

# The 10th annual AMSI Winter School Algebra, Geometry & Physics

The University of Queensland 29 June – 10 July 2015





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# Introduction

he Winter School is one of AMSI's flagship higher education vacation schools and outreach programs. The Hon. Jane Prentice MP opened the 10th annual event on Monday, 29 June at the University of Queensland's Science Learning Centre.

The Winter School aims to enable participants to broaden and deepen their mathematical knowledge, and build collaborative networks with other PhD students and early career researchers. This year's theme was on Algebra, Geometry and Physics, aimed at post-graduate students and postdoctoral fellows in the mathematical sciences and cognate disciplines. Four programs were offered over two-weeks, with a combination of introductory and advanced topics:

- Moduli Spaces and Symplectic Geometry
- Geometric Representation Theory
- K-Theory and its applications
- Moonshine Conjectures and Vertex Operator Algebras

The lectures were given by well established senior researchers from Australia and abroad, as well as experts in the field, many of whom received their PhD's from worldclass institutions overseas (for example Yale University, Massachusetts Institute of Technology and Stanford University) and who have now settled relatively recently at various Australian universities.

## Program



# Moduli Spaces and Symplectic Geometry

#### WEEK 1

Joan Licata The Australian National University

This course developed some beautiful constructions that illustrated one role of moduli spaces in contemporary symplectic geometry.

The program started with an overview of classical Morse theory on finite-dimensional manifolds, describing some of the key results and examining the technical considerations that make the theory work, and introducing some basic objects in symplectic geometry in order to define Lagrangian intersection Floer homology.

Further discussion focused on explaining the sense in which this construction is a natural infinite-dimensional analogue of Morse theory, as well as Heegaard Floer homology, a package of topological invariants of low-dimensional manifolds, which can be defined using Lagrangian intersection Floer homology.

#### WEEK 2

Brett Parker The Australian National University

A beautiful formulation of classical mechanics goes by the name of Hamiltonian mechanics. In Hamiltonian mechanics, the phase space has the structure of a symplectic manifold. It was demonstrated that any volume preserving transformation of phase space may be approximated (in some weak sense) by a Hamiltonian mechanical system, and that symplectic manifolds are very flexible.

"The courses were, as always, top notch. I cannot imagine being a mathematics student in Australia without AMSI's Winter School."

John Snadden, The Australian National University

# **Geometric Representation Theory**

epresentation theory, the study of (for example) matrix representations of groups, Lie algebras or quantum groups, plays a major role in areas such as algebra, algebraic topology, differential geometry and mathematical physics. One of the major trends in modern representation theory has been the discovery of geometric constructions of representations, which arise by applying technology such as cohomology, K-theory or sheaf theory to spaces such as flag varieties, quiver varieties and nilpotent cones. The benefits of such constructions range from canonical bases for ease of computation to theoretical understanding of higher-level categorical structure.

Ideas were introduced in some of their simplest forms, concentrating on the prototypical case of the general linear group and its associated Lie algebras and quantum groups.

#### WEEK 1

Masoud Kamgarpour The University of Queensland

Tony Licata The Australian National University

Topics covered included:

- Classical representation theory of the general linear group and its Lie algebra
- Borel-Weil construction of representations using line bundles on the flag variety
- Nakajima's construction of representations using cohomology of quiver varieties

#### WEEK 2

Anthony Henderson The University of Sydney

Topics covered included:

- Quantization and affinization of Lie algebras
- Lusztig's geometric Hall algebra construction of quantum groups
- Nakajima's construction of representations of quantum loop algebras

"The lecturers at the Winter School were really open to talk about the content of the courses as well as their research and their field in general. They all seemed genuinely interested in discussing and engaging with the students."

Jonathan Belletete, University of Montreal

# K-Theory and its applications

#### WEEK 1

An Introduction to K-Theory Vigleik Angeltveit The Australian National University

Using ideas introduced by Grothendieck, Atiyah and Hirzebruch defined topological K-theory from the set of vector bundles on a space X by applying group completion (also known as the Grothendieck construction). K-theory forms a multiplicative cohomology theory with an obvious geometric interpretation, and for many purposes it is a more useful invariant than ordinary cohomology.

