

**MENTAL
COMPUTATION:
A STRATEGIES APPROACH**

MODULE 5
fractions and decimals

Shelley Dole

Mental Computation: A strategies approach

Module 5 Fractions and decimals

Shelley Dole

This is one of a set of 6 modules providing a structured strategies approach to mental computation.

Module 1 Introduction

Module 2 Basic facts Addition and subtraction

Module 3 Basic facts Multiplication and division

Module 4 Two-digit whole numbers

Module 5 Fractions and Decimals

Module 6 Ratio and Percent

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Project Team:

Alistair McIntosh (University of Tasmania)
Jane Watson (University of Tasmania)
Shelley Dole (now at University of Queensland)
Rosemary Callingham (now at University of New England)
Rick Owens (ACT Department of Education and Training)
Annaliese Caney (APA (I) Doctoral Student, University of Tasmania)
Michael Kelly (Research Assistant, University of Tasmania)
Amanda Keddie (Research Assistant, University of Tasmania)

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Project Schools:

Charles Conder Primary School, ACT

Dominic College, Tas.

Holy Rosary School, Tas.

Lanyon High School, ACT

Lilydale District High School, Tas.

Norwood Primary School, Tas.

Key Teachers:

Marg Elphinstone Rachel O'Rourke

Liz Stanfield Martin LeFevre

Ros Wilson Dianne Ashman

Helen Cosentino Jill Wing

Jackie Conboy Tod Brutnell

John Rickwood Anna Wilson

Graphic Design: Clare Bradley

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CONTENTS

INTRODUCTION

	PAGE
Overview of Module 5	5

LEARNING ACTIVITIES

5.1	Exploring Unit Fractions (Developing part of a whole)	6
5.2	Exploring Equivalence (Simple fraction addition and subtraction)	8
5.3	Visualising Unit fractions within Collections	10
5.4	Visualising Non-unit fractions within Collections	12
5.5	Simple Fraction Multiplication	14
5.6	Visualising Fraction Division	16
5.7	Visualising Decimals Tenths	18
5.8	Decimal Multiplication	20
5.9	Visualising Decimal division	22

BLACKLINE MASTERS

Black Line Masters 5.1:	Circles	24
Black Line Masters 5.2:	One Third	25
Black Line Masters 5.3:	Fraction parts and wholes	26
Black Line Masters 5.4:	Fraction fifths	27
Black Line Masters 5.5:	Skip counting number lines	28
Black Line Masters 5.6:	Pictures for fraction division	29
Black Line Masters 5.7:	Tenths number lines	30
Black Line Masters 5.8:	Decimal fraction number lines	31
Black Line Masters 5.9:	Pictures for decimal fraction division	32

OVERVIEW OF MODULE 5

INTRODUCTION

This module is the fifth in a series of six that comprise a resource of activities for developing mental computation. The focus of this module is on mental computation of common and decimal fractions.

LANGUAGE

The terms common fraction and decimal fraction are used to distinguish between the symbolic representation of numbers written in fraction form (e.g., $\frac{1}{4}$) and in decimal form (e.g., 0.25) respectively. For simplification, in this module, common fractions are referred to as fractions and decimal fractions are referred to as decimals.

TEACHING SEQUENCE

The Activities presented in this module are based on two major principles for developing mental computation skills with fractions and decimals:

1. Building conceptual understanding to make calculation meaningful; and
2. Encouraging the use of mental images to assist in mental computation.

Rather than being a collection of ideas, strategies and games for mental computation, each **Activity** is a sequential teaching episode to develop a particular aspect of fraction/decimal knowledge and understanding for mental computation.

Each Activity is designed to stand-alone, but prior knowledge may assist students' performance in some cases, and such prior knowledge may be promoted through a previous Activity. Collectively, the Activities are not designed as a curriculum for fractions and decimals, but rather to augment and build students' knowledge of these topics to assist meaningful mental computation.

For each Activity, the *Aim* of the Activity is given, summarising the conceptual basis of the strategies presented. An *Overview* of the Activity is also provided, enabling the nature of the activity to be readily gleaned. *Materials* are indicated, with the letters **BLM** followed

by a reference number. Black Line Masters are located at the back of the booklet. *Teaching*

Points are presented in dot-point form for succinctness, serving as "reminder tips" that focus the key points of the Activity. Tips for assessing students' conceptual understanding, mental computation strategies, and mental computation performance are provided under the heading of *Assessing Performance*. Carefully selected *Practice Examples* are presented to indicate the types of calculations students would be expected to perform mentally. The practice examples are aimed to serve as a guide for devising further sets of calculations for consolidation purposes.

