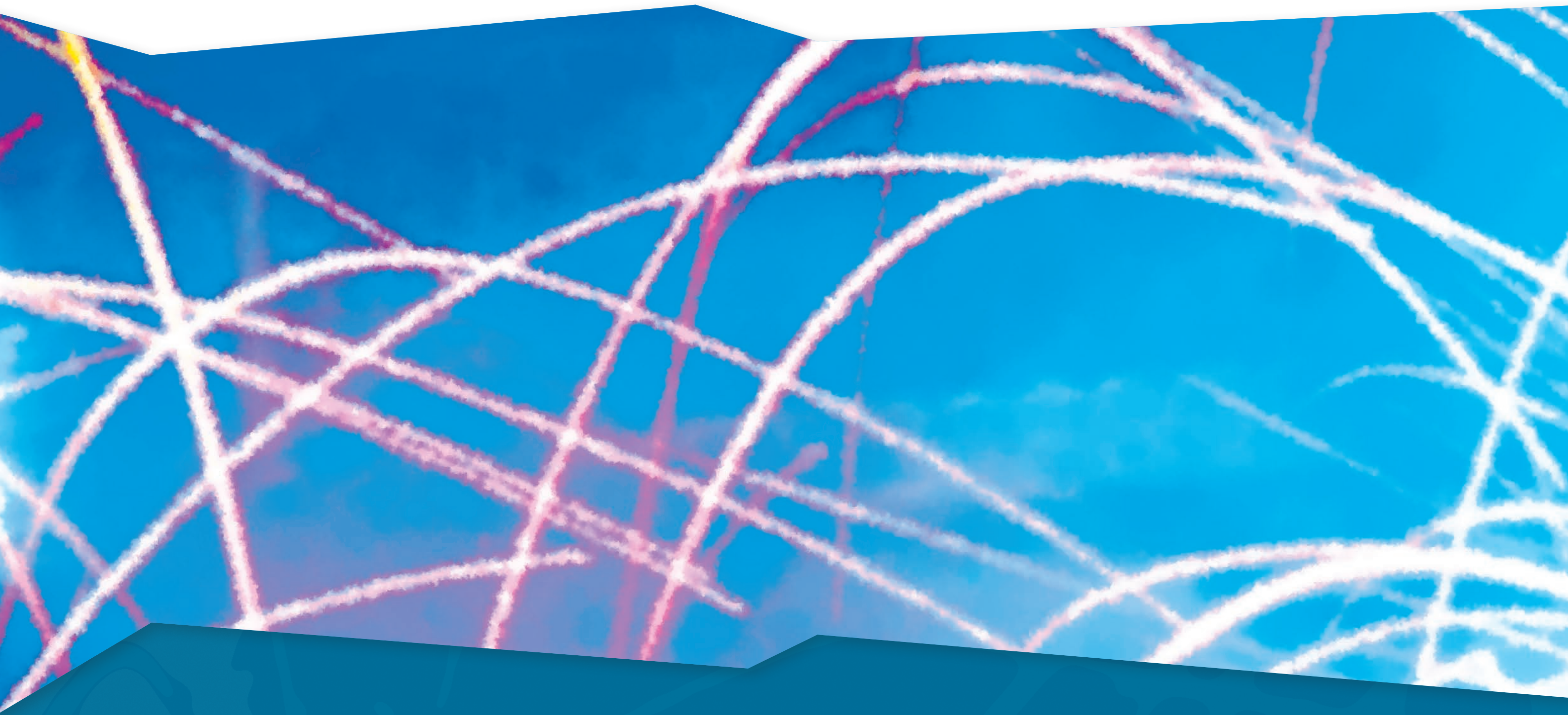
Mathematics Education for 21st Century Engineering Students



Literature Review

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Contents

Executive Summary

Four groups of strategies for teaching engineering students appear in the literature. These are problem-based learning (PBL), multidisciplinary approaches, computer-based methods and strategies that address student variability. Each of these addresses one or more of the issues facing 21st century mathematics educators. Specific issues addressed are: student variability, the need to enhance learning, the need to keep abreast of technological advancement and the need to acquire 'soft skills' and to relate learning experiences to workplace situations. Reports that include results of formal quantitative studies are limited. The effectiveness of computer-based methods and PBL have both been quantitatively analysed relative to learning enhancement. The integrated method's success has also been supported by formal studies. It is useful to combine or integrate computer-based approaches and PBL, with or without physical models.

1. Introduction

Like most other professions, engineering has evolved significantly in the last two decades. It is continually changing to reflect the needs of the 21st century. The needs of engineering students change similarly. Not only have students' needs changed but their needs are more varied as well.

The engineering profession is re-examining its relationship with its scientific roots. For the mathematics discipline, which impacts not only science but also business and management, this is an exciting time to invigorate the curriculum and its educational approaches. This will first require a review of current thinking by engineering and mathematics educators around the world. A picture of current trends emerges not only from published books and journals but also from websites of recognised centres of education. Therefore this review refers to formally published articles, reports from professional associations and to accounts of educational practices that are well documented on websites.

The curriculum and educational practices must be responsive to rapid economic and technological developments as well as emerging global environmental and social problems . Students' personal needs must be balanced against changing requirements of industry and professional accreditation bodies.

In the last five to ten years alone, some of these changes have affected Australian universities in general and engineering curricula in particular. Continuous increase in mineral and oil price resulted in further growth in an already healthy resources sector. This was coupled with a buoyant and fast growing Chinese economy and recovery in the Japanese market (both countries are heavy importers of LNG, for instance) that resulted in higher demand for petroleum, mining and process engineering graduates.

Enrolment data reflect a more diversified student population with varied needs. Consequently, university student support arrangements that were not essential before are now considered important. Examples of these are additional student financial aid and provision of childcare facilities. More students have full time or part time work commitments partly due to higher university fees. Although the Higher Education Contribution Scheme (HECS) was first introduced in 1989 **[1]**, its impact has been seen more during the last decade, in combination with other socio-economic factors. It is increasingly common for students to find employment after finishing high school, work for a few years, and then resume studies entering universities at more mature age.

In response to industry specifications, there has recently been a requirement for emerging professional topics to be included in the engineering curriculum. Examples of these are project management, human resource management and teamwork, communication skills, report writing skills, ethics, time management and environmental protection. These competencies are sometimes referred to as "soft skills"**.[2]**

It is therefore of paramount importance for educators of engineers in the 21st century to recognise changes in the engineering profession as well as changes in engineering students and to modify subjects to suitably cater for their new requirements.

The purpose of this literature review is to bring to light some of the more prominent teaching methods and strategies currently used to teach mathematics to engineering students in various universities worldwide. Emphasis is given to the effectiveness of these methods and strategies. Likewise of importance are extended learning environments such as drop-in centres and other associated learning support. The effectiveness of these methods, strategies and learning environments are discussed. Detailed discussion is found in Section 2 and a summary of key ideas is included in Sub-section 3.2.

A review of research on the teaching and learning of mathematics in Australian and New Zealand universities (from 2000-2003) inclusive found that most articles "contain little reflection, little evaluation or awareness of previous work in the area" **[89]**. There was much debate in the research about the use and extent of use of computer algebra (CA) systems in the mathematics community, however most statistics courses already rely heavily on computer software, with little work done by hand. The changing nature of students and the effect of this on the transition from high school to tertiary mathematics study is also widely discussed. There is also some research on the way students understand particular areas in mathematics. Wood **[89]** found gaps in the research, finding few articles researching the teachers, looking at higher level tertiary mathematics, cross-disciplinary teaching and learning, studies of graduates of mathematics and those who use mathematics, emotion and motivation in university teaching, alternative teaching and assessment methods.

This literature review is part of the initial stages of a scoping investigation to review teaching methods and learning environments employed in the teaching of mathematics to engineering students in Australian universities. Funded by the Carrick Institute for Learning and Teaching in Higher Education, this initial scoping project is headed by Philip Broadbridge of the University of Melbourne.

2. Why is Mathematics Education Important for Engineers?

Mathematics education is of such importance to the engineering curriculum because it helps to lay the foundation for good analytical and problem-solving skills often required in traditional engineering work. Mathematics subjects are prerequisite to a number of engineering subjects.

According to existing literature, mathematics skills are indispensable to engineering graduates. Furthermore, it is claimed that these mathematics skills are in fact not strong in many professional engineers and should be strengthened. This is hoped to be achieved through improved teaching methods. The need to reinforce these mathematics skills is further highlighted by a recognised engineering skills shortage not only in Australia but in many western countries. Methods used to enhance learning should also respond to issues such as diversified student population as well as technological and other changes that impact engineering mathematics education in the 21st century.

The importance of solid mathematical education for engineers is stressed by a number of authors **[3-4]** and argued by Blockley and Woodman **[5]**. It has been highlighted that mathematics is needed in engineering because "all mathematics is the ultimate form of logical rigour. It is the language of scientific communication, hence without a facility in mathematics engineers are cut off from scientific change and development." Most civil engineers believe that only few mathematical concepts are used at work, Kent and Noss propose an explanation **[6-7]** as to why this is so. It has been suggested that this is because mathematics has become wrapped up in engineering practice. As an example, geometry and trigonometry have become so much embedded in engineering practice such that a structural engineer tends to think about say, a simple plane curve, in terms of what they mean in structural terms rather than in mathematical terms.

Furthermore, Blockley and Woodman **[5]** claim that with the advent of computers, emphasis has changed from the ability to perform engineering mathematical calculations to the ability to interpret the meaning of mathematics in engineering. This is especially true in the use of computer software applications. Blockley and Woodman's idea is supported by Hadgraft **[8]** and Kent and Noss **[9-10]**. It has been established by Kent and Noss **[9]** that manipulative skill is less regarded than the "holistic" understanding or interpretation of mathematics and in identifying which areas need mathematical applications in an engineering context.