The course defined vector bundles, and generalised standard constructions such as direct sum, tensor product and exterior powers, from vector spaces to vector bundles. An explanation was given on how to define K-theory by considering formal differences of vector bundles. After establishing some formal properties of K-theory, the focus moved to the theory of characteristic classes.

The course concluded with an application to pure mathematics, using certain operations on K-theory, an easy proof was presented, due to Adams and Atiyah, of the Hopf Invariant One theorem. One easy consequence of this theorem is that there are no real division algebras in dimensions other than 1, 2, 4 and 8.

#### WEEK 2

Applications to Index Theory and Physics Pedram Hekmati **The University of Adelaide** 

The Atiyah-Singer index theorem is arguably one of the most significant achievements of 20th century mathematics. It draws on and bridges several branches of mathematics and its impact is felt strongly today. K-theory is the natural framework for proving the index theorem and historically these theories were developed in tandem. The aim of these lectures was to formulate the index theorem, sketch the K-theory proof and consider an application to physics.

The course began with an introduction to elliptic differential operators on compact manifolds and explored how their symbol determines a class in K-theory. Next, the notion of analytic index and topological index for elliptic operators was defined. The Atiyah-Singer index theorem is the statement that these two index maps are equal.

After explaining the basic idea behind the proof and deriving a cohomological formula, an application to symmetry breaking and particle physics was considered. The background in gauge theory, the mathematical theory underpinning particle physics was reviewed, and an explanation how the breaking of local gauge symmetry can be described by the index theorem.

"I most enjoyed the high quality of the lectures and the enthusiasm of all students who participated actively in the learning experience."

Pedram Hekmati, The University of Adelaide

# Moonshine Conjectures and Vertex Operator Algebras

#### WEEK 1

A General Introduction to Moonshine Nora Ganter **The University of Melbourne** 

An introduction was given on the the topic of moonshine, and generalized moonshine, with special attention on different interpretations of the role of phase factors (line bundle over moduli space and the categorical picture). Replicability, Hecke operators, Hecke monicity and Carnahan's approach to the genus zero property was discussed.

#### WEEK 2

*Newer Moonshines* Terry Gannon, **University of Alberta, Canada** 

The course reviewed the background on modular forms and finite group representation theory, and explored newer moonshines, including the Mathieu Moonshine.

#### WEEK 2

Introduction to Vertex Operator Algebras Geoffrey Mason **University of California, Santa Cruz, U.S.A.** 

Vertex Operator Algebras facilitate the construction of the Moonshine Module, and are valuable tools in 2-d Conformal Field Theory, String Theory and a variety of mathematical applications. Vertex Operator Algebras were introduced, demonstrating some of the fascinating results and applications associated with their structure.

Topics covered included:

- Axioms for vertex algebras: Definition of a vertex k-algebra, category of vertex k-algebras, modal endomorphisms, translation covariance, locality.
- Existence theorems: Field-theoretic characterizations, Heisenberg algebra (free-field theory), Virasoro algebra, vertex operator algebras (VOA), Heisenberg VOA, Virasoro VOA, VOAs associated to affine Lie algebras
- Characters and representations: Partition functions, modules over a VOA, finiteness theorems, modular functions, modular-invariance theorems, connections with Monstrous Moonshine



# Participation Statistics

Enrolments by Institution		
Monash University	1	
The Australian National University	9	
The University of Adelaide	1	
The University of Melbourne	3	
The University of New South Wales	2	
The University of Queensland	8	
The University of Sydney	9	
The University of Western Australia	1	
University of Cambridge	1	
University of Montreal	1	
University of Wollongong	2	
Other	1	
Total	39	



23%	ACI
33%	NSW
20%	QLD
3%	SA
10%	VIC
3%	WA
5%	International
3%	Undisclosed

		Breakdown by Residency Status
7	61%	Australian Citizen
	13%	Permanent Resident
	23%	Student Visa
	3%	Other

