MENTAL COMPUTATION:
A STRATEGIES APPROACH

MODULE 6
ratio and percent

Shelley Dole

Mental Computation: A strategies approach

Module 6 Ratio and percent

Shelley Dole

This is one of a set of 6 modules providing a structured strategies approach to mental computation.

Module 1 Introduction
Module 2 Basic facts Addition and subtraction
Module 3 Basic facts Multiplication and division
Module 4 Two-digit whole numbers
Module 5 Fractions and decimals
Module 6 Ratio and percent

Second Edition, Hobart Tasmania 2005

University of Tasmania
Department *of* Education Tasmania
Catholic Education Office Hobart
Department of Education and Training, Australian Capital Territory

This set of modules was prepared as part of Strategic Partnership with Industry – Research and Training (SPIRT) scheme project (C00107187): Assessing and Improving the Mental Computation of School-age Children.

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The contribution of all partners to the project is acknowledged, including
The Industry Partners:
Department *of* Education, Tasmania
Catholic Education Office, Hobart
Australian Capital Territory Department of Education and Training
and The Research Partner:
The University of Tasmania

The significant contribution of the Department *of* Education, Tasmania in terms of funding and time is acknowledged by all of the partners.

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Dominic College, Tas.
Holy Rosary School, Tas.
Lanyon High School, ACT
Lilydale District High School, Tas.
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Printed by: Printing Authority of Tasmania

Publisher: Department *of* Education, Tasmania, 2004 University of Tasmania

ISBN: 1 920865 25 X

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OVERVIEW OF MODULE 6

INTRODUCTION

This module is the sixth in a series of six that comprise a resource of activities for developing students' mental computation. The focus of this module is on mental computation involving ratio and percent.

TEACHING SEQUENCE

The Activities presented in this module are based on two major principles for developing mental computation skills in ratio and percent:

1. Building conceptual understanding to make calculation meaningful; and
2. Encouraging the use of visual images to assist in mental computation.

Rather than being a collection of ideas, strategies and games for mental computation, each Activity is a sequential teaching episode to develop a particular aspect of ratio and percent knowledge and understanding for mental computation.

Each Activity is designed to stand-alone, but prior knowledge may assist students' performance in some cases, and such prior knowledge may be promoted through a previous Activity. Collectively, the Activities are not designed as a curriculum for ratio and percent, but rather to augment and build students' knowledge of these topics to assist meaningful mental computation.

For each Activity, the *Aim* of the Activity is given, summarising the conceptual basis of the strategies presented. An *Overview* of the Activity is also provided, enabling the nature of the Activity to be readily gleaned. *Materials* are indicated, with the letters **BLM** followed by a reference number.

Black Line Masters are located at the back of the booklet. *Teaching Points* are presented in dot-point form for succinctness, serving as "reminder tips" that focus the key points of the Activity. Tips for assessing students' conceptual understanding, mental computation strategies, and mental computation performance are provided under the heading of *Assessing Performance*. Carefully selected *Practise Examples* are presented to indicate the types of calculations students would be expected to perform mentally. The practise examples are aimed to serve as a guide for devising further sets of calculations for consolidation purposes.

The sequence for each teaching episode is presented on the facing page under the heading: *Activity Outline*. The sequence is numbered to indicate the steps along which the teaching episode proceeds.

ACTIVITY 6.1 RATIO IMAGES

AIM
To provide students with a mental image of ratio equivalence to assist in mental computation.

OVERVIEW
In this activity, students use counters of two different colours to explore the multiplicative relationships within simple ratios.

MATERIALS
Approximately 40 counters (Unifix, cubes, or other suitable material) of two different colours (20 of each colour) per student.

TEACHING POINTS
- Encourage students to continue to display each ratio situation until they can visualise the solution.
- Discuss visual images and strategies students use to arrive at the answer.

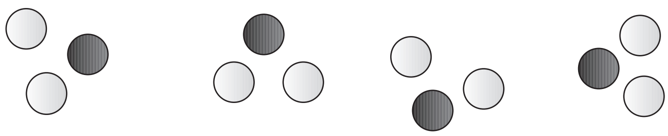
ASSESSING PROGRESS
- Students readily discuss their visual images of ratios.
- Students discuss appropriateness of various visual images of ratios, describing ratios in terms of parts in relation to the whole.
- Students' mental computation of simple ratios amounts becomes more accurate.

PRACTICE EXAMPLES

1 : 4	is	2:___	2 : 1	is	10 : ___
1 : 3	is	2:___	3 : 1	is	9 : ___
1 : 2	is	3:___	3 : 1	is	12 : ___
1 : 2	is	4:___	5 : 1	is	15 : ___
1 :4	is	5:___	4 : 1	is	12 : ___

ACTIVITY OUTLINE

1. Establish students' knowledge of the meaning of ratio, and their understanding of the symbolic recording (e.g., what does 1:2 mean?)
2. Ask students to use their counters to represent a ratio of 1:2 (for every red counter, there must be 2 whites).
3. Ask students to continue to arrange more groups of counter in this fashion until there are 4 red counters:



4. Guide students to see the
   multiplicative nature of ratio through such questions as the following:
   How many red counters? (4).
   How many white counters? (8).
   What is the pattern? (doubling).
   Why? (for the ratio of 1:2, need two times as many white counters as red counters).
5. Do the same for the ratio of 1:3. Encourage students to verbalise the relationship between the numbers in each group as a multiplicative relationship.
6. Do the same for the ratio of 4:1.
7. Practise some mental computation of unit ratios by asking students to close their eyes and imagine they are moving counters to determine solutions to the following:

   If red and white counters are in the ratio of 1:4, how many white counters will there be if there are 3 red counters?

   If red and white counters are in the ratio of 1:3, how many white counters will there be if there are 2 red counters?

   If red and white counters are in the ratio of 1:2, how many white counters will there be if there are 6 red counters?

   If red and white counters are in the ratio of 5:1, how many white counters will there be if there are 10 red counters?

   If red and white counters are in the ratio of 3:1, how many white counters will there be if there are 9 red counters?

8. Discuss students' mental images and strategies. Encourage students to describe their thinking in terms of multiplication.

ACTIVITY 6.2 SIMPLIFYING RATIOS

AIM

To link ratio simplification to the process of division, and to provide students with a mental image for performing ratio simplification in mental computation situations.

OVERVIEW

Students manipulate counters to enact ratio simplification. This activity is a reversal activity of 6.1.

MATERIALS

Approximately 40 counters (Unifix, cubes, or other suitable material) of two different colours (20 of each colour) per student.

TEACHING POINTS

- Encourage students to continue to display each ratio situation until they can visualise the solution.
- Discuss visual images and strategies students use to arrive at the answer.

ASSESSING PROGRESS

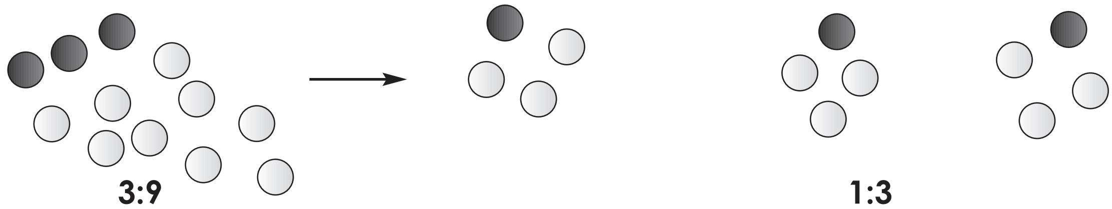
- Students readily discuss their visual images of ratios.
- Students discuss appropriateness of various visual images of ratios, describing ratios in terms of parts in relation to the whole.
- Students' mental computation of simple ratios becomes more accurate.

PRACTICE EXAMPLES

3 : 6	is	1:___	15 : 5	is	3 : ___
2 : 4	is	1:___	20 : 4	is	5 : ___
5 : 10	is	1:___	8 : 4	is	2 : ___
5 : 15	is	1:___	6 : 3	is	2 : ___
4 : 12	is	1:___	12 : 4	is	3 : ___

ACTIVITY OUTLINE:

1. Refresh students' knowledge of the meaning of ratio, and their understanding of the symbolic recording (e.g., what does 1:2 mean?).
2. Ask students to visualise red and white counters in the ratio of 1:3 and ask them to determine how many white counters if there are 3 reds (9). Ask students to state how many counters in total.
3. Ask students to place 12 counters to represent the situation visualised previously, and to name the ratio represented (3:9).