2.1 The Need for Improved Performance in Engineering Mathematics Education

Engineering students are seen to lack the necessary mathematics skills when they enter traditional technical occupations. According to Henderson **[11]**, surveys of widespread practices in software engineering show that many engineers do not have sufficient skills to use discrete mathematics and logic as tools in performing their work. In the case of Civil Engineering, Blockley and Woodman **[5]** found that while it is paramount for engineers to understand structural behaviour, practising civil engineers have been known to make incorrect assumptions in finite element modelling whereby non-existent boundary conditions and degrees of freedom have been set. This clearly illustrates a lack of understanding in interpreting mathematics as a tool to model physical and engineering conditions – a substantial part of what a young engineer's work may require. Pollock **[12]** and Springer *et al* **[13]** have also expressed concern over students' mathematical preparedness even as they enter higher education. This further highlights the need to strengthen engineering mathematics education at the university level.

In reference to the federally funded *Engineering Tomorrow's Engineers,* a national project funded by the Collaborative and Structural Reform Fund (CASR) designed to address the current skills shortage of engineers and build Australia's capacity to produce engineers with the skills required for the future. federal Education, Science and Training Minister, Minister Bishop was quoted saying:

"Australia's strong economy has led to increasing demands for skilled workers in certain industries. A number of the projects I have approved from the 2006 funding round will assist further in addressing these skills issues". **[14]**

The national engineering skills shortage has recently been formally recognised by the government. Funds have been directed to address this shortage and as a result additional engineering university places have been approved. In July 2006, Hon. Julie Bishop announced the addition of 510 new engineering places in universities from 2007. Prime Minister John Howard declared later in October 2006 an additional 500 Commonwealth-supported engineering places at universities from 2008. These extra places **[16]** would be invested over four years and will cost the government $56 million.

Tilli and Trevelyan **[15]** have highlighted the importance of engineering work due to its substantial contribution to any country's economy, indicating the major significance of an engineering skills shortage.

Projects that deal with an engineering skills shortage have also been accepted. One such project approved by the federal government to address the issue of engineering skill shortage is Australian Technology Network's (ATN) "Engineering Tomorrow's Engineers". **[14]**

Circumstances overseas do not appear to be very different. It has been reported **[17]** that the UK Technology Colleges Programme and the New Zealand Science and Technology Teacher Fellowship Scheme have both been developed with the purpose of advancing science and technology education, as well as responding to the impacts of globalisation in curriculum design. Attention to student enrolment in STEM (Science, Technology, Engineering and Mathematics) due to severe shortage of graduates in many Western countries is also identified by van Langen and Dekkers **[18]**. In a separate report, Kent and Noss **[9]** presented statistics of declining engineering enrolments in the UK from 1988 to 2000. Generally, industries in Europe have been said to be discontented due to the low number of engineering students passing mathematics exams. **[19]** Graduates of engineering in many developed countries are regarded to be low in number, with only 6.4% of graduates noted to be in the engineering field in the USA in 2001, while the UK produced 10.5% of its graduates for the engineering labour market. **[18]** Engineering accounts for 8% of total graduations in Australia. **[20]**

3. What are the Most Prominent Teaching Methods Currently in Use?

From a survey of recent (from 1995 up to the present time) literature in mathematics education for engineering students, the following are notable means for teaching and learning to adapt to 21st century needs and conditions:

- use advanced computer based methods – web based interactive, software applications, or both
- address student variability
- take a multidisciplinary approach
- use a Problem Based Learning (PBL) strategy

These methods and features are discussed in more detail in the succeeding sections. Variations and combinations of these teaching methods are also discussed. Where evidence of any method's success has been reported, this is included, together with the discussion of the method's features.

Active learning deserves special mention because it is related to many teaching methods including computer-based and problem-based learning.

In addition to the most prominent categories of teaching methods (listed above), formulating subject objectives with the use of learning outcomes may enhance learning by better defining objectives and specifying student activities.

3.1 Active Learning

Active learning broadly encompasses all learning driven by the learner **[21-22].** It is "learning by doing". It has been claimed that the more students participate in their learning, the more they achieve **[23].** In general terms, active learning is differentiated from a traditional lecture whereby students are passive listeners. Problem based learning (PBL)**,** "learning driven by problem" **[24]**, is classified as active learning **[25].** Computer based approaches, especially where interactive software is used by students, are also classified as active learning.

In their report, Sanz *et al* **[26]** discuss the development of a project at the University of Utah highlighting the benefits of an active learning environment. This project centres on a highly interactive multimedia module for science and engineering students where by numerous opportunities for "learning by doing" are highlighted with the use of graphics/animation, virtual labs and simulation software. Active learning is also advocated by other authors **[23, 27-28]** with various perspectives.

In the following sections, computer based learning and problem based learning will each be discussed. Due to the relationship of active learning to both of these methods, as each of them is discussed, active learning is also emphasized.

3.2 Advanced Computer Based Methods – Use of Web Based Interactive, Software Applications and Programs

With the rapid progress of computer technology in the last couple of decades, software applications and the web have become important elements of our daily activities. This reality is also evident in engineering mathematics education. Computer technology and its applications have been incorporated in engineering mathematics subjects at most universities in Australia. Hence, while one or more other teaching methods are employed, it may be observed that they are often used in conjunction with web technology or software applications.

In this section however, the application of computer technology in teaching mathematics to engineering students is discussed by itself.

Many authors **[7-9, 26, 28-29]** have either asserted that computer based teaching should be supported or highlighted the strong presence of computer software applications and various forms of computer and web-based interactive learning environments in the delivery of mathematics subjects to engineering students. This complements the equally large proportion of engineers' professional work being reliant on computer software applications. Taraban *et al* **[27]** highlight the importance of examining computer-based learning because it represents a leading progression in contemporary teaching. On the other hand, there are reasons why "engineering mathematics courses have not undergone wide scale changes towards IT. These are the high cost of moving 'chalk and talk' mathematics teaching out of lecture rooms and into computer laboratories, and the lack of a common grounding in mathematical technology in school mathematics curricula" **[9].**

Electronic learning (e-learning) is often thought to be the new way to communicate mathematics. It is also believed to enhance comprehension and stimulate interest **[19].** Particularly for engineering students and practising engineers, familiarity with a number of software packages is said to be vital for effective problem solving **[30].**

Some authors **[19, 27]** suggest that the use of interactive mathematics can provide more engaging learning materials. This will consequently attract more students to study mathematics (and engineering) and help stem the declining number of graduates **[9, 17].**

Aside from using packages, interactive mathematics may involve personalised, interactive documents on the web. It may also include the use of the web as an unconventional calculator using numeric, graphic and symbolic mathematics interactively **[19].** As well as assisting learning, computers can also be engaged in assessment by way of "computer aided assessment" (CAA). CAA has been reported to increase students' confidence and to reduce their stress levels **[12].** Furthermore, some authors **[4]** have suggested that it would be beneficial for educators to explore the potential of using technology (software tools) to improve the interrelation between mathematicians and engineers and for bridging their individual knowledge in a fresh manner.

3.21 Which Software program?

Brenner *et al* **[30]** assert that computer programs for engineering students must be simple to understand and easy to use. They should also be capable of achieving straightforward results. This will prevent the technical details of the computer program from taking away the focus of learning from the main subject matter. Waldvogel **[31]** has communicated along the same lines. He supports the appropriate use of modern mathematical software in teaching engineering but cautions that software cannot take the place of basic understanding. He is also a proponent of MATLAB, which is believed to be a transparent and versatile in numerical work **[33]**.

At the University of South Australia (UNISA) **[32]**, online study mode is accomplished via the internet where interactive technological tools are available. Most mathematics subjects for engineering students introduce students to MATLAB **[33]**.

Other notable users of MATLAB include the University of Canterbury in MATH/EMTH271 Mathematical Modelling and Computation 2 **[59, 62-64]** and the University of New South Wales at Australian Defence Force Academy (ADFA) in ECM1 Engineering Computational Methods 1 **[60-61].**

Curtin University **[34-35]** has incorporated the Maple programming language in its engineering program. It is intended that Maple will reinforce lecture topics for the students but they will also be made aware of Maple's limitations.

Kent and Noss **[4]** put forward Mathcad and MATLAB as useful tools for engineers for constructing models and for their own specialised applications. This is primarily because these tools allow accelerated prototyping. This means that, "initial modelling ideas can be investigated by the potential users, and the resulting interaction often leads to an improved match between the model and the users' requirements". Mathematica or Maple is thought to be less useful as they are more difficult to understand without thinking in "explicit mathematical terms" [4].

Proponents of Mathcad **[36]** refer to its ability to export and import data to external files. Users are also known to be able to build on previous projects by consolidating old and new models. Karady and Nigim **[37]** maintain that Mathcad is easily grasped by students. Part of the reason for this may be because formulas in Mathcad worksheets appear in the same form as they are seen in textbooks **[36-37].** MATLAB has been chosen at the Jerusalem College of Technology firstly because it is widely used in industry, and secondly because of its "pronounced Linear Algebra orientation" **[38].** MATLAB is believed to help make the theory more transparent to the engineering student by allowing the student to follow the theory's operation in a wider range of applications. Whereas mistakes in manual linear algebra exercises can slip away unnoticed, MATLAB also acts as an effective teaching tool by refusing to process incorrect statements. MATLAB is also said to be very useful when demonstrating graphs of functions and discussing limits **[39].**

Some universities **[37, 39]** have also chosen MATLAB or Mathcad due to their wide application in employment and industry.

Positive feedback has been obtained for a MATLAB guide **[39]** developed at the University of South Australia. The title of this 160-page guide is "A Focused Introduction to MATLAB". It aims to enhance the teaching and learning of mathematics and serves as a bridge between the software itself and the lecture material. Contents of the guide include an introduction to MATLAB features, worked examples and exercises. It is believed that the MATLAB worked problems are very effective. This is because they are problems that are taken from the textbook; therefore a student is able to compare the textbook solution to the MATLAB solution.

Keady *et al* [88] discuss the benefits of CAA systems, underpinned by Computer Algebra (CA) packages, used in the delivery of questions to students via the web in Australia. They argue that CAA systems become more common as departments have larger service classes. They focus on the benefits of s*tack* (free and open source, underpinned by Maxima), *AiM* (free and open source except for the CA, underpinned by Maple), *mapleTA, WebLearn* (commercial, both underpinned by Maple) and *CalMaeth* (commercial, underpinned by Mathmatica), but recognise that CAA systems will continually change. They are proponents of Maple as the underpinning CA and argue that sharing systems, question databases, etc is crucial for progress to be made and to allow the systems to be economically viable.