The sequence for each teaching episode is presented on the facing page under the heading: *Activity Outline*. The sequence is numbered to indicate the steps along which the teaching episode proceeds.



ACTIVITY 5.1 EXPLORING UNIT FRACTIONS (Developing part of a whole)**AIM**

To promote part/whole conceptual understanding and to assist students perform simple fraction mental computations through visualisation of a whole divided into equal parts.

OVERVIEW

In this activity, students explore one-third through dividing various wholes into three equal parts, and then create pictures of other unit fractions.

MATERIALS

BLM 5.1, **BLM 5.2** and **BLM 5.3** (one copy per student), scissors, glue.

TEACHING POINTS

- If implemented as a whole lesson, this would require approximately 30-45 minutes.
- Maintain the flow of the lesson by encouraging quick shading rather than elaborate and neat colouring.

ASSESSING PROGRESS

- Students readily discuss their visual images of fractions.
- Students discuss appropriateness of various visual images of fractions, describing how any “whole” can be divided into fraction amounts.
- Students’ mental computation becomes more accurate.

PRACTICE EXAMPLES:

$$1 - \frac{1}{3}$$

$$\frac{3}{4} + \square = 1$$

$$1 - \frac{1}{6}$$

$$\frac{2}{3} + \square = 1$$

$$1 - \frac{1}{10}$$

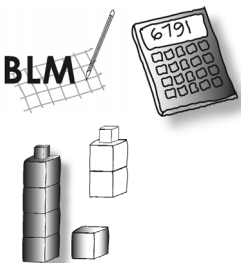
$$\frac{3}{5} + \square = 1$$

$$1 - \frac{1}{4}$$

$$\frac{4}{5} + \square = 1$$

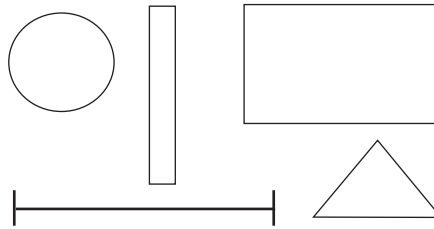
$$1 - \frac{1}{2}$$

$$\frac{7}{8} + \square = 1$$



ACTIVITY OUTLINE

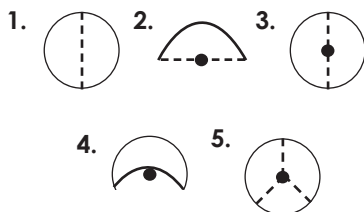
1. Present students with a series of shapes (drawn on the board) and ask them to describe how to show $\frac{1}{3}$ of each shape. Include a number line.



2. Ask key questions:
 How many parts?
 How many shaded?
 What does $\frac{1}{3}$ mean?

3. Create other unit fractions with various other shapes:

- Rectangular paper:
 show $\frac{1}{8}$ of a rectangle
 show $\frac{1}{4}$ of a rectangle
- Paper strips:
 show $\frac{1}{3}$
 show $\frac{1}{5}$
 show $\frac{1}{10}$
- Pre-cut circles (Use **BLM 5.1**):
 Fold to show $\frac{1}{8}$
 Fold to show $\frac{1}{3}$ (see below)
- Number lines:
 show $\frac{1}{9}$
 show $\frac{1}{7}$
 show $\frac{1}{12}$



Directions for folding a circle into thirds:

1. Fold down centre, open.
2. Turn, match lines and pinch centre.
3. Open and mark centre point.
4. Fold bottom edge to centre.
5. Open out and mark fold lines on circumference. Draw radii.
6. Cut into thirds. Fold each third for sixths.

4. Use **BLM 5.2** to consolidate, which contains various wholes divided into three equal parts. Direct students to write the fraction name in words (one-third) and symbols ($\frac{1}{3}$) on the various “parts” provided on the worksheet. The task for students is to cut out the parts of each shape at the bottom of the page and paste upon their corresponding “whole” at the top of the sheet.
5. Ask students to draw pictures to represent other unit fractions using a format of their choice:

e.g. $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{10}$.

6. Consolidate this activity by using **BLM 5.3** where fractions are represented on various shapes and number lines. To complete this worksheet, students must determine how many parts in total, what fraction of the shape is shaded and what fraction of the shape is unshaded.
7. Ask students to close their eyes and visualise certain fractions, such as $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{10}$ and so on. Ask them to describe the visual image that comes to their mind: Is it a circle, or a square, or a number line...or does it depend upon the fraction given?
8. Ask students to mentally calculate various part/whole calculations with unit fractions:

e.g., $1 - \frac{1}{3}$, $1 - \frac{1}{2}$, $1 - \frac{1}{6}$, $\frac{1}{3} + \square = 1$, $\frac{1}{4} + \square = 1$

ACTIVITY 5.2 EXPLORING EQUIVALENCE (Simple fraction addition and subtraction)

AIM

To promote simple addition and subtraction fraction mental computation through visualisation of a whole comprising a number of equal parts.