4. Ask students to distribute the counters evenly so that there is only one red in each group.
5. Discuss the process: the smaller group is separated and the other counters distributed evenly. Link this action to the process of division.
6. Practise with other counters in various groupings so that students can find the simplest form ratio:

   4 red and 8 white = 4:8 = 1:2

   3 red and 6 white = 3:6 = 1:2

   6 red and 18 white = 6:18 = 1:3

   5 red and 15 white = 5:15 = 1:3

   6 red and 3 white = 6:3 = 2:1

   20 red and 10 white = 20:10 = 2:1

   4 red and 16 white = 4:16 = 1:4

   3 red and 12 white = 3:12 = 1:4
7. Practice some mental computation of unit ratios by asking students to close their eyes and imagine they are moving counters to determine solutions to the following:

   If red and white counters are in the ratio of 3:6, how many white counters for every red counter?

   If red and white counters are in the ratio of 4:8, how many white counters for every red counter?

   If red and white counters are in the ratio of 5:15, how many white counters for every red counter?

   If red and white counters are in the ratio of 12:3, how many red counters for every white counter?

   If red and blue counters are in the ratio of 12:4, how many red counters for every white counter?
8. Discuss students' mental images and strategies. Encourage students to describe their thinking in terms of division.

ACTIVITY 6.3 RATIO GROUPS (not unit ratio)

AIM
To assist students visualise manipulation of a set when performing ratio mental computation.

OVERVIEW
In this activity students use counters of two different colours to explore the multiplicative relationship within non-unit ratios. This activity links to 6.1.

MATERIALS
Approximately 20 counters (Unifix, cubes, or other suitable material) of two different colours (20 of each colour) per student.

TEACHING POINTS
- Encourage students to continue to display each ratio situation until they can visualise the solution.
- Discuss visual images and strategies students use to arrive at the answer.

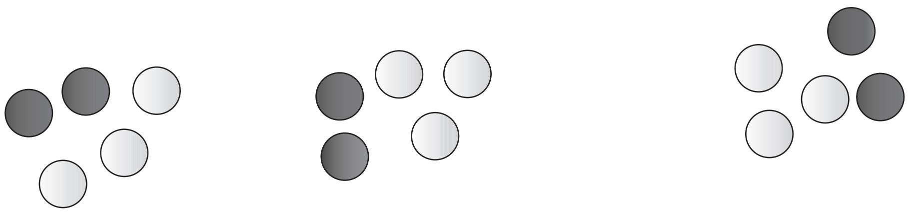
ASSESSING PROGRESS
- Students readily discuss their visual images of ratios.
- Students discuss appropriateness of various visual images of ratios, describing ratios in terms of parts in relation to the whole.
- Students' mental computation of ratios amounts becomes more accurate.

PRACTICE EXAMPLES

2 : 3	=	6 : ___	5 : 3	=	15 : ___
2 : 3	=	8 : ___	5 : 3	=	10 : ___
2 : 3	=	12 : ___	4 : 3	=	8 : ___
2 : 5	=	4 : ___	4 : 3	=	20 : ___
2 : 5	=	6 : ___	5 : 2	=	10 : ___

ACTIVITY OUTLINE

1. Establish students' knowledge of the meaning of ratio, and their understanding of the symbolic recording (e.g., what does 2:3 mean?)
2. Ask students to use their counters to represent a ratio of 2:3 (for every two red counters there are 3 blue counters).
3. Ask students to continue to arrange more groups of counters in this fashion until there are 6 red counters:



4. Pose guiding questions so that students see the multiplicative relationship within ratios:

   How many red counters? (6).
   How many white counters? (9).
   What is the pattern? (multiply each group by 3).
   Why? (to keep the ratio of 2:3 the same, need to have same amount of reds and whites within each group/collection).
   Link to unit ratios (see 6.1)

5. Do the same for the ratio 3:4. Encourage students to verbalise the relationship between the numbers in each group as a multiplicative relationship.
6. Do the same for the ratio of 2:5.
7. Practise some mental computations of non-unit ratios by asking students to close their eyes and imagine they are moving the counters to determine solutions to the following:

   If red and white counters are in the ratio of 2:3, how many white counters will there be if there are 6 red counters?
   If red and white counters are in the ratio of 4:3, how many white counters will there be if there are 8 red counters?
   If red and white counters are in the ratio of 2:5, how many white counters will there be if there are 6 red counters?
   If red and white counters are in the ratio of 5:3, how many white counters will there be if there are 15 red counters?
   If red and white counters are in the ratio of 2:3, how many white counters will there be if there are 6 red counters?

8. Discuss students' mental images and strategies. Encourage students to describe their thinking in terms of multiplication.

ACTIVITY 6.4 RATIO PART/WHOLE LINKS

AIM

To promote conceptual understanding of ratios that can be viewed as comprising their inherent parts, and also as parts in relation to the whole. Mental visualisation of ratios as comprising parts within a whole is the mental computation strategy promoted here.

OVERVIEW

In this activity, students use counters of two different colours to represent various ratios, and to explore the part/part/whole nature of resulting ratio situations.

MATERIALS

Approximately 40 counters (Unifix, cubes, or other suitable material) of two different colours (20 of each colour) per student.

TEACHING POINTS

- Encourage students to continue to display each ratio situation until they can visualise the solution.
- Discuss the visual images and strategies students used to arrive at the answer.

ASSESSING PROGRESS

- Students discuss appropriateness of various visual images of ratios, describing ratios in terms of component parts of the whole.
- Students readily discuss their visual images of ratios.
- Students' mental computation of component parts of ratios becomes more accurate

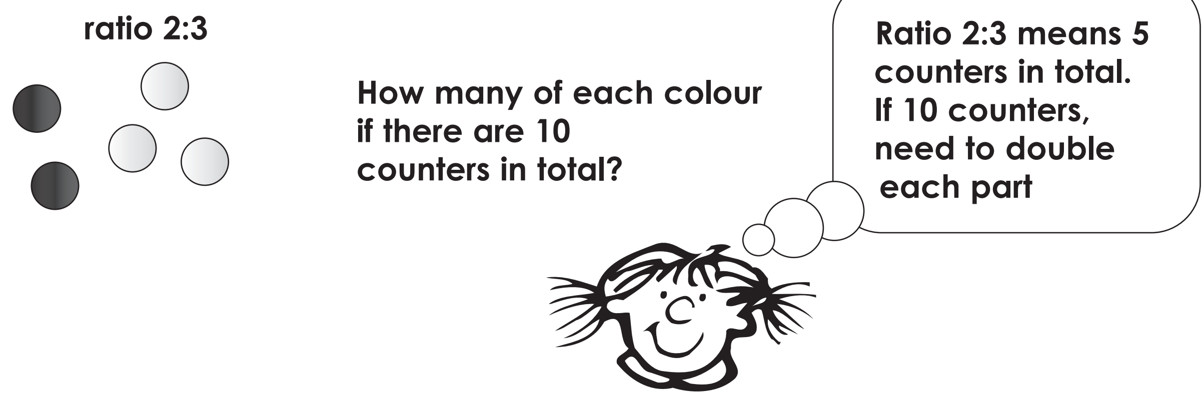
PRACTICE EXAMPLES

Ratio 1:2, how many of each colour if 6 counters?
Ratio 1:2, how many of each colour if 9 counters?
Ratio 1:3, how many of each colour if 8 counters?
Ratio 1:3, how many of each colour if 12 counters?
Ratio 3:1, how many of each colour if 8 counters?
Ratio 2:3, how many of each colour if 15 counters?
Ratio 2:3, how many of each colour if 10 counters?
Ratio 3:2, how many of each colour if 20 counters?
Ratio 4:1, how many of each colour if 10 counters?
Ratio 1:4, how many of each colour if 20 counters?