Brenner *et al* **[30]** and Morgenroth *et al* **[28]** envisaged the inclusion and effective use of modelling and simulation in the fields of environmental, chemical and biotechnology engineering through the application of POLYMATH. POLYMATH is a numerical computation package used for modelling and simulation, particularly created for engineering students and engineering professionals. It is generally used for interactive problem solving involving algebraic equation systems, differential equations and regressions. Its latest version allows automatic migration to Excel. With the application of a few elementary rules, POLYMATH models can be transformed to MATLAB functions.

POLYMATH is easy to understand and simple to use. "It requires only minimal user intervention in the technical details of the solution process." **[30]** POLYMATH is used in Ben Gurion University and in many other universities worldwide, including University of Adelaide and Curtin University of Technology, in Australia.

3.22 Does Computer Based Learning have any Evidence of Success?

Most authors and proponents of the use of software applications in engineering maths subjects do not have any formal quantitative empirical evidence of its success. At best, Brenner *et al* **[30]** offer some reasoning behind their conclusion stating the positive effects of the use of POLYMATH. They also demonstrate its effectiveness by describing how it was applied to the modelling and simulation of a classic problem in "Water Pollution Control". Similarly, Mtenga and Spainhour **[36]** provide sound arguments to support their claim about the benefits of using Mathcad. They also illustrate the functionality aspect of Mathcad by presenting a problem ("Stepped Column") where it has been implemented. Naimark **[38]** uses his observation to compare topics with and without MATLAB support. He explains that topics incorporating MATLAB are grasped well, with nearly all of the exercises solved correctly by the students. Colgan **[39]** bases his conclusions on student grades. But contrary to Naimark's results **[38]**, student grades indicate that mathematical knowledge and skills gained in subjects with MATLAB are comparable to those gained in subjects without MATLAB. It is the ability to program in MATLAB that increased significantly. Lee and Lin **[40]** showed how MATLAB might be integrated into an electrical engineering subject for the purpose of visualising and appreciating its application in a real environment, and only declared the associated relevance of measurement and analysis based on it. Educators generally make statements about how computer aided teaching and software applications enhance learning in engineering education, but do not provide any back-up quantitative information to support these claims **[26, 28].**

Taraban *et al* **[27]** recently conducted a case study comparing the effectiveness of software implemented interactive (active) learning compared to computer based plain text (passive) approach. The case study had twenty-five participants in a thermodynamics subject. Materials used were computer-based instructional supplements whereby screens display text, tables and charts, or interactive exercises and problem solving, modelling, simulation and online quizzes. As part of a method known as "verbal protocol analysis", students' verbalisations were captured as they viewed the computer screens. Verbalisations were associated with thought processes. Results of their case study showed significant increase in cognitive activity in interactive screens compared to text-based screens.

Results from the Taraban case study **[27]** indicate that computer based learning is much more effective when it is interactive than when it is passive. However, it is not categorically shown in their study that computer based interactive learning is superior to the traditional lecture and textbook based method.

The integrated approach has been used in a number of American universities. **[41]** This approach encourages active learning and combines different models mostly consisting of the physical model, mathematical model and computer simulation model. Electronic equipment, devices and machines, including computers, are widely used especially in measuring, collecting, storing and analysing data. Computer applications are part and parcel of this approach.

Nirmalakhandan *et al* **[23]** present recent findings involving independent evaluations of the success of using these active learning environments in engineering classes. Evidence shows that the integrated approach, promoting active learning, helps to improve engineering education with 81% of students confirming the effectiveness of the computer model.

3.3 Addressing Student Variability

Student variability has a number of implications. The student population is diverse in many ways. Due to the multiple needs and commitments of students, additional and diversified support is required. Examples of this are; flexible scheduling, childcare, financial aid, and support for students with disability. Diversity also means differences in students' learning style and implies a wide range of aptitude among student intakes.

It has been documented in Australia and elsewhere that student populations are becoming more diverse **[42].** For instance, women's enrolment in engineering has been reported to increase in many countries worldwide **[43].** Houghton and Dunne **[44]** and other authors **[42]** suggest that in order to address the challenges associated with this diverse population, students need to assume an active role in their learning. This may be associated with the belief that educators should pay attention to different learning styles as this is crucial to the success of any teaching and learning method **[45].** These ideas are related because when students actively participate, they have more opportunity to activate whatever learning style is suitable for them.

Townend **[46]** makes another proposal, specifically dealing with varying abilities of engineering students, to make the students' mathematical encounters more "user friendly". Croft and Ward **[3]** present a similar solution to take into account the individual needs of students with the use of an open environment and include components with flexible study arrangements that are considered to enhance and stimulate learning.

One such open environment is the Maths Learning Centre **[47]** at Loughborough University, which provides, among other things, one-to-one teaching and 24 hour, 7-day on-line support. The presence of learning centres is supported by Blair-Editor [**42**] and Fuller **[48]**. It is asserted that students of mathematics need a supportive extended learning environment such as learning centres, tutoring labs, counsellors, support for students with disabilities and others.

Bamforth *et al* **[49]** describe a pre-sessional course run for electrical engineering students at the University of Loughborough with non-conventional maths backgrounds (without A-Level) to help and retain students at risk of underachievement. Aside from mathematics lessons, this course also covers team and key skills. The course is a good starting point because it helps at-risk students become aware of the facilities and support available to them at the university and it also helps them begin the process of improving their main skills. It has been noted that student feedback has been largely affirmative.

As a whole, the provision of varied learning resources and support such as those found at Loughborough University and Central Queensland University **[48]** gives more opportunity for a variable student population to learn in the way that best suits them.

In order to address the issue of students having multiple life demands such as study, work and family, the University of South Australia offers flexible study arrangements **[50]** to students to help them cope with the demands of studies and of other areas in their life. Students are able to take subjects on campus, externally, online or a combination of these. Online students (in a computer science subject) have been shown **[51]** to perform at least as well as students in a traditional lecture. In a separate report, Hadgraft **[8]** has specifically suggested that skill development in engineering subjects should be supported by online assessment in order to provide more flexibility for students to study and obtain assessment at their own pace.

The University of Western Australia (UWA) is claimed to have a diverse student population. Based on its website **[52]**, its student population is said to be diverse in relation to cultural background, age, gender, race, disability, sexual orientation and socio-economic status. According to UWA's Best Practice Pathways Database **[53]** completed in 1998 and 2000, the Department of Mathematics and Statistics takes into consideration the diverse learning needs of students by using different forms of delivery such as; lectures, computer-based learning (including videoconferencing) and use of the web. Web-based materials can be accessed anywhere in the world at a time convenient for the students. Examples of materials placed in the web are subject notes, assignment solutions, software packages and assessments. The Department also uses various forms of assessment such as; written assignments, tests and quizzes, computer-based tests, reports and essays. Students are often required to use computer packages and the computing lab. The Mathematics Learning Centre is another option for students.

Based on information provided by Professor Mario Zadnik **[54]**, Dean of Teaching and Learning at Curtin University**,** it can be seen that Curtin University has recognised the variability in knowledge and competencies of students entering the engineering program. They have therefore introduced two Mathematics subject groupings or series, one intended for students who achieved better marks in their Year 12 studies (Engineering Maths 110 then 130 series with more challenging and enriching contents) and another series (Engineering Maths 120 then 140) for students whose marks were lower.

Similar streaming is being implemented at Edinburgh University **[55]**, with one stream provided to students from average academic background and another stream functioning at remedial level. Different teaching approaches and different assessments are used to support the streaming being implemented at the University of Liverpool **[56]**. At ETH Zurich **[31]**, streaming is being implemented with mathematics taught to first and second year engineering students. Students are however able to choose between a fast Stream A and a slow Stream B and this is certainly contributing to additional flexibility of their programs.

Although the success of University of Liverpool's streaming by ability is only gauged informally through staff-student forums, they have generally been judged to be very good **[56]**.

3.4 Multidisciplinary Approach

The LTSN Maths Team **[57]** recommended that mathematics subjects to engineering students be delivered within an engineering context. One way to help students integrate their knowledge and be able to see mathematics in the context of engineering is to design a subject where collaboration between engineering and maths departments (and other staff) is a fundamental principle. Additionally, Haryott **[58]** suggests that for a good working engineering syllabus to succeed, collaboration should be accomplished not only among academics but it should also include accrediting bodies and recent engineering graduates.

Maths and engineering collaboration is undoubtedly consistent with the arrangement in modern industrial working practices **[4].** It has been identified (and success noted from student comments) that interface between engineering and mathematics subjects can be achieved through the multidisciplinary approach. This was evidenced by the noted success (based on student feedback) of this program at the Edinburgh University **[55]** and at Canterbury University **[59]**.

In Australia and in New Zealand, Dr. Steve Barry (ADFA) **[60-61]** and Prof. Graeme Wake (formerly from University of Canterbury) **[59, 62-64]** have independently helped introduce the multidisciplinary approach in teaching Numerical Methods. Groups usually working together include mathematics, engineering and either statistics or computer science academics.

In the Canterbury model **[59, 62-64]**, each week generally contains lecture and lab sessions handled by the Department of Maths and Stats. Lab sessions are used for problem solving. However four out of the twelve weeks (four weeks are non-consecutive) are devoted to engineering case studies that are taught by staff with an engineering background. The case studies are carried out in groups of two students.

The ADFA model **[60-61]** is different in the way the interfaces are structured. The whole subject consists of six engineering problems based either on real world occurrences or on more advanced engineering subjects. It may be said that the subject has strong elements of PBL, although individual students do assignments. Development and delivery of the subject is accomplished jointly. Different departments present each lecture and each lab session jointly. Positive feedback has been received from student survey results as well as from staff-student review meetings. Although fundamentally multidisciplinary due to the collaboration of different departments in the subject's delivery, the teaching strategy employed in this subject in Numerical Methods is a mix of lectures, computer based laboratory and "real world" engineering problems/case studies (PBL). There is substantial interface between mathematicians, engineers and computer scientists in ADFA's model **[60-61]** with each lecture and each lab session jointly handled by three educators. On the other hand, Canterbury's model **[59, 62-64]** has additional support for students in the form of dedicated tutors, regular drop-in help classes, consultation periods, availability of web-based assistance and help with administrative issues. Both subjects have used MATLAB and both have claimed success based on student feedback. Both subjects have also met a number of challenges related to implementation. Both subject designs have been newly implemented (2003) and no measurable analysis has been done yet on their effectiveness.