OVERVIEW

In this activity, students create a set of fraction materials and perform various calculations through manipulation of the fraction pieces. Through simple addition and subtraction exercises using fifths, students will encounter fraction equivalence, improper fractions and simplification.

MATERIALS

BLM 5.4 (one copy per student), scissors.

TEACHING POINTS

- Encourage students to refer back to their fraction pieces if their solutions are incorrect.
- Encourage students to visualise the process of creating wholes using their fraction parts, and how many whole shapes are created when the solution is greater than one.
- Encourage students to discuss their strategies for solution to the mental calculations.

ASSESSING PROGRESS

- Students readily discuss their visual images of fraction parts and wholes when adding or subtracting.
- Students describe how the sum of parts can be greater than the whole.
- Students' mental computation becomes more accurate.

PRACTICE EXAMPLES

$$\frac{1}{5} + \frac{2}{5}$$

$$\frac{1}{3} + \frac{1}{3}$$

$$\frac{1}{4} + \frac{2}{4}$$

$$\frac{6}{7} - \frac{2}{7}$$

$$\frac{4}{5} - \frac{1}{5}$$

$$\frac{6}{4} = \underline{\quad}$$

$$\frac{5}{3} = \underline{\quad}$$

$$\frac{5}{2} = \underline{\quad}$$

$$\frac{8}{3} = \underline{\quad}$$

$$\frac{6}{3} = \underline{\quad}$$

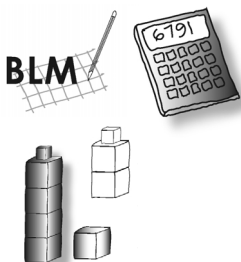
$$\frac{3}{4} + \frac{3}{4}$$

$$\frac{4}{5} + \frac{2}{5}$$

$$\frac{3}{4} + \frac{2}{4}$$

$$\frac{5}{6} + \frac{2}{6}$$

$$\frac{2}{3} + \frac{2}{3}$$



ACTIVITY OUTLINE

- Three of the circles on **BLM 5.4** are clearly divided into fifths. Establish students' part/whole fraction knowledge - that each part is recorded as $\frac{1}{5}$. Instruct students to label each fraction piece ($\frac{1}{5}$) and to cut out the three circles so that they have a total of fifteen fifths. Students may also like to write the fraction name in words (one-fifth) on the back of each fraction piece. Instruct students to cut out the whole circle and label 'one whole'.
- Use fraction pieces to model various addition and subtraction calculations: (for each calculation, ensure students are placing the fifths in a way to make complete circles):

$$\begin{array}{ll} \frac{1}{5} + \frac{2}{5} & 1 - \frac{1}{5} \\ \frac{2}{5} + \frac{2}{5} & 1 - \frac{3}{5} \\ \frac{2}{5} + \frac{5}{5} & 1 - \frac{5}{5} \\ \frac{1}{5} + \frac{1}{5} & \frac{4}{5} - \frac{2}{5} \\ \frac{4}{5} + \frac{1}{5} & \frac{3}{5} - \frac{2}{5} \end{array}$$

Use the one whole as a reference point. Put fifths on top to cover the whole.

- Explore improper fractions and mixed numbers. Encourage students to state the answer in terms of complete 'wholes' and remaining fifths:

$$\begin{array}{ll} \frac{8}{5} = \underline{\quad} & \frac{4}{5} + \frac{2}{5} \\ \frac{10}{5} = \underline{\quad} & \frac{3}{5} + \frac{2}{5} \\ \frac{5}{5} = \underline{\quad} & \frac{2}{5} + \frac{5}{5} \\ \frac{12}{5} = \underline{\quad} & \frac{3}{5} + \frac{4}{5} \\ \frac{9}{5} = \underline{\quad} & \frac{4}{5} + \frac{4}{5} \end{array}$$