ACTIVITY OUTLINE

1. Establish students' knowledge of the meaning of ratio, and their understanding of the symbolic recording (e.g., what does 2:3 mean?)
2. Ask students to use their counters to represent a ratio of 2:3 (for every 2 red counters there must be 3 whites).
3. Focus on the part/part/whole nature of the representation through asking questions such as:

   How many red? (2)
   How many white? (3)
   How many counters in total? (5)
   If I wanted a total of 10 counters in the same ratio, how many of each colour would there be? (4 red and 6 white)
   Why?



4. Have students continue to use the counters to explore the part/part/whole nature of other ratios:

   Ratio of 2:3, how many of each colour if there are 15 counters?
   Ratio of 1:4, how many of each colour if there are 15 counters?
   Ratio of 5:1, how many of each colour if there are 18 counters?
   Ratio of 3:4, how many of each colour if there are 14 counters?
   Ratio of 1:3, how many of each colour if there are 20 counters?
5. Practise some mental computation of part/part/ ratio situations by asking students to close their eyes and visualise each situation and solution:

   Ratio of 1:3, how many of each colour if 8 counters?
   Ratio of 1:3, how many of each colour if 12 counters?
   Ratio of 1:3, how many of each colour if 20 counters?
   Ratio of 2:3, how many of each colour if 10 counters?
   Ratio of 2:3, how many of each colour if 20 counters?
6. Ask students to describe their thinking and visual images.

ACTIVITY 6.5 PERCENT COMPLEMENTS

AIM
To consolidate conceptual understanding of percent as a special ratio with a reference base of 100, and to promote mental computation of percent parts and complements through linking to two-digit mental computation to 100.

OVERVIEW
In this activity, students represent real-world percent situations on 10x10 grids, exploring the part/whole nature of percent in relation to 100.

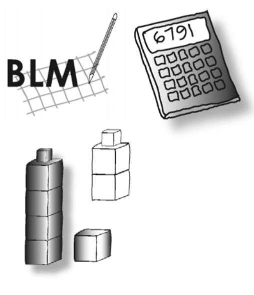
MATERIALS
2 copies of **BLM 6.1** and one copy of **BLM 6.2** per student

TEACHING POINTS
- Keep the pace of the activity if students find this very easy, by moving to mental calculations early.
- Deter students who may wish to ‘colour’ their grids: encourage efficient shading.
- Discuss thinking strategies and actively link to calculation bonds to 100.

ASSESSING PROGRESS
- Students readily discuss their visual images of percentage parts and complements.
- Students link their mental strategies to two-digit whole number mental addition and subtraction computation of numbers to 100.
- Students’ mental computation becomes more accurate

PRACTICE EXAMPLES
What is the complement of:

25%	$33\frac{1}{3}$ %
70%	42%
16%	78%
29%	13%
45%	22%



ACTIVITY OUTLINE

1. Establish students' understanding of 100% as one whole.
2. Distribute **BLM 6.1** (10x10 grids) and ask students to shade 50% of one grid. Ask how much is unshaded. Ask students to label their diagram as an addition sum: eg., 50% + 50% = 100%
3. Have students shade other grids and label as above:

20% shaded, 80% unshaded	(20% + 80% = 100%)
48% shaded, ___% unshaded	(48% + ___% = 100%)
16% shaded, ___% unshaded	(16% + ___% = 100%)
96% shaded, ___% unshaded	(96% + ___% = 100%)
35% shaded, ___% unshaded	(35% + ___% = 100%)
23% shaded, ___% unshaded	(23% + ___% = 100%)
78% shaded, ___% unshaded	(78% + ___% = 100%)

4. Distribute **BLM 6.2** and ask students to shade the grid within each diagram to represent the percentages of various substances within each item.
5. Using a second copy of **BLM 6.1**, ask students to represent percentage components within the following items:

Mayonnaise	25% vinegar, the rest cream
Gloves	75% wool, the rest rayon
Cordial drink	$33\frac{1}{3}$% cordial, the rest water
Cup of coffee	$12\frac{1}{2}$% coffee, $12\frac{1}{2}$% milk, the rest water
Shorts	equal amounts of cotton and rayon
Socks	equal amounts of rayon and nylon
Sausages	beef, pork, and lamb in equal amounts
Yoghurt	twice as much yoghurt as fruit

6. Practise mental part/complement percentages by asking student to close their eyes and visualise solutions to the following:

   If I drank 25% of the milk, what percent is left?

   Twenty-eight percent of the smarties were green, what percent were not green?

   The jacket was discounted 25%; what percent of the original price did I have to pay?

   Rump steak is 92% beef; what percent is fat?

   Soap contains 0.5% perfume, 37% pure soap, and the rest is fat. What percent is fat?

   The human body is about 78% water. What percent is made up of other "stuff"?

7. Discuss students' mental strategies and images.

ACTIVITY 6.6 PERCENT - FRACTION - DECIMAL EQUIVALENCE

AIM
To promote equivalence understanding of percents, fractions and decimals.

OVERVIEW
In this activity students represent a variety of percentages on 10x10 grids to explore equivalence between percents, fractions and decimals.

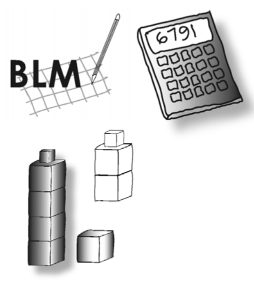
MATERIALS
BLM 6.1

TEACHING POINTS
- Check that students are shading percents as blocks, not individual squares.
- Maintain flow of activity by encouraging efficient shading in one colour, not "careful" colouring using many colours.
- Have students close their eyes after each picture to visualise fraction equivalence.

ASSESSING PROGRESS
- Students readily discuss their thinking strategies.
- Students readily state a fraction, decimal or percent equivalence for any given fraction, decimal, percent less than 1.

PRACTICE EXAMPLES

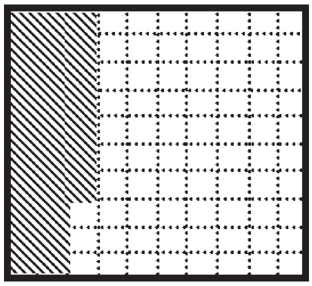
Name the fraction that is equivalent to:	Name the percentage that is equivalent to:	Name the percentage that is equivalent to:
14%	$\frac{29}{100}$	0.75
27%	$\frac{7}{100}$	0.23
99%	$\frac{43}{100}$	0.04
2%	$\frac{9}{10}$	0.59
48%	$\frac{61}{100}$	0.89



ACTIVITY OUTLINE

1. Distribute 10x10 grid pages (**BLM 6.1**) to students.
2. Ask students to select their favourite grid of the 8 on the page, and trace around the outside of this grid with their finger. Make the students state as they are doing this: "This is one whole square."
3. Ascertain that all students are aware that each grid is divided into 100 equal parts.
4. Ask students to shade their selected square to give the best possible picture of 27%.
5. Instruct students to label their diagram using percent, fraction and decimal symbolic representation: e.g., 27% = $\frac{27}{100}$ = 0.27.
6. Ask students to describe what 27% looks like e.g.

   27% is less than 50%
   27% is almost 25%
   27% is 2 x 10% and 7% more
   27% is not close to one whole (100%)



7. Identify different shading representations that students have produced, and generate discussion as to the most effective picture of 27% (i.e., one that shows 27% is made up of 2 x 10% and 7% more). Discuss why shading in this way is a more powerful visual picture than 27 individual squares shaded in a random pattern.
8. Repeat steps 5, 6 and 7 using other percents:

   Shade a picture of 89%

   Shade a picture of 11%

   Shade a picture of 1%

   Shade a picture of $\frac{1}{2}$%

   Shade a picture of 25% (to show it is 2 x 10% and 5% more)
9. Pose other percentages and ask students to visualise them, stating whether they are close to 1% or close to 100%
10. Practise some mental percent – fraction – decimal equivalence.
11. Extension:
    Discuss pictures for percentages greater than 100%.
    Discuss the inappropriateness of using 2 grids for percents greater than 100% (gives a false image of the whole as consisting of 200 parts).

ACTIVITY 6.7 PICTURING PERCENT BENCHMARKS

AIM
To assist students perform simple percent calculations mentally through reference to common percent benchmarks such as 25%, 50%, $33\frac{1}{3}$% and so on.