These approaches may be effective because materials are presented in different ways (lecture, lab and case studies) within a short time frame and this enables the students to obtain thorough and in-depth knowledge/skills of the subject matter. The presence of regular lab work (weekly) is useful because the students' lab experience will support and complement what has been learned in the lecture each week. This will also reinforce learning and provide hands-on opportunity with MATLAB and other computer packages.

There are various ways in which collaboration between maths teachers and engineering departments benefits students at the ETH Zurich **[31]**. In this university, engineering departments are consulted and their agreement sought when determining topics for second year engineering maths subjects. Practical examples given in class are also taken from engineering fields. Taking practical examples and problems from engineering has also been suggested for technological colleges in Japan **[65]**, although this idea is taken further by proposing to revisit these problems at a later time. Waldvogel **[31]**, in his presentation at the IDEA League 'Workshop on Mathematics in Engineering' proposed additional improvement by focusing on and extending linear algebra and numerical analysis with possible inclusion of discrete mathematics, due to the fact that practical engineering problems "are rarely solvable in closed form".

Since 2004, Zadnik **[54]** of Curtin University of Technology has chaired a committee composed of representatives from maths and engineering. The committee aims to tackle issues in mathematics teaching, to bridge the maths-engineering divide and to make proposals for improvement in the following areas:

- Incoming students' variable knowledge and competencies
- Engineering maths subject contents and learning outcomes
- Who teaches
- Guest lectures by engineering staff reinforcing the importance of maths

Incorporating guest lectures by engineering staff is an innovative teaching practice for mathematics subjects and this can potentially improve engineering students' appreciation of mathematics **[54]**. This is because the engineering staff can reinforce the importance of maths and better relate mathematics to the engineering concepts, practical problems and even work requirements.

3.5 Problem Based Learning (PBL)

Active learning is "learning by doing". This occurs when students play an active role in their learning. The problem based approach is a form of active learning where learning is driven by the problem. Some authors **[42, 44]** suggest that in order to address the challenges associated with a diverse student population, students need to assume an active role in their learning.

Although it has been identified from Kent **[66]** and Tilli and Trevelyan's **[15]** research that engineering practice is not clearly understood, some authors **[9, 27]** believe that engineering has a creative quality. Furthermore, Haryott **[58]** advocates that in order to support the creative nature of engineering, the use of problem-based learning should be explored further for teaching mathematics to engineering students.

PBL has been claimed **[67]** to be the "natural technique" to use in teaching engineering because it duplicates work situations most engineers find themselves in. Workplace situations hold similarities with small group, PBL in the classroom. This is evidenced by the fact that in engineering work, people team up in small collaborative groups **[13]** and encounter problems that are open-ended and often with conflicting elements.

Litzinger *et al* **[68]** propose a revision of engineering curricula to include multiple learning experiences that challenge students to cultivate self-directed learning skills. PBL subjects have been studied in connection with this and positive correlation was obtained between PBL and readiness for self-directed learning. Aside from multiple learning experiences, some authors **[67]** have suggested PBL based programs include more student participation. It has been proposed **[69]** that there are many opportunities for engineering students to work on projects requiring their collaborative skills. This is demonstrated by the Faculty of Engineering at the University of South Australia **[39]** and ADFA **[60-61]**; group work and report writing are included in mathematics subjects for engineering students. These subjects are intended to be problem-based, with students encouraged to acquire additional knowledge on top of what was covered in lectures. They also require group work and report writing to assist in developing skills associated with working cooperatively, time management and investigating "real applied problems". This is supported by the PBL experience described by Johnson **[70]** in a hydraulic engineering class.

Although benefits of PBL have been reported to be positive **[71]**, Johnson **[70]** has pointed out that there is considerable time associated with the overall implementation of this strategy. Contrary to this, Jonassen **[72]** argues that online environments are able to provide a platform for designing, developing and implementing PBL with minimal support needed.

3.51 Does Problem Based Learning have any Evidence of Success?

Morgenroth *et al* **[28]** have suggested the usefulness of PBL in understanding and appreciating mathematical modelling. This claim is supported by Thomas **[73]** with the success (based on feedback) of a game show format for PBL classes in mathematical modelling. Projects undertaken by students have been argued to enhance learning; however the claim is usually based only on direct students' feedback **[70, 74-76]**.

Karady and Nigim **[37]** report on a proposed problem solving method in power engineering at Arizona State University which has been tested for four years, however no quantitative data is put forward. They claim that this method, which also utilises Mathcad, has helped improve grades and has also lifted student interest in the subject matter. It is postulated that the success of this problem solving method can be attributed to the interactive nature of the process. This signifies that it is effective for students to discover the trends, meanings and interconnections in an interactive way.

An integration of studies from 1980 to 1999 was accomplished by Springer *et al* **[13]**. This pertains to small group learning's effectiveness with undergraduate students of science, mathematics, engineering or technology (SMET) in North American educational institutions. This integration study analysed 39 reports using the standardised mean difference (d-index) effect size. The d-index was measured for the three most prominent learning outcomes found in SMET education literature. These are: academic achievement, persistence (or retention) and attitude.

As guidance for the interpretation of the d-index, a "d" value of 0.20 is considered to have a small effect, d=0.50 can be taken to have moderate effect, and if the value of "d" is 0.80 then this means that the effect is large. An effect size of 0.33 is considered to be the minimum required to confirm significance, and typical values range from 0.25 to 0.50 **[13]**.

"The main effect of small group learning on achievement, persistence, and attitudes among undergraduates in SMET was significant and positive" **[13]**. Results of the integration study are detailed below:

- Achievement "d" value of 0.51, more than the average value 0.40, would move a student from 50^{th} percentile to 70^{th} percentile;
- Persistence "d" value of 0.46 would reduce attrition in SMET subjects by 22%;
- Attitude "d" value of 0.55 exceeds the average value of 0.28 for classroom-based educational interventions on affective outcome measures.

The significance of this analysis to PBL cannot be ignored if small group PBL is utilised.

Taraban *et al* **[27]** performed a case study on 25 science and engineering students of thermodynamics which yielded positive results for effective use of software implemented active learning. The case study examined cognition when students were placed in a rich visual, auditory and print environment. Students' verbalisations are sampled while they interacted with and solved problems using software. While this experiment has e-learning elements associated with it, it is actually the interactive aspect of it that is being examined. The method used is considered to be PBL because problem solving was accomplished while learning. The kinds of learning materials used support theories of skill progression, which require that students have the knowledge first and then the technique to translate that knowledge into skill by applying it to problems. The materials are different from a traditional lecture in their ability to preoccupy senses and give immediate feedback to student input. The more interactive the material is, the more student participation is required.

It is noted that while interactive and PBL are both forms of active learning, they are not exactly the same as each other but only contain common elements. As shown in the experiment mentioned in the previous paragraph, they can be designed to blend together.

Innovative teaching methods being used in a number of American universities **[41]** are consistent with an integrated approach characterised by active learning environments. This approach encourages "learning by doing". Projects are often required of students working in small groups, challenging them to "rationalise, reconcile, predict and validate" theoretical knowledge against the physical model **[23]**. Active learning can take several forms and one of these is PBL. This integrated approach can therefore be thought of as an extension of PBL.

Nirmalakhandan *et al* **[23]** present recent findings involving independent evaluations of the success of using these PBL based learning extension in engineering classes. Evidence shows that this integrated method helps to improve engineering education, with 92.1% of respondents attesting to the effectiveness of this teaching strategy.

3.6 Variations and Combination of Methods

3.61 Design-Based Learning

The general concept used in Design Based Learning (DBL) is similar to that of Problem Based Learning (PBL). In PBL, the problem drives learning, that is, the problem is posed first before learning can begin. Similarly in DBL, the need for learning is met only when it is necessitated by design. The reasoning behind both approaches is the same: Students are more motivated to learn if they know 'Why' they are doing it.

Kent and Noss **[9]** suggest that mechanisms for change are ripe, they promote a so called "pull-based mathematics" and "design-based approach to engineering subjects":

"Perhaps common ground can be gained by a constructive dialogue on two fronts: on the mathematical topics in the curriculum, and on delivery and pedagogical approaches. These two issues are intertwined, and consideration of one without the other leads inevitably to misunderstanding and inertia. Engineering subjects are tending towards 'design-based' approaches (employing design as an organising principle), with a decreasing use of 'chalk and talk' pedagogy, and mathematics subjects will have to accommodate themselves to this trend (this does not necessarily entail a radical departure from existing mathematics teaching). Although a number of methodologies are being chosen for design-based learning, the effect on mathematics is similar: the need for analysis is 'pull, not push' – the need can emerge where design requires it, not pushed into the student prior to having a meaningful context for it" **[9]**.

3.62 Projects-Based Learning

In 2003 "Projects Based Learning in Engineering" (PBLE) published a guide **[77]** that advocates PBLE. Due to their similar descriptions and acronym, PBLE and PBL can be easily confused with each other. Educators' implement PBLE with the "use of projects in their work with students", and not just any type of problem. Projects may include environmental impact assessment, design portfolio and simulated public enquiry. In a separate report **[71]**, it has been suggested that projects based learning will be more straightforward for academics to adopt due to their familiarity with projects.

3.63 The "Integrated" Approach

The integrated approach has been found to be used in a number of American universities **[41]**. This approach encourages active learning and combines different models mostly consisting of physical, mathematical and computer simulation models. Electronic equipment, devices and machines, particularly computers, are widely used especially in measuring, collecting, storing and analysing data. Projects are often required of students working in small groups**,** challenging them to "rationalise, reconcile, predict and validate" theoretical knowledge against the physical model **[23]**.