- Ask students to close their eyes and visualise solutions to similar fraction exercises before they record their answers:

$$\begin{array}{llll} \frac{1}{5} + \frac{3}{5} & \frac{4}{5} - \frac{2}{5} & \frac{7}{5} = \underline{\quad} & \frac{3}{5} + \frac{2}{5} \\ \frac{1}{5} + \frac{1}{5} & 1 - \frac{4}{5} & \frac{5}{5} = \underline{\quad} & \frac{4}{5} + \frac{2}{5} \\ \frac{2}{5} + \frac{1}{5} & \frac{3}{5} - \frac{1}{5} & \frac{9}{5} = \underline{\quad} & \frac{1}{5} + \frac{5}{5} \\ \frac{4}{5} + \frac{1}{5} & \frac{2}{5} - \frac{1}{5} & \frac{10}{5} = \underline{\quad} & 1 + \frac{1}{5} \\ \frac{2}{5} + \frac{2}{5} & 1 - \frac{3}{5} & \frac{8}{5} = \underline{\quad} & \frac{4}{5} + \frac{6}{5} \end{array}$$

- Explore similar operations with other fraction denominators:

$$\begin{array}{llll} \frac{1}{4} + \frac{1}{4} & 2 - \frac{1}{8} & \frac{7}{6} = \underline{\quad} & \frac{3}{4} + \frac{2}{4} \\ \frac{1}{4} + \frac{2}{4} & \frac{9}{10} - \frac{4}{10} & \frac{3}{2} = \underline{\quad} & \frac{5}{6} + \frac{2}{6} \\ \frac{7}{8} - \frac{3}{8} & \frac{1}{6} + \frac{1}{6} & \frac{5}{2} = \underline{\quad} & \frac{9}{10} + \frac{4}{10} \\ \frac{1}{3} + \frac{2}{3} & 1 - \frac{3}{4} & \frac{5}{4} = \underline{\quad} & \frac{3}{2} + \frac{3}{2} \\ \frac{7}{10} + \frac{2}{10} & \frac{6}{7} - \frac{3}{7} & \frac{7}{4} = \underline{\quad} & 2 - \frac{2}{3} \end{array}$$

ACTIVITY 5.3 VISUALISING UNIT FRACTIONS WITHIN COLLECTIONS

AIM

To promote mental computation of a unit fraction within a set through visualisation. This activity also links to basic division facts (e.g., $20 \div 4$; $\frac{1}{4}$ of 20).

OVERVIEW

In this activity, a number of counters are used to represent one whole, and students explore parts within a set (or collection of objects).

MATERIALS

Approximately 20 counters per student (Unifix, cubes, or other suitable material).

TEACHING POINTS

- Continue posing problems with the material until students no longer need the material to achieve the solution.
- To assist students' focus on the whole, have students place their counters on a piece of coloured paper or card with all other counters away to the side of the desk.
- Encourage students to discuss their strategies for solution to the mental calculations.

ASSESSING PROGRESS:

- Students readily discuss their visual images of a set of material that is divided into equal groups.
- Students' mental computation becomes more accurate.

PRACTICE EXAMPLES:

$$\frac{1}{2} \text{ of } 12$$

$$\frac{1}{2} \text{ of } 16$$

$$\frac{1}{4} \text{ of } 8$$

$$\frac{1}{3} \text{ of } 12$$

$$\frac{1}{5} \text{ of } 15$$

$$\frac{1}{3} \text{ of } 9$$

$$\frac{1}{6} \text{ of } 24$$

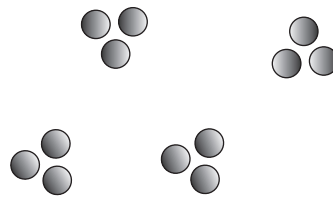
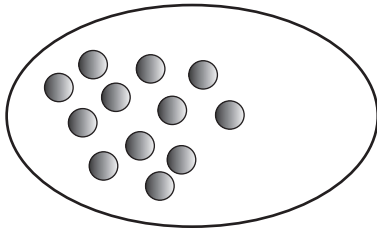
$$\frac{1}{5} \text{ of } 20$$

$$\frac{1}{3} \text{ of } 9$$

$$\frac{1}{4} \text{ of } 20$$

ACTIVITY OUTLINE

- Instruct students to put 12 counters in front of them.
- Ask students to encase their counters with their hands, and say to the teacher, "This is one whole."
- Ask students to identify $\frac{1}{4}$ of the counters and discuss their solution strategies. Reflect on the process required - that the 12 counters had to be arranged into four groups, and each group represented one-quarter. Taking one-quarter of the whole in their hand, they could see that they had 3 counters. Therefore, one-quarter of 12 is 3.