OVERVIEW
In this activity, students represent common percent benchmarks on 10x10 grids, exploring their link to fractions

MATERIALS
2 copies of **BLM 6.1** and 1 copy of **BLM 6.3** per student.

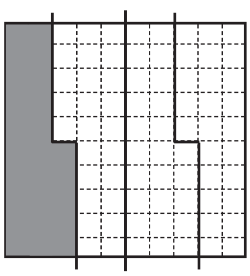
TEACHING POINTS
- Check that students are shading percents as blocks, not individual squares.
- Maintain flow of activity by encouraging efficient shading in one colour, not "careful" colouring using many colours.
- Have students close their eyes after each picture to visualise percent, fraction, decimal equivalence.

ASSESSING PROGRESS
- Students readily discuss their thinking strategies.
- Students state that their calculation method is linked to the amounts given (e.g., finding 25% of 80 is to divide by 4, but finding 25% of 106 is to find half then half again).
- Students' mental computation of percentage amounts becomes more accurate.

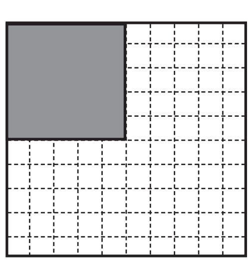
PRACTICE EXAMPLES

50% of 200	25% of 400	$33\frac{1}{3}$% of 60
50% of $1.80	25% of 80	$33\frac{1}{3}$% of 9
50% of 75	25% of 100	$33\frac{1}{3}$% of 120
50% of 20	25% of 27	$12\frac{1}{2}$% of 800
50% of $17.50	25% of 16	$33\frac{1}{3}$% of 48

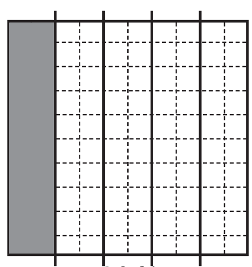
Pictures of percent benchmarks:



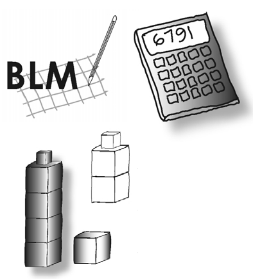
25% as 2 x 10% + 5%



25% as $\frac{1}{4}$

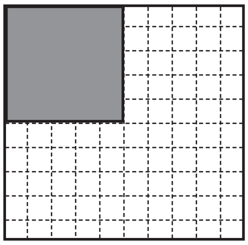


20% as $\frac{1}{5}$



ACTIVITY OUTLINE

1. Distribute 10x10 grid pages (**BLM 6.1**) to students.
2. Ask students to select their favourite grid of the 8 on the page, and trace around the outside of this grid with their finger. Make the students state as they are doing this: This is one whole square.
3. Ascertain that all students are aware that each grid is divided into 100 equal parts.
4. Shade a picture of 25% to show 2 x 10% + 5% (refer to pictures of percent bench-marks)
5 Ask students to draw another picture of 25%, but this time make it represent one-quarter.



6. Ask students to represent the following percents on other grids, making clear links to fraction equivalents:

   Shade a picture of 50% to show that it is one half

   Shade a picture of 75% to show that it is three-quarters

   Shade a picture of 20% to show that it is one-fifth

   Shade a picture of 10% to show that it is one-tenth

   Shade a picture of $33\frac{1}{3}$% to show that it is one-third

   Shade a picture of $12\frac{1}{2}$% to show that it is one-eighth

   Shade a picture of 100%
7. Ask students to represent percentages of various amounts by using **BLM 6.3.**
8. Practise linking percents to fractions by asking students to calculate percentages of various amounts and ask students to describe their thinking strategies:

   50% of 12

   50% of 20

   50% of 30

   50% of 200

   50% of $5.00

   25% of 800 (possible strategy - divide by 4)

   25% of 90 (possible strategy - divide in half and then half again)

   $12\frac{1}{2}$% of $80 (possible strategy - divide by 8)

   $33\frac{1}{3}$% of $9000

ACTIVITY 6.8 DEVELOPING 10%

AIM

To develop understanding of 10% as dividing by 10 and to promote 10% as a benchmark.

OVERVIEW

In this activity, students explore finding 10% through dividing a number by 10. Students use a calculator to focus on the effect of dividing numbers by powers of 10 and visualising movements of digits across place values. The accompanying worksheet for this activity has been designed to consolidate this understanding and to assist mental computation of 1% and also 5%.

MATERIALS

BLM 6.4 (one copy per student), calculators

TEACHING POINTS

- Encourage students to use the calculator until they have “internalised” the pattern.
- Encourage students to discuss their strategies.
- Reinforce the visual notion of the number moving across the places, not the decimal point moving when a number is divided by 10.
- Frequently iterate the strategy for finding 5% is by halving 10%.

ASSESSING PROGRESS

- Students’ mental computation of 10% of an amount becomes more accurate.
- Students readily discuss their thinking strategies.

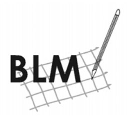
PRACTICE EXAMPLES

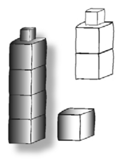
Finding 10%

10 % of 1000	10% of 1 000 000
10 % of 480	10% of 36 000
10 % of $16.50	10% of 48.6
10 % of $27	10% of 9.2
10 % of $20	10% of 0.37

Finding 5%

5% of 80	5% of 4.8
5% of 65	5% of $2.50
5% of 250	5% of 42
5% of 480	5% of 37
5% of 2 000	5% of $250 000





ACTIVITY OUTLINE

1. Draw a place value chart on the board using columns to separate the places. Highlight the decimal point at the bottom of the letter O in the ones place

Tth	Th	H	T	O.	t	h	th

2. Ask students how the number 4600 would be displayed on the place value chart.

Tth	Th	H	T	O.	t	h	th
	4	6	0	0.			

3. Ask students to enter this number on the calculator.
4. Compare the difference between the calculator representation and the place value chart (i.e., the calculator does not show the places).
5. Ask students to divide this number by 10 on the calculator.
6. Display this new number on the place value chart, indicating action (÷ 10) to the far left of the chart. Highlight the decimal point at the bottom of any digit in the ones place.

	Tth	Th	H	T	O.	t	h	th
		4	6	0	0.			
÷ 10			4	6	0.	0		

7. Discuss the representation using the place value chart and the fact that the four digits can be represented in this way, but on the calculator, the last zero disappears.
8. Do some more examples:
   e.g., 25 000 ÷ 10
   34 ÷ 10
   0.57 ÷ 10
9. Distribute **BLM 6.4** and focus students' attention on the instructions at the top of the sheet. Ask students to complete the worksheet in a manner similar to that demonstrated on the board.
10. As the majority of students complete the worksheet, discuss what happens when a number is divided by 10 (i.e., it moves one place to the right across the places in the place value chart; the decimal point does not move).
11. Discuss how this assists in finding 10% of an amount (i.e., 10% means dividing by 10. To find 10% of 400, think of 400 moving one place to the right across the place values. Therefore, 10% of 400 is 40). Also discuss strategies for finding 5% of an amount, as per examples on the worksheet (find 10% by visualising the number moving across places, then divide the number in half).
12. Practise some mental calculations of 10%.

ACTIVITY 6.9 BUILDING ON 10%

AIM
To provide students with concrete materials that can be manipulated mentally to assist mental computation.

OVERVIEW
This activity links to 6.8. In this activity, students use 10% as a benchmark to find other percents (e.g., 60% is 10% x 6). This activity also links to 5.4 where students use counters to find a fraction part of a set.

MATERIALS
Approximately 40 counters (Unifix, cubes, or other suitable material) for each student.

TEACHING POINTS
- Allow students plenty of time to visualise the solution and to mentally carry out all the steps.
- If students are still having difficulty, revise calculating 10% (see 6.8 Developing 10%)
- Allow sufficient time for students to share their solution strategies with discussion on efficiency of particular strategies.

