers in our example above) and we're interested in arithmetic in these rings of algebraic integers. In the Gaussian integers, every element can be factorised uniquely into a product of prime elements. However, in many other examples, this unique factorisation fails, for example in the ring  $\mathbb{Z}[\sqrt{-5}]$ .

This obstacle can be overcome by moving from elements to ideals – it turns out that every ideal in an algebraic number ring can be factorised uniquely into a product of prime ideals.

This is the starting point for a very rich theory in which we will study these rings (more precisely, a class of rings called Dedekind rings), their groups of units and ideal classes modulo principal ideals, which form a finite group called the ideal class group.

Number theory is famous for borrowing techniques from all other branches of mathematics. The exact topics we will study will determine the techniques we will use, and will be decided based on the interests and backgrounds of the students.

### 2. Week-by-week topic overview

Topics covered will include the following:

- Number fields
- Dedekind rings, unique factorisation of ideals into prime ideals
- Minkowski's geometry of numbers and finiteness of the ideal class group
- Dirichlet's Unit Theorem.



**Learning Outcome Descriptors at AQF Level 8**

**Knowledge**

K1: coherent and advanced knowledge of the underlying principles and concepts in one or more disciplines

K2: knowledge of research principles and methods

**Skills**

S1: cognitive skills to review, analyse, consolidate and synthesise knowledge to identify and provide solutions to complex problem with intellectual independence

S2: cognitive and technical skills to demonstrate a broad understanding of a body of knowledge and theoretical concepts with advanced understanding in some areas

S3: cognitive skills to exercise critical thinking and judgement in developing new understanding

S4: technical skills to design and use in a research project

S5: communication skills to present clear and coherent exposition of knowledge and ideas to a variety of audiences

**Application of Knowledge and Skills**

A1: with initiative and judgement in professional practice and/or scholarship

A2: to adapt knowledge and skills in diverse contexts

A3: with responsibility and accountability for own learning and practice and in collaboration with others within broad parameters

A4: to plan and execute project work and/or a piece of research and scholarship with some independence

## 5. Learning resources

The course will follow lecture notes by Matthew Baker

(<https://sites.google.com/view/mattbakermath/publications#h.uuedvamjgl11>)

There are many good books on Algebraic Number Theory, I recommend in particular:

- Paul Pollack: "A Conversational Introduction to Algebraic Number Theory", AMS Student Mathematical Library, vol 84.
- Pierre Samuel: "Algebraic Theory of Numbers", Dover

More advanced textbooks include:

- Serge Lang: "Algebraic Number Theory", Springer Graduate Texts in Mathematics vol 110.
- Jürgen Neukrich: "Algebraic Number Theory", Springer
- James Milne: "Algebraic Number Theory", free course notes at <https://www.jmilne.org/math/CourseNotes/ant.html>

## 6. Assessment

Exam/assignment/classwork breakdown					
Exam	50 %	Assignment	50 %	Class work	0 %
Assignment due dates		28 August 2023	9 October 2023		
Approximate exam date				November 2023	

### Institution honours program details

Weight of subject in total honours assessment at host department	10 units of 80 total
Thesis/subject split at host department	40 units of 80 total
Honours grade ranges at host department	
H1	85 - 100 %
H2a	75 - 84 %
H2b	65 - 74 %
H3	50 - 64 %

### Institution masters program details

Weight of subject in total masters assessment at host department	Click here to enter text.
Thesis/subject split at host department	Click here to enter text.
Masters grade ranges at host department	
H1	Enter range %
H2a	Enter range %
H2b	Enter range %
H3	Enter range %