	Breakdown by Academic Status
679	7% PhD
80	3% Honours
159	5% Masters
20	2% Academic
30	8% Early Career Researcher
5°	5% Other





## **Event Highlights**



#### Women in Maths Networking Event

AMSI Winter School 2015 hosted a very successful Women in Maths Networking Event on Thursday, 2nd July with over 80 people attending. The event was held in conjunction with the AustMS Women in Mathematics Special Interest Group (WIMSIG). The event supports the network and raises awareness about issues for women in mathematics. This year, female mathematicians from The Australian National University, SilverRail, National Australia Bank and The University of Melbourne opened the event by talking about their experiences in studying Mathematics and their subsequent careers. The presentations were followed by lively discussion among attendees.

"One of the real highlights again this year was the Women in Maths Networking Event."

> Professor Peter Bouwknegt, Director of the Mathematical Sciences Institute The Australian National University

#### **Travel Grants**

AMSI offers travel grants for students to attend higher education flagship events. This year, 20 students received full travel awards, and 4 students received partial travel awards to attend the AMSI Winter School 2015 in Brisbane.

#### Welcome BBQ

A welcome BBQ was held at Emmanuel College on the evening before the conference started. This was a great opportunity for the participants to register, pick up their delegate bags, and to meet each other ahead of the Winter School.

#### **Participant talks**

Winter School attendees presented a 15 minute talk on the area of their research (or related field), linked to the theme of the school.

These participant talks give participants experience in presenting to their peers, and an opportunity to find out about related research, prompting discussion among the group.

A slightly competitive edge is added, with students voting for the best presentations within a small group, and talk finals held in the second week, with a book prize up for grabs!

Congratulations to winner Brett Chenoweth from The University of Adelaide for his talk entitled *"Flexibility and Rigidity in Complex Analysis"*.

#### **Queensland Brain Institute tour**

For the second consecutive year, Professor Geoff Goodhill from School of Mathematics and Physics and Queensland Brain Institute (QBI) at the University of Queensland, invited participants on an interactive tour of the research facilities at QBI.

#### **Social Dinner**

The Winter School social dinner provided an informal environment for participants and lecturers to socialise and relax at the end of the first week.

#### **Conference Dinner**

The conference dinner is hosted by the Winter School team to celebrate the end of another event. This year, Tom Forbes, cofounder of Biarri, gave a well received talk on the significance of mathematics in the workplace and internship opportunities in commercial mathematics. Speeches from Professor Kerry Mengersen (ACEMS) and Dr Graham Chen (QCIF) were also highlights throughout the night.



### **Public Lecture**





Professor Arun Ram The University of Melbourne

he AMSI Winter School Public Lecture is run annually in conjunction with BrisScience, and engages a broad audience linked to the subject matter of the Winter School program.

This year, Professor Arun Ram hosted a virtual tour of a toy store with friends Maria Callas, Alexander Grothendieck and Hermann Hesse. This showed the audience pleasant games with glass beads, athletic games skiing the moguls, and violent games where everything gets smashed. There were crystals, hurricanes and, of course, a few polynomials. Throughout the lecture, Arun shared some stories related to his current research in symmetry, and the topic of the concurrent International Centre for Mathematical Sciences research workshop on the Algebraic Lie Theory.

Arun Ram grew up in a small town in New Mexico, before moving to Boston to attend university at MIT. After deciding that he needed a lifestyle that enabled him to travel and sit in coffee shops, he found it best to get a PhD in Mathematics. After obtaining his PhD from University of California, San Diego, he progressed through a sequence of junior positions before landing a position at The University of Wisconsin in 1999. After just under a decade, in 2008, he moved to The University of Melbourne where the mathematics, the weather, and the city suit him well.

# Student Profile

# Time, Space and Mathematics

rom time travel, wormholes and warping space to the mathematics of bubbles. Two-time AMSI Winter School attendee, Ross Ogilvie followed a childhood passion for science fiction into the mind-bending world of differential geometry.