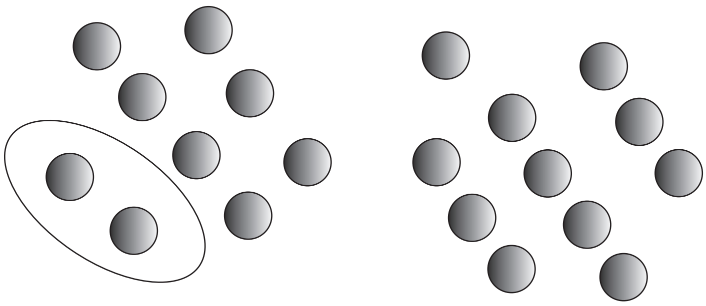
ASSESSING PROGRESS
- Students readily discuss their thinking strategies.
- Students can readily articulate the relationship between 10% and other multiples of 10% to assist in mental computation.
- Students' mental computation becomes more accurate.

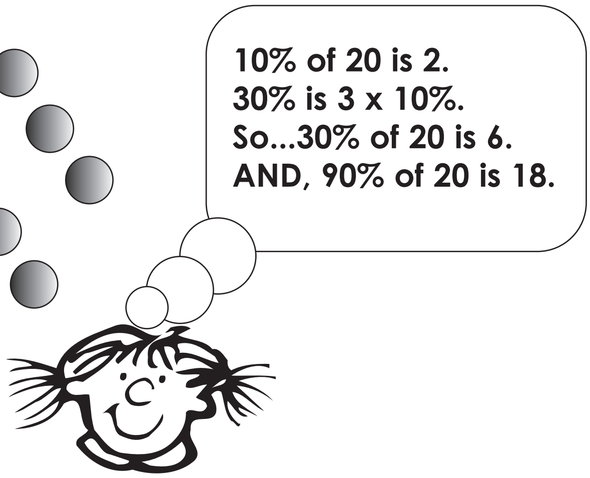
PRACTICE EXAMPLES

70% of 40	60% of 90
30% of 80	70% of 200
90% of 60	30% of 55
20% of 50	40% of 600
20% of 450	40% of 200

ACTIVITY OUTLINE

1. Instruct students to place 20 counters in front of them.
2. Students encase their counters with their hands and say to the teacher, “This is one whole.”
3. Ask students, in pairs, to identify 10% of the counters. Ask students to discuss their strategies for finding 10% (i.e., 10%, divide by 10).
4. Ask students how they might identify 30% (multiply 10% by 3)





10%

Discuss how they might find 90% of 20.

5. Make a collection of 30 counters, and repeat the activity (find 10%, find 30%, find 90%).
6. Repeat using 40 counters.
7. Ask students to close their eyes and visualise 70 counters. Ask them to calculate 10% of 70. Ask them to calculate 30% of 70. Discuss strategies.
8. Pose larger numbers for students to visualise 10%

   e.g., 10% of 500

   70% of 500 = 7 x 50 = 350
9. Practise some mental calculations of 10%, 20%, 30%, 40%, 60%, 70%, 80%, 90%.
10. Discuss with students their strategies for finding 20% and 40%. Do they divide by 5, or find 10% first?

ACTIVITY 6.10 VISUALISING 1%

AIM
To promote meaning for dividing a number by 100 and to promote mental computation and facility for 1%.

OVERVIEW
This activity links to 6.8. In this activity, students explore finding 1% through dividing a number by 100. Students use a calculator to focus on the effect of dividing numbers by powers of 10 and visualising movements of digits across place values. The accompanying worksheet for this activity has been designed to consolidate this understanding and to assist mental computation of 1%.

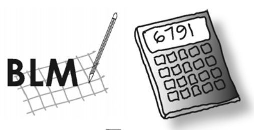
MATERIALS
BLM 6.5 (one copy per student), calculators

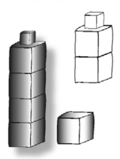
TEACHING POINTS
- Encourage students to use the calculator until they have "internalised" the pattern.
- Practice sets of mentals in groups of 10, with students correcting each set of 10.
- Encourage students to discuss their strategies.
- Reinforce the visual notion of the number moving across the places, not the decimal point moving when a number is divided by 100.

ASSESSING PROGRESS
- Students readily discuss their thinking strategies.
- Students' mental computation of 1% of an amount becomes more accurate.

PRACTICE EXAMPLES

1 % of 40	1 % of 2 000 000
1 % of 200	1 % of 85 000
1 % of 450	1 % of 96 000
1 % of 4.5	1 % of 4 600
1 % of 2000	1 % of 240





ACTIVITY OUTLINE

1. Draw a place value chart on the board using columns to separate the places. Highlight the decimal point at the bottom of the letter O in the ones place.

Tth	Th	H	T	O.	t	h	th

2. Ask students how the number 3500 would be displayed on the place value chart.

Tth	Th	H	T	O.	t	h	th
	3	5	0	0.			

3. Ask students to enter this number in the calculator.
4. Compare the difference between the calculator representation and the place value chart (i.e., the calculator does not show the places).
5. Ask students to divide this number by 100 on the calculator.
6. Display this new number on the place value chart, indicating action (÷100) to far left of chart. Highlight the decimal point at the bottom of any digit in the ones place.

Tth	Th	H	T	O.	t	h	th
	3	5	0	0.			
			3	5.	0	0	

7. Discuss the representation using the place value chart and the fact that the four digits can be represented in this way, but on the calculator, the last zero disappears.
8. Do some more examples:

   e.g., 25 000 ÷ 100
   34 ÷ 100
   0.57 ÷ 100

9. Distribute **BLM 6.5** and ask students to complete the worksheet in a manner similar to that demonstrated on the board.
10. As the majority of students complete the worksheet, discuss what happens when a number is divided by 100 (i.e., it moves two places to the right across the places in the place value chart, the decimal point does not move).
11. Discuss how this assists in finding 1% of an amount.
12. Practise some mental calculations of 1%.

ACTIVITY 6.11 PERCENT DISCOUNTS (whole complement)

AIM

To provide students with an opportunity to practise discount calculations.

TEACHING POINTS

- As the process of finding the discount and the sale process is a two-step procedure, allow students recourse to pen and paper to assist memory.

ASSESSING PROGRESS

- Students readily discuss their thinking strategies.
- Students' mental computation of percentage discounts becomes more accurate.

PRACTICE EXAMPLES

25% discount on \$50.	Amount of discount: ___	Sale price: ___
25% discount on \$400.	Amount of discount: ___	Sale price: ___
25% discount on \$100.	Amount of discount: ___	Sale price: ___
$33\frac{1}{3}$% discount on \$90.	Amount of discount: ___	Sale price: ___
$33\frac{1}{3}$% discount on \$18.	Amount of discount: ___	Sale price: ___
10% discount on \$400.	Amount of discount: ___	Sale price: ___
30% discount on \$400.	Amount of discount: ___	Sale price: ___
10% discount on \$4000.	Amount of discount: ___	Sale price: ___
75% discount on \$400.	Amount of discount: ___	Sale price: ___
10% discount on \$40,000.	Amount of discount: ___	Sale price: ___

ACTIVITY OUTLINE

1. Revise finding a percentage of an amount:

   e.g. 50% of 60 = ___

   25% of 40 = ___

   25% of 90 = ___

   10% of 70 = ___

   10% of 900 = ___

2. Establish students' understanding of "discount" as a reduction in reference to shopping activities.
3. Ask students to mentally calculate a discount of 25% on an item priced at $40 ($10 discount). Calculate the new price ($30)
4. Practise calculation of some other discounts:

25% discount on $60.	Amount of discount: ___	Sale price: ___
25% discount on $80.	Amount of discount: ___	Sale price: ___
25% discount on $120.	Amount of discount: ___	Sale price: ___
$33\frac{1}{3}$% discount on $120.	Amount of discount: ___	Sale price: ___
$33\frac{1}{3}$% discount on $30.	Amount of discount: ___	Sale price: ___
10% discount on $200.	Amount of discount: ___	Sale price: ___

5. Encourage students to describe their thinking strategies and mental images (if any).

ACTIVITY 6.12 INCREASES OF GREATER THAN 100%

AIM

To assist students visualise the effect of increases of greater than 100%.

OVERVIEW

In this activity, students use counters to explore the effect of percent increases on the original whole, and how this relates to multiplication.