Nirmalakhandan *et al* **[23]** present recent findings involving independent evaluations of the success of using active learning environments in engineering classes. Hydraulic engineering classes totalling 131 students over 5 semesters were taken for the evaluation. The effects of an integrated approach promoting active learning using a combination of physical, mathematical, and computer simulation models were measured using student surveys and student performance. Collated end-of-semester survey results from five semesters indicated that 90.6% of students attested to the effectiveness of the physical model, 80.6% of students confirmed that of the computer models and 92.1% affirmed that the teaching approach helped improve their problem solving skills. The percentage of students that received A, B or C also increased to about 86±4.7% after using this approach for 5 semesters. This is an acceptable value compared to the percentage before using this strategy, which was 70±1.3%. Although this evidence may be thought to be limited, they show that the integrated approach promoting active learning helps to improve engineering education.

In a separate report, Duran *et al* **[78]** show that a learner-centred approach highlighting student satisfaction has a positive effect on learning. The Software-based methodology (SBM) has been designed, implemented and formally tested on students of Electrical Machines and Installations at the University of Seville. SBM has distinct similarities with the method tested by Nirmalakhandan *et al* **[23]**. Both methods are fundamentally integrations of physical, theoretical and computer-based models. The software simulations in SBM provide visual tools that help students understand theoretical concepts. Real examples in the laboratory complement the virtual scenarios. SBM is highly interactive not only with respect to the computer simulation but also in terms of promoting discussion and brainstorming. The effects on approximately eighty-five students (different pre- and post test) were formally tested using questionnaires and cognitive tests. Statistical evaluation yields positive results for this integrated method.

3.64 The Interdisciplinary, Multi-semester Integration

While strategies such as the integrated approach discussed in the previous section, the ADFA model **[60-61]** and the Canterbury model **[59, 62-64]** achieve the link "laterally" across theory, computer applications and physical models in one subject, Avitabile **[75-76, 79]** and his group's approach at the University of Massachusetts Lowell integrates materials both "laterally" across modules/projects, and "vertically" from earlier subjects to later subjects across several semesters. The simultaneous lateral and vertical integrations result in interweaving of learning materials. This interdisciplinary, multi-semester approach concludes with a final year project in a Dynamic Systems subject. The strategy uses modules that have been developed for inclusion in a number of lower year pre-requisite subjects.

The strategy can be thought of as having a large project spanning several semesters. The preceding phases of this project are the pre-requisite subjects from 2nd year to 3rd year and its final phase is the Dynamic Systems subject. All skills and knowledge obtained in the preceding phases culminate and are utilised in dynamic systems.

This innovative engineering teaching strategy has various modules integrating basic materials from pre-requisite subjects and problem solving materials in later subjects **[75-76, 79]**. Modules have been set up in pre-requisite subjects such as:

- Differential Equations (2nd Year)
- Mechanical Laboratory (3rd Year)
- Numerical Methods (3rd Year)

with topics including:

- Numerical Integration/Differentiation
- Visualisation Tools for Understanding 1st and 2nd Order System Response Characteristics
- Understanding Complex Frequency Response Characteristics
- Development of a Virtual Measurement System.

In order to make the connection between the interrelated subjects, modules have been developed and deployed to the preceding subjects **[75-76, 79]**. The injection of these modules enables the students to see the connection between earlier subjects and later subjects. The strategy is thus fundamentally a "vertical" integration of earlier materials and later materials achieved through the use of modules.

It should be stressed that as integration is achieved between lower year subjects and higher year subjects ("vertical"), to some extent different materials/modules are also being combined in each year ("lateral"). An example of this is the combination of virtual measurement system (PBL, computer-based), actual measurement system (PBL, computer-based, physical model) and supporting tutorial material **[76]**. Furthermore, the same mass-spring-dashpot system (physical model) used for actual measurement is also used in teaching second-order linear differential equations.

There are in fact two examples of projects that are being studied from different perspectives for four successive semesters. These are the simple RC circuit and the mass-spring-dashpot system. All of these result in an interweaving effect on students' learning and appreciation of the relationships between materials from 2nd year, 3rd year and through to final year engineering **[79]**.

It is also worth noting that the overall strategy of interweaving materials ("vertical"/"lateral" integration) relies on the use of PBL and computer applications for its actual implementation. Additionally it also incorporates physical models in the subjects and has extensive use of electronic equipment and devices, GUI (graphic user interface), as well as hands-on student activities **[75-76, 79]**.

One such hands-on module is the "virtual measurement system", which can be considered as preparation for the actual experimental project. Another such module focusing on real-world (real data) measurement is R.U.B.E. (Response Under Basic Excitation). This module is an actual measurement system having variable mechanical parameters and is available as an online experiment. "In this module, students collect and process data to numerically integrate and differentiate displacement and acceleration measurement. The measurement system forces the students to address issues related to real-world measurements" **[76]**. The actual system has variable mechanical parameters in that it is able to change data for every operation resulting in all data sets being different from each other.

Assessment of the first few semesters when this hands-on project was available shows that students developed more knowledge and understanding of the subject matter. This is evidenced by students' direct comments presented by Avitabile **[76]**.

Integration has been achieved in the Differential Equations subject **[79]** by comparing an analytically obtained solution to one attained by the use of modelling. The computer-based tools allow students to study particular systems that have previously been defined by theory. Both of these are validated against the physical model including actual measured data. The differential equations module is in fact being investigated again from various other viewpoints in three more semesters.

Theory from earlier subjects can be compared with application in later subjects **[79]**. Lecturers likewise remind students of materials studied in earlier subjects and discuss how these materials relate to current topics. This is achieved by comparing topics and modules and by highlighting particular techniques learned in earlier subjects that are crucial in later subjects. Early years' subjects include projects in integration, differentiation and regression analysis (with MATLAB-based graphical user interface tool) that are vital for application in later years **[75]**. In the subject itself (for example, Regression Analysis), the traditional analytical method (hand calculations) can be compared to software-based methods of using MATLAB and Excel. Focus on distortions present in real-world data is achieved with the use of graphical user interface (GUI). GUI is useful for graphical presentation of data that can be manipulated by the students.

All of these material overlaps occurring in various subjects throughout the engineering degree program help to effectively integrate learning.

3.65 The Four-Leaf Clover Model with Emphasis on Mathematical Modelling

Ernest **[80]** has warned to exercise caution before accepting the effectiveness of the integrated or modelling approach. Mathematical modelling has been identified by Hadgraft **[8]** as a necessary constituent of engineering education particularly when applied to real world problem solving (combination of PBL and computer-based approach). It has been stressed further that reinforcing mathematical modelling skills can be effectively achieved by practising this technique many times. "Students need to be able to move from the modelling task to skill development and back again". The skill development learning model is extended further to a four-leaf clover education model, where the additional elements include "learning from others" and "learning from literature". Designed in consideration of situations usually encountered in industry, the four leaf clover is a comprehensive representation of what is required in engineering education namely: modelling skills, mathematical technique skills, collaborative learning skills and research skills.

The four-leaf clover engineering education model **[8]** highlights modelling in a PBL-computer based combination method that provides learning experiences said to be close to workplace situations. The incorporation of various elements being addressed in this education model is a strong feature, as well as the emphasis on repetition and practice.

3.7 Structuring by Learning Outcomes

One way in which mathematics can be related to engineering subjects and engineering applications is by incorporating this statement in the learning outcomes **[34-35, 81-82]**. A direct and simple expression of a goal (Outcome) will necessitate certain activities to be designed in order to achieve the desired outcome. Examples of such learning outcome statements can be found in Curtin University's Unit Outlines for Engineering Mathematics 140, 120, 130 and 110 **[83]**. Below are examples of actual statements:

- The ability to perform routine vector and matrix manipulations which arise in engineering
- The ability to sketch and visualise elementary mathematical functions routinely used in engineering analysis
- Understanding the concept and role of functions in an engineering context
- Identifying the role of mathematics in your own and related discipline area.

Some examples of how engineering applications have been covered are:

- Learning Outcome of adding, subtracting and taking scalar multiples of vectors are being applied to situations such as determining angles between different members of a truss
- Learning Outcome for Applications of Integration includes determining the work done in compressing/stretching springs and in lifting objects as well as determining moments and centres of mass/centroids **[34-35].**

Subject guides **[84-86]** for mathematics at UWA indicate that to some extent, Outcomes-Based Learning is being utilised in those subjects. Lectures, group work, lab, practice classes and the use of computer applications are the main features and learning environments used.

Based on publications and other materials covered in this review, there have been no studies made showing that specification of outcomes leads to better student learning than more traditional curriculum description.

4. How Are the Findings Summarised?

4.1 Impression of the Body of Literature

Although articles collected for the literature review of this research project are mainly accredited published papers, also included are publications by legitimate professional organisations, subject materials from universities, presentations to scholarly workshops, official communication between academics and materials from academically recognised websites. Published papers are predominantly from 1995 to 2007.

A large number of published papers are about computer-based/technologically up-to-date methods. However it is useful to note that in many of these articles, the use of technology is combined with one or more other methods. It is common to find computer based methods intrinsically mixed with active learning (specifically PBL – usually small group).

There are also many papers written about PBL. Springer *et al's* **[13]** paper on small group learning has been considered to be related to the PBL approach. This is because small collaborative groups characterise a common form of PBL.

The variability within the student population has been recognised by a number of authors. This has again been linked to active learning (or specifically PBL – usually small group). The multidisciplinary approach is the least written about, however this method has been gaining ground in Australian and New Zealand universities.

Most of the articles surveyed present a specific teaching method or discuss aspects of a particular method. Approximately half of these works include some discussion about the method's success (or failure) usually as related to learning enhancement, however this information is often obtained informally through student feedback or through the authors' experience and observation. Four papers provided some formal quantitative measure of performance or effectiveness of a method or a mix of methods. All four papers have been peer-reviewed. Three of these were published independently in January 2007. The first paper written by authors from University of Seville **[78]** presented results of a software-based teaching methodology but has strong elements of active learning. The second paper, written mostly by staff of Texas Tech University **[27]** examined the effect of an interactive screen for software implemented active learning. The third paper is from New Mexico State University educators and research staff **[23]**, although their experiment was structured fundamentally around the integrated approach (physical, mathematical and computer models combined), the experimental results can also be established for effectiveness of computer based learning and active or PBL independently. The effect of small-group learning has also been determined quantitatively, and a meta-analysis performed with reports from 1980 to 1999. This was summarised in a highly cited paper (47 citations from 1999) written by Springer *et al* **[13]**.