At seven, with no idea what a mathematician was, Ross wanted to be a scientist. Now the avid hiker and rock climber who admits to an obsession with computers, is studying the mathematics behind general relativity to help describe spaces and their geometries. When asked to explain his work, he uses bubbles, not the familiar glossy spheres from childhood, but instead harmonic tubes (tubes with as little surface area as possible).

"If you look at the same equations that describe harmonic surfaces in other geometric spaces (not the type of 3D space we live in) then the bubble can assume all sorts of interesting shapes. I'm trying to classify certain types of bubbles and determine which ones can be deformed (continuously bent and stretched) into one another," Ross explains.

While primarily a piece of pure mathematics, this work forms part of a broader family of equations. These have a number of scientific applications including particle physics, the study of matter, the nature and make up of particles and the laws that control the physical universe.

Given this year's theme, Algebra, Geometry and Physics, it should come as no surprise the University of Sydney PhD student made the trek to AMSI Winter School for a second time in 2015. Having also attended in 2012, it is an experience he is quick to recommend to other students.



Ross Ogilvie The University of Sydney

"The same thing attracted me to Winter School both times, the impressive courses that were available. The first time, I was at the beginning of my PhD and figuring out where to take my research. I was really excited to have the opportunity to explore so many different fields and the questions within them."

Now in the late stages of his studies and with a clearer sense of direction and understanding, Ross discovered a different side to the program in 2015.

"This year I mastered some really clever techniques and saw how they were applied to active questions of the field. I came away with great insights into how experts across the field thought about the problems they were tackling, which gave me ideas for my work." He said.

While still considering the future beyond his PhD, for Ross one of the greatest challenges facing mathematicians is the need to see what they do to the broader community and incorporate it into society in a way that is seen as useful and appealing. A challenge, he believes AMSI is vital to addressing.

"When I tell people what I am studying, it is alarming how often they say 'oh I hated mathematics at school'. I can't think of another profession that elicits such a reaction. That's what makes AMSI so important. It does a great job at being two different things: both a link between mathematics and the outside world (industry and the broader community) and a mixer within the mathematics community."



#### Overall, the Winter School was well organised

STRONGLY AGREE

AGREE

NEUTRAL

DISAGREE

STRONGLY DISAGREE





I would recommend this event to others	
STRONGLY AGREE	
AGREE	
NEUTRAL	
DISAGREE	
STRONGLY DISAGREE	

#### Director's Report

The AMSI Winter School 2015 was a collaboration with the Australian National University Mathematical Sciences Institute and was part of the Special Year of Geometry and Physics program of events. Professor Peter Bouwknegt, Director of the ANU Mathematical Sciences Institute was integral in identifying and recommending speakers in the various topics.

nce again, the AMSI Winter School provided an excellent learning opportunity for Australia's Mathematical Sciences students to advance their subject knowledge and to make connections with their peers. The school attracted a wide range of students from across Australian Universities, and two international students from University of Cambridge and the University of Montreal. It should be noted that the school is attracting more overseas self funded participants as its reputation for excellence continues to grow.

The Algebra, Geometry and Physics theme appealed to a wide student base, including students from physics. The participant contingent came from a number of different universities across the country, which is an excellent indicator of how well recognised the AMSI Winter School has now become. The school also attracted three paying participants from overseas – the two students highlighted above, as well as one visiting academic from the Australian National University.

Students really enjoyed being exposed to fields outside of their specialised research area, and found it a good networking opportunity.

One of the real highlights again this year was the Women in Maths Networking Event. We chose to go with a panel discussion this year, which proved to be highly entertaining. Having speakers from various backgrounds contributed to a good opportunity for attendees to learn about the various



career paths and opportunities for women in the field of mathematics.

The second notable highlight was the Public Lecture, given by Professor Arun Ram from The University of Melbourne at the State Library. Arun is a very dynamic guest speaker and the topic was both informative and entertaining. The supper provided by the UQ Science Faculty and BrisScience allowed for plenty of discussion and networking after the event.