MATERIALS

Approximately 20 counters (Unifix, cubes, or other suitable material) per student; **BLM 6.6** (one copy per student)

TEACHING POINTS

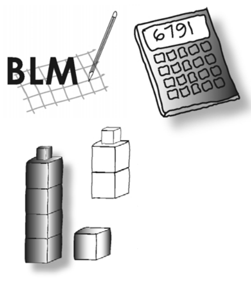
- Make sure the material is a separate material (such as counters) so that students do not join the material together, e.g., as may be the case with Unifix cubes.
- Encourage students to discuss their thinking, and the difficulty they may experience in overcoming the natural tendency to think of, for example, a 200% increase as 2 times the original amount.

ASSESSING PROGRESS

- Students' mental computation of increases of greater than 100% becomes more accurate.
- Students readily discuss their thinking strategies.
- Students can verbalise why, for example, a 200% increase means an increase of 3 times the original, and explain why it is not simply double.

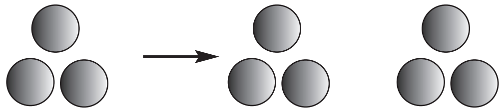
PRACTICE EXAMPLES

Increase	70 by 100%	8 by 300%
	16 by 100%	20 by 300%
	30 by 200%	15 by 300%
	8 by 200%	12 by 400%
	15 by 200%	7 by 500%



ACTIVITY OUTLINE

1. Put out 3 counters. Increase by 100%



New Total = 6

New group is 2 times the original group. A 100% increase = 2 times the original

2. Try other examples:
   a) *Start with 4 counters*
      Increase by 300%.
      How many counters now? (16)
      How many times bigger is the new group compared to the old group? (4 times)
   b) *Start with 3 counters*
      Increase by 400%.
      How many counters now? (15)
      How many times bigger is the new group compared to the old group? (5 times)
   c) *Start with 2 counters*
      Increase by 500%.
      How many counters now? (12)
      How many times bigger is the new group compared to the old group? (6 times)
   d) *Start with 5 counters*
      Increase by 400%.
      How many counters now? (25)
      How many times bigger is the new group compared to the old group? (5 times)
3. Encourage students to generalise what happens when a collection is increased by an amount greater than 100%. Link this to multiplication, and draw attention to the fact that a 200% increase is not the same as multiplying by 2.
4. Distribute **BLM 6.6** to enable students to practice calculating increases of greater than 100%.
5. Practise some mental calculations of amounts greater than 100% by asking students to close their eyes and visualise increasing groups by various percentages:

   4 counters, increase 100%
   6 counters, increase 200%
   10 counters, increase 200%
   5 counters, increase 300%
6. Try larger collections:

   50 counters, increase 100%
   45 counters, increase 100%
   200 counters, increase 200%
   400 counters, increase 200%
   8000 counters, increase 300%

Encourage students to discuss their solution strategies.

ACTIVITY 6.13 PERCENT INCREASES AND DECREASES

AIM
To extend students' conceptual understanding of percent and to provide a mental image to assist in mental computation.

OVERVIEW
In this activity, students use counters to explore percent increases and decreases and the effect this has on the original whole amount.

MATERIALS
Approximately 20 counters (Unifix, cubes, or other suitable material) per student; **BLM 6.7** (one copy per student)

TEACHING POINT
- Make sure the material is a separate material (such as counters) so that students do not join the material together, e.g., as may be the case with Unifix cubes.
- Encourage students to visualise the collection before and after each percent increase and decrease when they are performing mental computation.
- Encourage students to discuss their thinking, and the difficulty they may experience in overcoming the natural tendency to think of, for example, that increasing something by 25% means that you could reduce it by the same percent to get back to the original amount.

ASSESSING PROGRESS
- Students' mental computation of increases and decreases becomes more accurate.
- Students readily discuss their thinking strategies.
- Students can verbalise why, for example, a 25% increase is not the same as a 25% decrease.

PRACTICE EXAMPLES

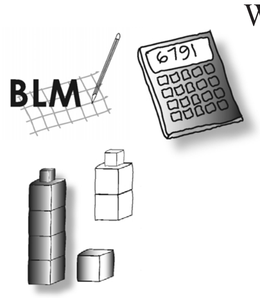
What is the new total when 6 is increased by 50%? ___
What percent must it be reduced by to get back to the original? ___

What is the new total when 8 is increased by 25%? ___
What percent must it be reduced by to get back to the original? ___

What is the new total when 9 is increased by $33\frac{1}{3}$%? ___
What percent must it be reduced by to get back to the original? ___

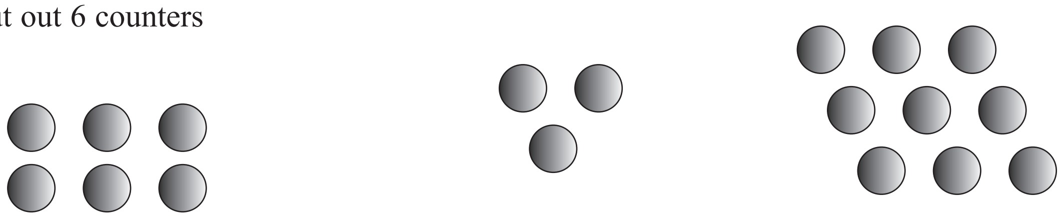
What is the new total when 6 is increased by $33\frac{1}{3}$%? ___
What percent must it be reduced by to get back to the original? ___

What is the new total when 20 is increased by 100%? ___
What percent must it be reduced by to get back to the original? ___



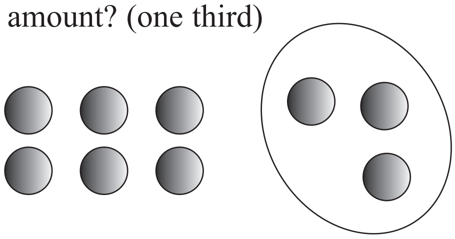
ACTIVITY OUTLINE

1. Put out 6 counters

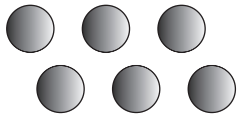


Increase by 50% New total = 9

2. Reverse: What fraction of the new amount must be removed to get back to the original amount? (one third)



Remove $\frac{1}{3}$ (33 $\frac{1}{3}$%)



An increase of 50% means we must reduce by 33$\frac{1}{3}$% to get back to the original

3. Distribute **BLM 6.7**. Allow students to explore various size increases in this fashion, using the accompanying worksheet. The worksheet enables students to explore the following relationships:

An increase of 50% means a reduction of 33$\frac{1}{3}$% to get back to the original.

An increase of 25% means a reduction of 20% to get back to the original.

An increase of 33$\frac{1}{3}$% means a reduction of 25% to get back to the original.