From the literature review, there are no reports that compare the performance of the different methods of teaching & learning in a measurable and controlled manner. Admittedly, this would be difficult to achieve. Like Wood [89], we also found limited research on the teachers of mathematics to engineering students, papers on the teaching of higher level mathematics - most research focuses on first and second year service teaching and research on alternative teaching and assessment methods in the teaching of mathematics to engineers in Australian universities.

Included in the literature reviewed are a small number of highly cited and well-cited papers. One highly cited paper on small group learning has already been mentioned. Also referred to are six papers that are on average cited once every year (5 citations from 2002) and four other papers that have been cited once every other year on average (2 citations from 2003). The number of highly cited papers in this field is surprisingly low.

The work of Springer *et al* **[13]** has been cited by numerous writers and researchers due to the relevance and extensiveness of its analysis. This paper provides very useful material because it involves quantifiable information synthesized in a systematic way. In addition, it is relevant because it serves as a summary integrating studies from two decades.

4.2 Discussion of Key Ideas

This review of literature indicates that two categories of teaching methods are most frequently claimed to be effective. These are Advanced Computer Based Method and use of PBL.

Computer Method's claimed advantages are that it stimulates interest and it enhances comprehension. While many authors **[26, 28, 40]** argue this to be the case, most do not offer quantitative evidence to support their claim **[30, 36, 38, 39]**. At best, reasoning and conclusions are obtained by means of observation, experience and informal student feedback. Although few papers with empirical evidence have been found **[23, 78]**, they show vital information for engineering maths teaching research. Computer based method also supports students' introduction to the profession, as most industries are reliant on computer applications **[28]**.

An important aspect of computer-based method is that it is often used in conjunction with other methods. As open-ended engineering problems can be given to students and while the computer application being adopted is based on an interactive platform, computer method blends with the problem based learning approach.

Nirmalakhandan *et al* **[23]** provide interesting insight with the results of their study of 131 students of engineering over 5 semesters. Their study has measured the effectiveness (in terms of learning enhancement) of physical and computer models independently, and they have also quantified the effectiveness of integrating these models with the theoretical (traditional) model. However, the results of this study may have some shortcomings due to limited statistical analysis.

The experiment presented by Taraban *et al* **[27]** is about the comparison between interactive (active) and passive learning with respect to learning enhancement. Both of these learning environments are on online platforms. Although they have used established techniques utilising cognition and learning research as well as statistical analysis, data has been obtained indirectly and this may affect the validity of their final results. They have also used a smaller sample of 25 students.

Another experiment by Duran *et al* **[78]** shows the effectiveness of computer based approach in aPBL. The authors claim that increased student satisfaction leads to the success of its implementation. Termed Software-Based Methodology (SBM), their method utilises virtual scenarios for theoretical explanations, all in a highly interactive environment where real examples and simulations are shown in lectures. Questionnaires and cognitive tests have been statistically analysed proving the effectiveness of this method in terms of learning enhancement.

Three computer software applications have been noted to be effectively used in universities as well as in industry. These are MATLAB, Mathcad and POLYMATH. MATLAB is perceived to be the most useful **[4, 38, 39]**. Mathcad is also well regarded **[36, 37]** and POLYMATH has started to gain a lot of support **[30]**.

PBL is claimed to be a method that effectively addresses diversity **[42, 44]**. It has also been identified to support the creative nature of engineering **[58]**. Like computer-based methods, PBL is a good way of assisting students' initiation into the workforce because PBL environment is close to what engineers might encounter in their profession **[67]**. This is to do with the presence of challenging problems usually with disorganised and conflicting requirements **[13]**. If small-group learning is utilised in PBL, this is also similar to what engineers might experience at work, that is, being part of a small collaborative team.

Challenges associated with teaching a diverse student population can be assisted by using methods characterised by active learning **[42, 44]**. Active learning is also advocated by other authors **[23, 27, 28]** from various perspectives. Students are thought to achieve more when they actively participate. Studies that measured the effectiveness of active learning environments support this claim **[23, 27]**. PBL is mostly defined by active learning and there is therefore an additional benefit of this method. Small group PBL may be promoted based on a meta-analysis performed by Springer *et al* **[13]** on small group learning's effectiveness (with respect to learning enhancement) that yielded positive results. In addition, computer based methods are more effective when based on an interactive platform.

Integrated approaches, combining various methods, particularly physical, computer and theoretical (traditional) models are being employed in the US **[41]**. Nirmalakhandan *et al* [23] show quantitatively that the integrated approach can be effective in enhancement of learning.

The main advantage of integrating methods is that it allows educators to mix methods and capture the benefits of each one. Computer-based method easily blends with small-group PBL (or any other method) and there are some areas where multidisciplinary approach may be injected as well. Universities that use integration in the US also include the physical model. The presence of physical models provides real benefit, as shown by results from Nirmalakhandan *et al* **[23]**.

The inclusion of physical models in a projects based small-group PBL utilising computers and electronics applications is providing unmistakable advantage to learning approaches represented in many American universities.

Another advantage of integration is the flexibility it provides. As students "see" a particular problem in different perspectives (via different models), they are given more than one opportunity to grasp the subject matter **[75-76, 79]**. Furthermore, this approach is able to address diversity, with students' differing learning styles catered for by various models. The third benefit of integration is its unifying effect of being able to relate theoretical concepts to physical and computer simulation models. This will assist students see mathematics in the context of engineering and to appreciate the practical applications of theory.

While "lateral" integration of learning materials may be achieved across theory, computer applications and physical models in one subject, as evidenced by the ADFA model **[60-61]** and Canterbury model **[59, 62-64]**, Avitabile *et al* **[75-76, 79]** have also shown that simultaneous "lateral" and "vertical" integration is effective. Their approach at the University of Massachusetts Lowell integrates materials both "laterally" across modules/projects, and "vertically" from earlier subjects to later subjects across several semesters. This strategy when applied to an assortment of modules and topics results in interweaving of learning experiences. This interdisciplinary, multi-semester approach concludes with a final year project in a Dynamic Systems subject.

In order to make the connection between the interrelated subjects and topics, modules have been developed and deployed in the preceding subjects. The injection of these modules enables students to see the connection between earlier materials and later subjects. The strategy is thus fundamentally a "vertical" integration of earlier materials and later materials achieved through the use of modules. However it should be stressed that to some degree, materials and modules are also being combined in each year. As subject elements get interwoven, so do the teaching methods/models employed. These often involve the use of PBL and computer applications. Some physical models are also utilised. Applications of GUI and electronic equipment and devices to projects and hands-on student activities are extensive. Documented direct students' feedback has been positive **[75-76, 79].**

In Australia and in New Zealand, Barry (ADFA) **[60-61]** and Wake (formerly from University of Canterbury) **[59, 62-64]** have independently helped introduce the multidisciplinary approach in teaching Numerical Methods. In addition to collaboration of different departments, the teaching strategy contains a mix of lectures, computer based laboratory and "real world" engineering problems/case studies (PBL). Problem solving is performed through MATLAB. There is substantial departmental cooperation in the ADFA model with three or more academics teaming up to deliver each lecture and each lab session. On the other hand, the Canterbury model has additional support for students in the form of dedicated tutors, regular drop-in help classes, consultation periods, availability of web-based assistance and help with administrative issues.

Above-mentioned multidisciplinary-based subjects share a number of attributes. Both subjects encountered administrative and other challenges, however based on student feedback they have both been very successful in enhancing student learning **[59-64]**. The two subject designs have been newly implemented (2003) and no measurable analysis has yet been performed on their effectiveness.

Curtin University's multidisciplinary feature involves guest lectures by engineering staff reinforcing the importance of mathematics **[54]**.

Focus on mathematical modelling skills in a combined PBL-computer based method further emphasises the need to practise and highlights its closeness to what engineers might expect to experience in the workplace. Referred to as "four leaf clover", this education model developed by Hadgraft **[8]** also incorporates "learning from others" and "learning from literature", together with "modelling task" and personal skill development". The involvement of these four major elements is a strong feature, as well as emphasis on practice and repetition.

Subject and curriculum designers and learning support centre proponents have looked at the variability of students **[47, 87]** and how this may be addressed. Various models and initiatives have been started as a result of this. Streaming students according to their ability is one of them **[31, 55-56]**. Students' variability manifests in their knowledge and ability even prior to entering university. In order to address this, universities such as Curtin **[54]** and Edinburgh **[55]** have introduced subject groupings that enable students to be streamed according to their academic standing. ETH Zurich **[31]** has a fast and a slow stream providing students a choice of subjects.

Variability likewise relates to students' multiple needs and commitments as well as different learning styles. Flexible subjects and support for students with special needs (such as disability or specific health conditions requiring special attention) are useful in this respect. Loughborough University **[47]** and Central Queensland University **[48]** are involved in numerous special programs aimed at supporting the broad engineering student population in their learning endeavours. Some of these projects at Loughborough University are the Maths Learning Support Centre **[47-48, 87]**, the HELM project and the Engineering Subject Centre **[Refer Appendix B]**. These projects are varied and far-reaching. Serving to assist as many students as possible, the programs have given considerable attention to students' varied needs.

5. Conclusions

An engineering mathematics subject should be a part of a well-designed engineering curriculum for its full benefit to be realised. This has been suggested by the European Society for Engineering Education's (SEFI) proposed hierarchical engineering curriculum **[81]**.

It can also be seen from the literature that if based primarily on learning enhancement, a combination of various effective teaching methods may be the best strategy in teaching mathematics to engineering students since that can capture the value of each method. Some combinations reported to be effective are PBL-computer based **[23, 78]**, multidisciplinary-computer based-part PBL **[59]** and PBL-multidisciplinary-computer based **[61]**. It is essential to be able to combine the methods effectively; but in an effective subject design, these methods (computer based, multidisciplinary and PBL) merge well. Most of the reported use of PBL is small group PBL, and with evidence presented for small-group learning's success **[13]**, it is worthwhile noting the benefits of using small groups.