The AMSI Winter School was a great success on many levels and has proven to be an outstanding opportunity for Australia's next generation of mathematicians and other cognate researchers.

> Phil Isaac AMSI Winter School 2015 Director The University of Queensland

# Beautiful maths, beautiful physics

BRISBANE, QUEENSLAND

Friday, 26 June 2015

When someone says they are a mathematician or describes their research, it is natural to wonder why it is useful. Why do we need it? How will it affect me?

What may seem like an abstract study today may end up being part of the cure for cancer tomorrow or new wi-fi technology in five years.

The 10th annual AMSI Winter School, at The University of Queensland in Brisbane, reminds us of the importance of theoretical mathematical research — that beautiful mathematics often turns out to be useful mathematics.

Mathematicians in the 1860s were not thinking about computer graphics when studying two-dimensional differential geometry. And in 1822, how could Joseph Fourier have known his research into heat flow would transform the way we process, store and transmit information. This led to a transformation in the way we live as profound as that caused by the Industrial Revolution. It has also resulted in huge advances in medical diagnostic therapies such as MRI and PET.

As in the 1800s, humans today cannot see into the future; we cannot begin to imagine the infinite possibilities discoveries in fundamental mathematics may have in centuries to come.

The famous astronomer and polymath Galileo Galilei said that the book of nature is written in the language of mathematics. So, by developing an understanding of symmetry, structure, geometry and other mathematical constructs we may be able to reveal the patterns of nature.

Einstein's 1915 theory of general relativity asserted that the presence of mass distorts the geometry of space and time in a way described by the mathematics developed by Bernhard Riemann sixty years earlier. A critical experimental test of this geometrical theory of gravity required the occurrence of a solar eclipse.

While the development of physics and mathematics may proceed along different paths, each fundamental theory in physics has a corresponding specific mathematical structure, for general relativity this is Riemannian geometry and for quantum mechanics it is the Hilbert space.

These descriptions of nature are works of mathematical beauty and affect our everyday lives. We couldn't decode the human genome, build aeroplanes or have millions of people talking on their phones across the world simultaneously without mathematics.

## A Winter School on Algebra, Geometry and Physics to grow tomorrow's Einsteins

The AMSI Winter School gives Australian students the chance to expand their skills in the mathematical sciences and build collaborative networks with other students and early career researchers, while focusing on Algebra, Geometry and Physics at this years event. They learn from leading international experts in the field, from United States and Canada, as well as domestic experts from across the nation.

The event also hosts a Women in Maths evening designed to highlight the contribution of women in mathematics and provide a forum for discussion of career paths, as well as a Public Lecture, featuring Professor Arun Ram and his current research in symmetry.



AMSI wishes to acknowledge the generous donation of time and scientific advice of the following committees - without their contribution this event would not be a success.

#### **Organising Committee**

Phil Isaac University of Queensland AMSI Winter School 2015 Director

Peter Bouwknegt Australian National University

Simi Henderson Australian Mathematical Sciences Institute

Jon Links University of Queensland

Andree Philips University of Queensland

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# ON BIOLOGICAL & ENVIRONMENTAL MODELLING

4-15 JULY | THE UNIVERSITY OF QUEENSLAND

OUR "SECOND BRAIN": MODELLING ITS DEVELOPMENT & DISEASE Kerry A Landman, The University of Melbourne

USING A.I., NETWORKS THEORY & BUTCHERS PAPERS TO CONSERVE SPECIES Eve McDonald-Madden, The University of Queensland

THE MATHEMATICAL MODELLING OF CHEMOTAXIS Graeme Pettet, Queensland University of Technology

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THE DYNAMICS OF CALCIUM: THE INTERACTION OF MODELLING & EXPERIMENTS James Sneyd, The University of Auckland

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IMAGE: OVERLAPS BETWEEN KNOWN BIOLOGICAL PROCESSES VILLE-PETTERI MÄKINEN. SAHMRI

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