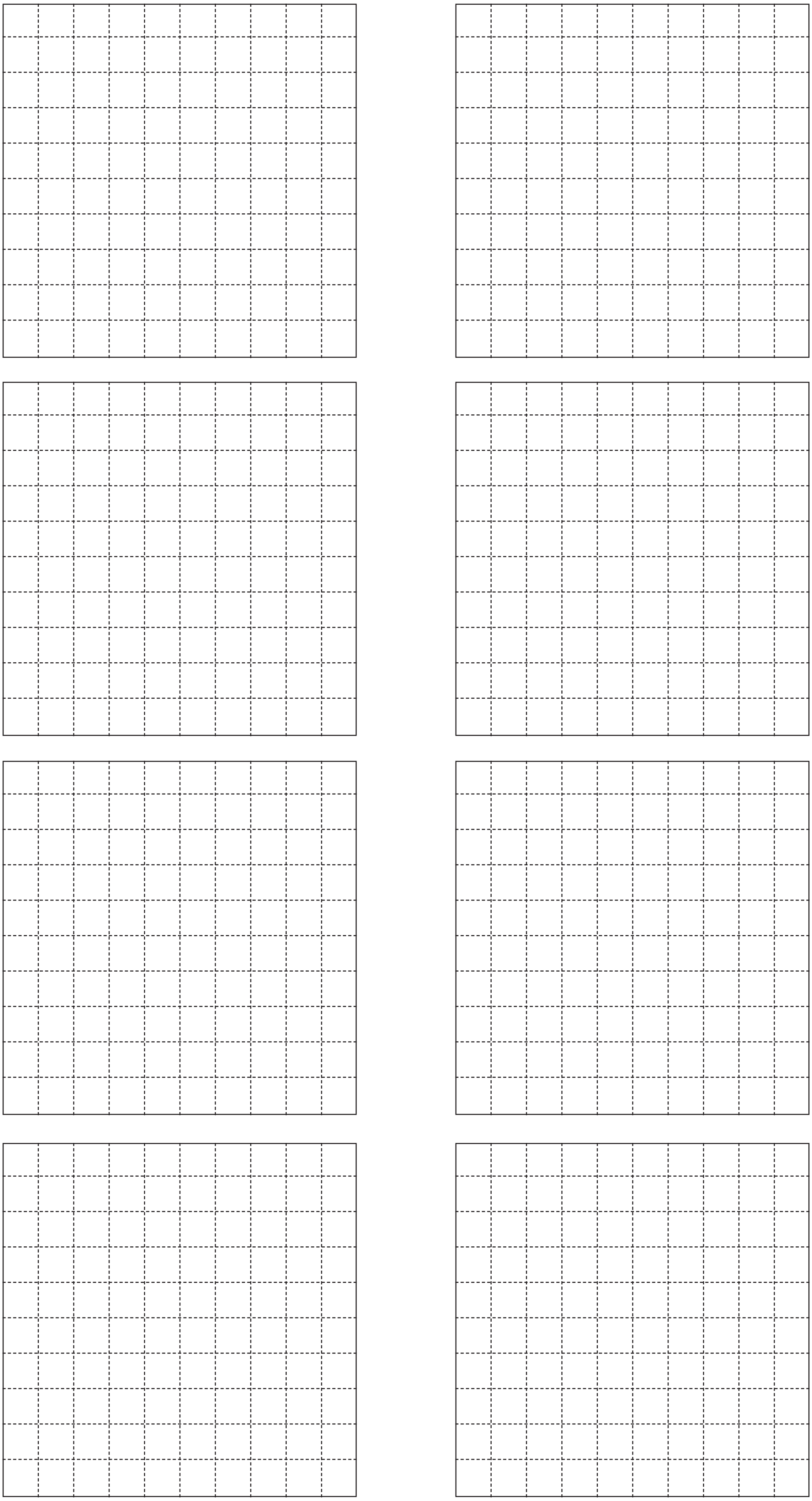
An increase of 100% means a reduction of 50% to get back to the original.

4. Practice some mental calculations of increasing an amount by a given percent. Ask students to close their eyes and visualise the process:

e.g., *Increase 8 by 50%. What percent must it be reduced by to get back to the original?*

BLM 6.1

PERCENT GRIDS



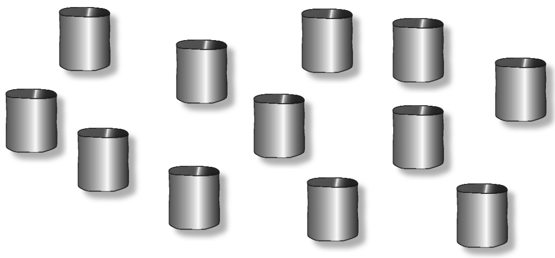
BLM 6.2

PERCENT PART/WHOLE PICTURES

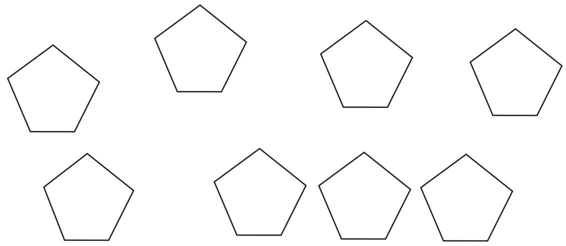
Jumper 80% wool 20% cotton	**Milk** 96% milk 4% fat	**Sausages** 60% beef 40% pork
Steak 91% meat 3% fat 6% bone	**Socks** 40% rayon 60% cotton	**Cordial drink** 72% water 28% cordial
Human body 78% water 22% other	**Chocolate** 25% cocoa 25% sugar rest milk	**(Draw one of your own)**

BLM 6.3

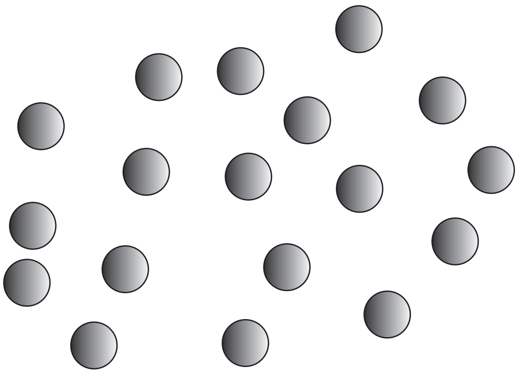
PERCENTAGES OF COLLECTIONS



25% of 12 = ______



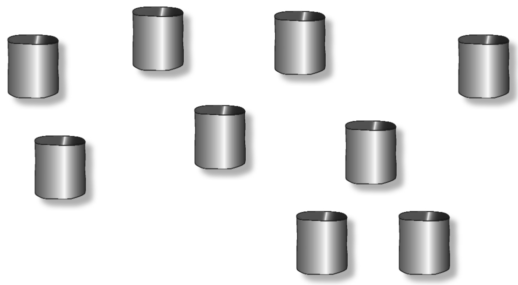
50% of 8 = ______



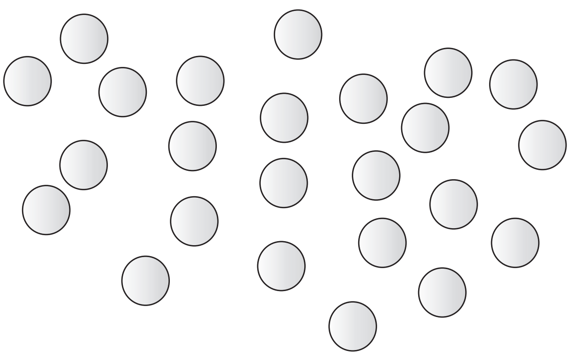
$33\frac{1}{3}$ % of 18 = ______

25% of 20 = ______

25% of 16 = ______



$33\frac{1}{3}$ % of 9 = ______



25% of 24 = ______

25% of 16 = ______

BLM 6.4

DIVIDING BY 10

Materials: Calculator

1. For each question below, enter the number on the calculator as well as in the appropriate columns in the place-value chart given. Then divide the number by 10, and record the new number on the chart. The first one has been done for you.

	M	Hth	Tth	Th	H	T	O.	t	h	th
1. a) 4600				4	6	0	0.			
b) ÷ 10					4	6	0.	0		
2. a) 250										
b) ÷ 10										
3. a) 7 500 000										
b) ÷ 10										
4. a) 320										
b) ÷ 10										
5. a) 35										
b) ÷ 10										
6. a) 525										
b) ÷ 10										
7. a) 2.4										
b) ÷ 10										
8. a) 0.56										
b) ÷ 10										

Describe what happens when you divide a number by 10:

BLM 6.4 (continued)

DIVIDING MONEY BY 10

2. Complete the following table in a similar fashion as the previous one.

	M	Hth	Tth	Th	H	T	O.	t	h
	$	$	$	$	$	$	$	c	c
1. a) $250									
b) ÷ 10									
2. a) $160									
b) ÷ 10									
3. a) $500 000									
b) ÷ 10									
4. a) $42.00									
b) ÷ 10									
5. a) $10.00									
b) ÷ 10									

3. Without using your calculator, find 10% of the following:

a) 10% of 550 =______ d) 10% of 300 =______ g) 10% of $2 000 =______

b) 10% of 20 =______ e) 10% of $20 =______ h) 10% of $22.50 =______

c) 10% of 235 =______ f) 10% of 300 =______ i) 10% of $56 =______

4. What about 5%? Try working these out in your head

a) 5% of $4.00 = ______ c) 5% of $180 = ______ e) 5% of $80 = ______

b) 5% of $36 = ______ d) 5% of $75 = ______ f) 5% of $22.00 = ______

5. SALE – 10% DISCOUNT

Find the sale price after a 10% discount on the following prices. Do not use your calculator and only write down your answer.