Taking this approach of combining methods to another level, many American universities have integrated theoretical, physical and computer models in a dynamic, projects-based (PBLE) approach. This is demonstrated by mathematics laboratories of University of Delaware and others **[41]**. The distinguishing feature of this strategy is the presence of physical models. Moreover, having three models which can be viewed in three different perspectives, helps students better recognise the interrelationships between the problem itself, the theory and the physical and computer models.
This improves their understanding especially if the integration is accomplished over several semesters, similar to the multi-semester project **[75-76, 79]** at the University of Massachusetts Lowell. At the University of Delaware, the MEC Lab was used first as a "capstone" course in the fourth year of undergraduate study. In earlier years, there are mathematics subjects that require abstract concept development. We are not aware of any study that investigates how much time is required for individual contemplation of abstract concepts. Britton *et al* **[90]** attempted to create an index to measure the ability of science students to transfer mathematics, this was later modified by Roberts *et al* **[91].** They found that, while students who scored high marks in the in-context questions in the test were able to transfer mathematical knowledge to out-of-context mathematics question, a large proportion of those who scored average marks in the in-context mathematics question were unable to answer the out-of-context questions. This is a worrying finding and indicates that teaching mathematics in context to weaker students limits their ability to use the mathematics in any other context.

Integration may be achieved "laterally" in various degrees. Examples of this are subject designs presented in **[23, 59-64]**. In this case, there is usually only one subject involved, with different modules utilised in the subject to convey various viewpoints. Integration has also been found to be successfully implemented in an interwoven fashion, that is, both "vertically" and "laterally" at the same time. This is the strategy developed **[75-76, 79]** at the University of Massachusetts Lowell. Lateral integration takes place across various modules in the same year, while vertical integration is achieved with interconnected materials from most years of the undergraduate program. Interweaving results from simultaneous lateral and vertical integrations, especially when applied to an assortment of modules or projects. Positive effects of this strategy are evidenced by documented students' direct comments and responses to questionnaires, but not by formal quantitative study.

The multidisciplinary approach in teaching Numerical Methods introduced in Australia and New Zealand **[59-64]** is relatively new but is very encouraging. Although fundamentally based on the collaboration of different departments in the subject's delivery, it is similar to other strategies reported in that it uses a combination of various teaching methods. The Canterbury model also has additional support **[59, 62-64]** for students in the form of dedicated tutors, regular drop-in help classes, consultation periods, availability of web-based assistance and help with administrative issues. This may be likened to Math Centres **[47-48]** of some European universities.

Providing additional support and resources through avenues similar to Loughborough University's Math Centre (Math Centre includes a drop-in centre and a Math Centre website) further helps to enhance student learning **[48]**. The provision of varied resources also addresses flexibility requirement and responds to students' varied learning styles. This is achieved through the provision of materials and resources ranging from traditional hardcopy handouts and exercises to downloadable video tutorials from the internet, and from personal one-to-one tutorial at the centre to remote 24-hour web-based support.

It would be interesting to investigate more closely the combinations of approaches. A better comparison of their achievements can then be performed. It is also worthwhile considering a comparison of the effectiveness of two or more individual methods particularly in a measurable and controlled manner. No literature reviewed has evidenced such investigations.

It may also be valuable to note that no papers or studies encountered have discussed in detail the impact of cost, time and resources in determining which teaching methods to use. While small-group PBL, computer method or integrated approach may be most effective in enhancing learning, their full implementation may be limited by their associated costs, implementation time and resources.

Although many articles argue the merits of innovative methods of teaching, it is not suggested that the traditional lecture has unconditionally lost its usefulness. Naturally, there needs to be some innovative content in peer reviewed publications so there is not so much material on the most effective styles for conducting traditional lectures and tutorials.

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Appendices

Appendix A

Definition of Terms

Mathematics/Statistics Department: an academic unit whose chief responsibility is the teaching of mathematics and/or statistics. This may be a team within a school or a faculty.

Engineering Department: an academic unit whose chief responsibility is the teaching of one or more branches of engineering. This may be a team within a school or a faculty.

Subject: a study of a particular set of topics usually over a period of 12 to 14 weeks which is assessed as an individual element within a degree program.

Engineering Degree Program: the complete 3 to 4 year study program majoring in any strand of engineering.

Engineering Mathematics subject: a subject that is at least 50% mathematical in content taken by students undertaking an undergraduate engineering degree program.

Appendix B

Mathematics Education for Engineering Students in Europe and the United States

The provision of mathematics education to engineering students faces many challenges. These challenges are being addressed by various groups worldwide, including:

- European Society for Engineering Education (SEFI) based in Brussels [http://www.sefi.be/]
- Loughborough University [http://www.lboro.ac.uk/] in Leicestershire, UK and other proponents of Mathematics Support Centres (Coventry University and University of Leeds) [http://mathcentre.ac.uk/]
- Helping Engineers Learn Mathematics (HELM) consortium [http://helm.lboro.ac.uk/ and http://helm.lboro.ac.uk/pages/consortium.html]
- School of Mathematics Science & Technology, Institute of Education, University of London have undertaken various projects and studies [http://ioewebserver.ioe.ac.uk/ioe/cms/get.asp?cid=54]
- The Ove Arup Foundation based in the UK, particularly the May 2003 report "Mathematics in the University Education of Engineers" ([http://www.theovearupfoundation.org/pages/index.cfm]
- The IDEA League (composed of **I**mperial College London, **D**elft University of Technology, **E**TH Zürich, University of **A**achen and Ecole Polytechnique de Paris) in their London Workshop in April 2006 (devoted to Mathematics in Engineering) [http://www.idealeague.org/news/newsletter/2006/news29/workshop]
- The University of Massachusetts Lowell's interdisciplinary project [http://dynsys.uml.edu/]
- University of Delaware [http://www.math.udel.edu/MECLAB/] and eight other American universities supporting MEC Labs (Courant Institute New York, Massachusetts' Institute of Technology, New Jersey Institute of Technology, Pennsylvania State University, University of North Carolina at Chapel Hill, University of Arizona, North Carolina State University and Georgia Institute of Technology)
- The Connected Curriculum Project (CCP) at Duke University, Montana State University and California Polytechnic State University, San Luis Obispo. [http://www.math.duke.edu/education/ccp/aboutccp.html]

Europe

Europe has in the past decade addressed the broader issue of the teaching and learning of mathematics (for all disciplines, but with particular focus on engineering students). For example, see the description of the LTSN MathsTEAM Project. On the other hand, some engineering organizations (e.g. SEFI) have shown particular interest in mathematics education.

European Society for Engineering Education (SEFI)

SEFI proposed a Core Curriculum based on learning outcomes with a hierarchical structure arranged in four levels [81]. This curriculum assumes a Core Zero prerequisite knowledge that is an essential foundation to move at least to the first level. If this (Core Zero knowledge) has not been achieved upon entry into the program, then Core Zero topics may be taught in support classes running in parallel. Level 1 is Core Knowledge to be covered in Year 1 of the engineering course. This comprises fundamental material essential to every engineering student (with the exception of Computer and Software Engineering students whose curricula follow a different route). Level 2 consists of more advanced knowledge and skills particular to the specific engineering discipline. Practical engineering examples characterise the teaching at this level. Level 3 involves highly specialist knowledge and skills and requires a comprehensive but condensed integration of mathematics and engineering. Generally speaking, the curriculum specifications on mathematics are quite expansive.

Loughborough University

Loughborough University [http://www.lboro.ac.uk/] in Leicestershire, UK, has by way of its initiatives and projects, shown considerable commitment to the teaching and learning of mathematics. Its Mathematics Learning Support Centre (housed at the Mathematics Education Centre) [http://mlsc.lboro.ac.uk/] holds many different kinds of learning resources and it also provides one-to-one assistance to students studying maths.

Below are some of the main innovative features of Loughborough University's Maths Learning Support Centre [87] :

- HELM Project – delivery of engineering maths through self study open learning
- Engineering student Support Desk – offers web based materials to students and a Power Mac G5 for video and audio processing
- Support available for students with special needs – examples of this type of one-to-one tutoring is given to dyslexic and dyscalculic students
- Videos that show the use of mathematics in modelling engineering problems
- Computers and software – network of 5 PC's; packages perform calculation in algebra, calculus and others, packages also perform graph plotting
- Computer Assisted Learning (CAL)
- Math Centre's dedicated website

- Other resources include leaflets, posters, handouts, workbooks, textbooks, one-to-one tutorial, specialist statistical help, lunchtime refresher courses, study sheets, diagnostic test, graphical calculators and overhead projectors.

Helping Engineers Learn Mathematics (HELM)

The HELM Project was a major curriculum development project undertaken from October 2002 to September 2005 by an alliance of five universities (Loughborough, Manchester, Reading, Hull and Sunderland) [http://helm.lboro.ac.uk/]. It was funded by Higher Education Funding Council for England (HEFCE) and was intended to improve the mathematical education of engineering students by the provision of learning resources with emphasis on computer aided learning and assessment. HELM's resources include:

- Fifty workbooks of explanations, worked examples and cases studies
- Web-based Computer Aided Learning (CAL) packages
- Two modes of Computer Aided Assessment (CAA); web-delivered and CD version.

This was a remarkable outcome on a modest budget.

MathCentre

Mathematics Support Centres are housed by a number of universities in the UK. The collaboration between these universities is exemplified by Math Centre [http://mathcentre.ac.uk/], a group consisting of universities that run maths support centres (Loughborough University [http://mec.lboro.ac.uk/], Coventry University [http://www.mis.coventry.ac.uk/maths_centre/] and University of Leeds [http://www.maps.leeds.ac.uk/]) and representatives from the Educational Broadcasting Services Trust and representatives from UK Learning and Teaching Support Networks (LTSN).

MathCentre provides mathematics learning materials free of charge to anyone. The website [http://mathcentre.ac.uk/] has learning materials for mathematics learners from different disciplines, including Engineering. The success of MathCentre is evidenced in the high ratings received from the monitored student feedback section of this website.