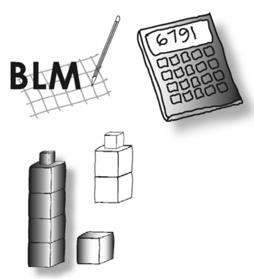
a) Item costs $25. Sale price after 10% discount = __________

b) Item costs $900. Sale price after 10% discount = __________

c) Item costs $45. Sale price after 10% discount = __________

d) Item costs $380. Sale price after 10% discount = __________

e) Item costs $1000. Sale price after 10% discount = __________



BLM 6.5

DIVIDING BY 100

Materials: Calculator

1. For each question below, enter the number on the calculator as well as in the appropriate columns in the place-value chart given. Then divide the number by 100, and record the new number on the chart. The first one has been done for you.

	M	Hth	Tth	Th	H	T	O.	t	h	th
1. a) 4600				4	6	0	0.			
b) ÷ 100						4	6.	0	0	
2. a) 250										
b) ÷ 100										
3. a) 7 500 000										
b) ÷ 100										
4. a) 320										
b) ÷ 100										
5. a) 35										
b) ÷ 100										
6. a) 525										
b) ÷ 100										
7. a) 2.4										
b) ÷ 100										
8. a) 0.56										
b) ÷ 100										

Describe what happens when you divide a number by 100:

BLM 6.5 (continued)

DIVIDING MONEY BY 100

2. Complete the following table in a similar fashion as the previous.

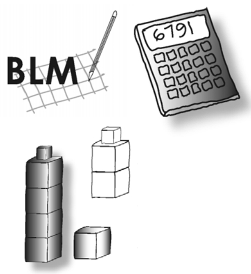
	M	Hth	Tth	Th	H	T	O.	t	h
	$	$	$	$	$	$	$	c	c
1. a) $250					2	5	0		
b) ÷ 100							2.	5	0
2. a) $160									
b) ÷ 100									
3. a) $500 000									
b) ÷ 100									
4. a) $42.00									
b) ÷ 100									
5. a) $10.00									
b) ÷ 100									

3. Without using your calculator, find 1% of the following:

a) 1% of 550 =______
b) 1% of 20 = ______
c) 1% of 235 =______
d) 1% of 300 =______
e) 1% of $20 =______
f) 1% of 300 =______
g) 1% of $2 000 =______
h) 1% of $22.50 =______
i) 1% of $56 =______

4. What about 5%? Try working these out in your head

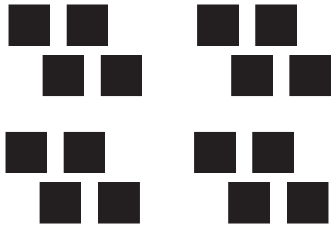
a) 5% of $4.00 =______
b) 5% of $36 =_______
c) 5% of $180 =______
d) 5% of $75 =______
e) 5% of $80 =______
f) 5% of $22.00 =______



BLM 6.6

INCREASES OF GREATER THAN 100%

Increase 300%



How many times bigger than the original is the group now?

1. Start with 3 counters.
   Increase by 100%. How many counters now? _____
   How many times bigger is the new group compared to the old group? _____
2. Start with 4 counters.
   Increase by 200%. How many counters now? _____
   How many times bigger is the new group compared to the old group? _____
3. Start with 3 counters.
   Increase by 300%. How many counters now? _____
   How many times bigger is the new group compared to the old group? _____
4. Start with 2 counters.
   Increase by 500%. How many counters now? _____
   How many times bigger is the new group compared to the old group? _____
5. Start with 5 counters.
   Increase by 400%. How many counters now? _____
   How many times bigger is the new group compared to the old group? _____

What can you say about increases of greater than 100% and the number of times bigger the new group is compare to the old group?

....

....

....

Complete the following

6. Increase by 100%
a) 15 _____
b) 23 _____
c) 200 _____
d) 45 _____
e) 700 _____

7. Increase by 200%
a) 4 _____
b) 30 _____
c) 50 _____
d) 300 _____
e) 450 _____

8. Increase by 150%
a) 68 _____
b) 100 _____
c) 50 _____
d) 460 _____
e) 600 _____

9. Increase by 25%
a) 16 _____
b) 24 _____
c) 200 _____
d) 48 _____
e) 800 ___

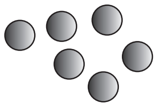
10. Increase by 50%
a) 4 _____
b) 30 _____
c) 50 _____
d) 300 _____
e) 450 _____

11. Increase by $33\frac{1}{3}\%$
a) 66 _____
b) 90 _____
c) 6 _____
d) 51 _____
e) 600 _____

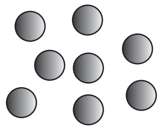
BLM 6.7

PERCENT INCREASES AND DECREASES

Start with 6 counters



increase $33\frac{1}{3}$%



The size of the group is now 8 counters.
To get back to the original amount, decrease by 25%

Try these:

1. a) Start with 6 counters.
   Increase by 50%.
   How many counters now? _____
   What fraction do you have to remove to get back to the original amount? _____
   What percent is this? _____

   b) Start with 4 counters.
   Increase by 50%.
   How many counters now? _____
   What fraction do you have to remove to get back to the original amount? _____
   What percent is this? _____

   c) Start with 10 counters.
   Increase by 50%.
   How many counters now? _____
   What fraction do you have to remove to get back to the original amount? _____
   What percent is this? _____

   d) Start with 12 counters.
   Increase by 50%.
   How many counters now? _____
   What fraction do you have to remove to get back to the original amount? _____
   What percent is this? _____

   e) Start with 8 counters.
   Increase by 50%.
   How many counters now? _____
   What fraction do you have to remove to get back to the original amount? _____
   What percent is this? _____

Complete the following statement:

An increase of 50% means a reduction of _____% to get back to the original.

2. Increases of 25%

   a) Start with 8 counters. Increase by 25%. How many counters now? _____
   What fraction do you have to remove to get back to the original amount? _____
   What percent is this? _____

   b) Start with 4 counters. Increase by 25%. How many counters now? _____
   What fraction do you have to remove to get back to the original amount? _____
   What percent is this? _____

BLM 6.7 (continued)

c) Start with 16 counters. Increase by 25%. How many counters now? _____
What fraction do you have to remove to get back to the original amount? _____
What percent is this? _____

d) Start with 24 counters. Increase by 25%. How many counters now? _____
What fraction do you have to remove to get back to the original amount? _____
What percent is this? _____

e) Start with 12 counters. Increase by 25%. How many counters now? _____
What fraction do you have to remove to get back to the original amount? _____
What percent is this? _____

An increase of 25% means a reduction of _____% to get back to the original.

3. Increases of $33\frac{1}{3}$%

a) Start with 6 counters. Increase by $33\frac{1}{3}$%. How many counters now? _____
What fraction do you have to remove to get back to the original amount? _____
What percent is this? _____

b) Start with 9 counters. Increase by $33\frac{1}{3}$%. How many counters now? _____
What fraction do you have to remove to get back to the original amount? _____
What percent is this? _____

c) Start with 3 counters. Increase by $33\frac{1}{3}$%. How many counters now? _____
What fraction do you have to remove to get back to the original amount? _____
What percent is this? _____

d) Start with 12 counters. Increase by $33\frac{1}{3}$%. How many counters now? _____
What fraction do you have to remove to get back to the original amount? _____
What percent is this? _____

An increase of $33\frac{1}{3}$% means a reduction of _____% to get back to the original.

4. Increases of 100%

a) Start with 4 counters. Increase by 100%. How many counters now? _____
What fraction do you have to remove to get back to the original amount? _____
What percent is this? _____

b) Start with 7 counters. Increase by 100%. How many counters now? _____
What fraction do you have to remove to get back to the original amount? _____
What percent is this? _____

c) Start with 20 counters. Increase by 100%. How many counters now? _____
What fraction do you have to remove to get back to the original amount? _____
What percent is this? _____

d) Start with 12 counters. Increase by 100%. How many counters now? _____
What fraction do you have to remove to get back to the original amount? _____
What percent is this? _____

An increase of 100% means a reduction of _____% to get back to the original.