On the MathCentre website, twelve main engineering mathematics topics are covered including complex numbers, sequences & series, matrices and vectors. Each of these main topics has subtopics that are linked to ‘teach-yourself booklets’. These booklets have been rated excellent by users [http://mathcentre.ac.uk/]. Aside from ‘teach-yourself booklets’, the engineering sub-topics also generally contain online exercises and diagnostic tests, as well as video tutorials some of which can be downloaded to an iPod. All of these other learning materials have likewise received high ratings from users. Other disciplines have links to online refresher booklets as well.

The MathTutor [http://www.mathtutor.ac.uk/index.shtml] is a set of seven DVD-Rom disks written to assist students of mathematics. MathTutor was created by the same group who created MathCentre and was funded by the HEFCE Fund for the Development of Teaching & Learning and the Gatsby Technical Education Project/Higher Education Academy. The disks contain eighty topics with summary texts and exercises, diagnostics and video tutorials.

Engineering Subject Centre

Engineering Subject Centre [http://www.engsc.ac.uk/us/what_engsc/index.asp] is part of the Higher Education Academy (HEA) and contains two thousand teaching and learning resources. It is based in the Faculty of Engineering at Loughborough University. It was formed in May 2004 from the merger of the Institute of Learning and Teaching in Higher Education (ILTHE), Learning and Teaching Support Network (LTSN) and the Teaching Quality Enhancement Fund National Coordination Team (NCT). The Engineering Subject Centre is the "national centre for all engineering academics in the UK", it administers support to UK engineering higher education primarily by sharing good practice.

LTSN MathsTEAM Project

The LTSN MathsTEAM Project (funded by the LTSN) carried out an in-depth survey, which led to the publication of three booklets:

- *Maths Support for Students* [http://www.mathstore.ac.uk/mathsteam/packs/student_support.pdf]
- *Maths for Engineering and Science* [http://www.engsc.ac.uk/downloads/mathsteam/engineering_science.pdf]
- *Diagnostic Testing for Mathematics* [http://www.mathstore.gla.ac.uk/index.php?pid=60]

Each booklet provides a comprehensive collection of case studies, intended to assist and inspire educators of mathematics. Contributing authors discuss current teaching practice and the barriers and enablers to setting up these learning initiatives. The booklets are intended to assist in the transfer of knowledge within higher education communities informing mathematics educators of effective new and innovative teaching and learning practices. The LTSN scheme has now been superceded by the Higher Education Acadamey's Subject Centres, including the Maths, Stats and OR Network (http://www.mathstore.ac.uk/).

The IDEA League

The IDEA League [http://www.idealeague.org/about/index] is an association composed of five leading universities. They are:

- **I**mperial College London
- **D**elft University of Technology, Netherlands

- **E**TH Zurich, Switzerland
- University of **A**achen, Germany
- Ecole Polytechnique de Paris

The main objective of IDEA League is to provide a platform for discussing common issues of teaching and curricula and exchange experiences and best practice in those areas. In April 2006, the IDEA League held a workshop 'Mathematics in Engineering'. This was hosted by Imperial College and included topics such as 'Mathematics Teaching with Respect to the Diversity of Engineering Subjects' and 'Computer Use in Mathematics'.

Jorg Waldvogel [http://www.math.ethz.ch/~waldvoge/], in his presentation at the IDEA League 'Workshop on Mathematics in Engineering' [31] talked about the various methods in use at ETH Zurich to cope with diversity of engineering students. These include; motivating engineering students, streamlining study, collaboration between mathematics and engineering staff, using practical engineering examples, using software applications in teaching & other innovative ideas such as teaching numerical analysis along with calculus.

The United States

Mathematics and Engineering Departments at several universities in the United States utilise an integrated form of active learning. Universities examined include the University of Massachusetts Lowell (Interdisciplinary Multi-semester Project on Dynamic Systems) and most universities that support laboratories such as University of Delaware (MEC Lab).

Innovative teaching methods used in these universities are consistent with an integrated approach characterised by active learning environments (noting that problem based learning is one form of active learning). This approach encourages active learning "learning by doing" and combines different models mostly consisting of the physical, mathematical and computer simulation model. Electronic equipment, devices and machines, including computers, are widely used especially in measuring, collecting, storing and analysing data. Projects are often required of students working in small groups [23], challenging them to "rationalise, reconcile, predict and validate" theoretical knowledge against the physical model.

The Modelling, Experiment and Computation Laboratory (MEC Lab)

At the University of Delaware's Department of Mathematical Sciences, innovative strategies are used to enhance the educational experience of mathematics students. The MEC Lab [http://www.math.udel.edu/MECLAB] supports projects where mathematics has direct connection with a physical science phenomenon or experiment, hence its wide scope in engineering. A number of engineering students complete a mathematics minor by taking a 500-level subject with mathematics students in the laboratory.

The MEC Lab is a place where students, teachers and even visitors carry out research and project work related to applied mathematics. It is equipped with PCs running PASCO's Data Studio datalogging software [http://www.pasco.com/datastudio/] allowing the use of various sensors and recording information directly to the PC. Data Studio also has some data processing capabilities. In addition to the PC setups, MEC Lab has a range of machine tools, video equipment, power supplies and other miscellaneous materials. Past laboratory activities have been associated with courses such as partial differential equations, mathematical methods, asymptotic methods and fluid dynamics. Examples of engineering concepts handled are magnetic fields, harmonic motion, electrostatic deflections and many others. Research and project work based at the MEC Lab are usually published in the MEC Lab website. Teaching staff are encouraged to incorporate MEC Lab activities in their courses.

Other universities that have similar laboratories in their applied mathematics departments are the following:

- Courant Institute, New York - The AML, Applied Mathematics Lab
- Massachusetts Institute of Technology - Fluids Lab
- New Jersey Institute of Technology - Capstone Lab

- Pennsylvania State University - Fluids Lab
- University of North Carolina at Chapel Hill - Fluid Lab
- University of Arizona - The Applied Math Lab
- North Carolina State University - Instructional and Research Lab
- Georgia Institute of Technology - The ACE Lab

Connected Curriculum Project (CCP)

The Connected Curriculum Project (CCP) is a coordinated effort to develop interactive learning materials for mathematics and mathematics applications, including Engineering Mathematics. The project involves the Mathematics Departments at Duke University, Montana State University and California Polytechnic State University [http://www.math.montana.edu/frankw//ccp/modeling/topic.htm - continuous and http://www.math.duke.edu/education/ccp/aboutccp.html. These interactive materials combine the Web with specific computer algebra systems such as Mathcad, Mathematica and Maple and contain the following additional features and tools:

- Hypertext Links
- Java Applets
- Sophisticated Graphics
- Realistic Scenarios
- Thought-provoking Questions
- Summary Questions

CCP materials are structured into modules, which are single-topic units [http://www.math.duke.edu/education/ccp/aboutccp.html]. These modules require varying amounts of time to complete. This structure makes them flexible to use. They can be used singly or in combination with each other or in combination with other teaching and learning methods. They can also be integrated into courses or used as supplements to usual classroom activities. Lastly, they can be used by individual students and by group of students alike.

Interactive learning materials such as the ones developed by CCP are particularly suited to students of today. The flexibility offered by electronic environment allows designers to incorporate various forms of materials in a way that can convey optimum information, be effective and be interesting to the students. It gives teachers another learning method which can support classroom activities and which places students in an environment in which they are quite comfortable (today's students are comfortable and well-versed with e-learning). The electronic environment allows learning materials to cater to students' varying abilities, needs and situations. It is also a platform for incorporating materials and information that engineering graduates expect to find useful when they join the workforce.

Interdisciplinary, Multi-semester project on Dynamic Systems

Students often do not clearly understand how materials in earlier courses relate to higher level courses. Dr. Peter Avitabile of Mechanical Engineering Department, University of Massachusetts Lowell, and his team attempted to address this by introducing an interdisciplinary, multi-semester project integrating mathematics and engineering **[75-76, 79]**. This has been applied to a Dynamic Systems course by progressing a series of modules. These modules integrate basic materials from pre-requisite courses, and problem solving materials in later courses. The modules have been set up in pre-requisite courses such as:

- Differential Equations (2nd Year)
- Mechanical Laboratory (3rd Year)
- Numerical Methods (3rd Year)

While topics covered include:

- Numerical Integration/Differentiation
- Visualisation Tools for Understanding 1st and 2nd Order System Response Characteristics
- Understanding Complex Frequency Response Characteristics
- Development of a Virtual Measurement System.

"These modules have been inserted into the pre-requisite courses to enable the students to develop skills on a continuing project that is threaded through the development of the material across several related courses. This culminates in a Dynamic Systems project that forces the students to integrate many previously learned skills in a meaningful manner" **[76]**

Appendix C

Summary of Reported Quantitative Studies on the Effects of Innovative Teaching Methods

A summary of quantitative studies reported regarding the effects of innovative teaching methods are presented in the following table:

A Summary of Reported Quantitative Studies on Effect of Innovative Teaching Methods

Author/Paper	Number of Respondents/ Length of Time	Main Method Studied	Evaluation Method	Comments/Notes
Nirmalakhandan et al, Jan 2007 (Refereed)	131 in 5 semesters	Integrated Active Learning Approach (Physical, Mathematical & Computer Simulation Models Combined)	Student Evaluations (Surveys), Student Performance	Excellent Affirmation (90% of Respondents), Improved Final Grades (% of Students who passed increased from 70% to 86%)
Taraban et al Jan 2007 (Refereed)	25 (includes science students)	Software Implemented Active Learning (Interactive vs Plain Text both on Online Platform)	Students' verbalisations while completing task (verbal protocol analysis)	Mean verbalisation per screen of 4.30 for Interactive VS approx 2.85 for Plain Text
Duran et al Jan 2007 (Refereed)	Approx 85 (different pre- and post test)	Increased Learner Satisfaction by combination of real examples and virtual simulations (software-based) in a highly interactive PBL context	Pre- and Post-Implementation Questionnaires and Cognitive Tests	Questionnaire – increment positive in 14 of 17 items Cognitive Test – increment positive for all different dimensions
Springer et al Spring 1999 (Highly Cited) (Refereed)	39 reports from 1980 to 1999	Small Group Learning	Standardised mean difference (d-index) effect size	Achievement value 0.51 versus average 0.40; Persistence value 0.46 reduce indolence by 22% ; Attitude value 0.55 versus average 0.28
END OF ENTRIES				