This is one whole

Four equal groups

- Continue in this fashion with counters of various totals:

8 counters, show $\frac{1}{2}$	16 counters, show $\frac{1}{4}$
20 counters, show $\frac{1}{2}$	12 counters, show $\frac{1}{3}$
10 counters, show $\frac{1}{5}$	12 counters, show $\frac{1}{4}$
15 counters, show $\frac{1}{3}$	16 counters, show $\frac{1}{8}$
9 counters, show $\frac{1}{3}$	15 counters, show $\frac{1}{5}$
- Ask students to close their eyes and mentally calculate various unit fractions of a group:

$\frac{1}{5}$ of 15	$\frac{1}{3}$ of 9
$\frac{1}{4}$ of 12	$\frac{1}{3}$ of 12
$\frac{1}{4}$ of 8	$\frac{1}{6}$ of 12
$\frac{1}{5}$ of 20	$\frac{1}{10}$ of 20
$\frac{1}{4}$ of 20	$\frac{1}{3}$ of 15
- Ask students to explain their thinking and the mental images that come into their head.

ACTIVITY 5.4 VISUALISING NON-UNIT FRACTIONS WITHIN COLLECTIONS

AIM

To build on the visualisation strategy for finding unit fractions in a set (e.g., $\frac{1}{4}$ of 20) to finding non-unit fractions in a set through multiplication (e.g., $\frac{3}{4}$ of 20 = $\frac{1}{4}$ of 20 x 3).

OVERVIEW

Linking to 5.3, in this activity students use counters to explore non-unit fractions within a set (or collection).

MATERIALS

Approximately 20 counters per student (Unifix, cubes, or other suitable material).

TEACHING POINTS

- To assist students' focus on the whole, have students place their counters on a piece of coloured paper or card with all other counters away to the side of the desk.
- Ensure students are at a level of competence in finding unit fractions of the set before moving on to other fractions.
- Discuss with students the extra level of difficulty required to find fractions of a set that are not unit fractions.
- Encourage students to discuss their strategies for solution to the mental calculations.

ASSESSING PROGRESS

- Students readily discuss their visual images of a set of material that is divided into equal groups.
- Students' mental computation becomes more accurate.

PRACTICE EXAMPLES

$$\frac{3}{4} \text{ of } 12$$

$$\frac{4}{5} \text{ of } 20$$

$$\frac{2}{3} \text{ of } 15$$

$$\frac{2}{3} \text{ of } 9$$

$$\frac{2}{5} \text{ of } 20$$

$$\frac{3}{4} \text{ of } 8$$

$$\frac{2}{3} \text{ of } 6$$

$$\frac{3}{5} \text{ of } 20$$

$$\frac{4}{5} \text{ of } 25$$

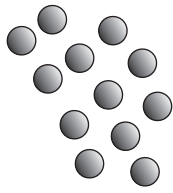
$$\frac{3}{4} \text{ of } 20$$

ACTIVITY OUTLINE

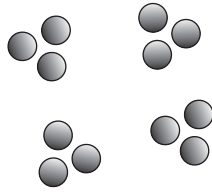
- Follow a similar sequence as in 5.3, but draw students' attention to the extra level of difficulty required to find a fraction that is more than a unit fraction.

Begin with 12 counters. Show $\frac{3}{4}$.

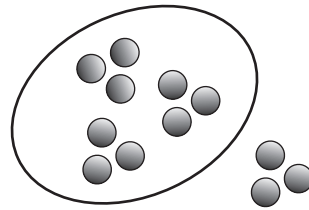
Discuss the procedure required - that the 12 counters had to be arranged into four groups, and each group represented one-quarter. They were required to select three of the four groups, and that gave a total of 9.



12 counters



4 groups



3 of these groups equals 9

- Continue in this fashion with counters of various totals:
 - e.g., 12 counters, show $\frac{2}{3}$
 - 20 counters, show $\frac{3}{5}$
 - 10 counters, show $\frac{2}{5}$
 - 15 counters, show $\frac{4}{5}$
 - 20 counters, show $\frac{3}{4}$
- Ask students to close their eyes and mentally calculate various non-unit fractions of a group:

$\frac{2}{5}$ of 15	$\frac{3}{4}$ of 16
$\frac{3}{4}$ of 12	$\frac{2}{3}$ of 12
$\frac{3}{5}$ of 20	$\frac{2}{3}$ of 6
$\frac{2}{3}$ of 9	$\frac{2}{5}$ of 10
$\frac{3}{4}$ of 20	$\frac{3}{4}$ of 24
- Ask students to explain their thinking and the mental images that come into their heads.

ACTIVITY 5.5 SIMPLE FRACTION MULTIPLICATION

AIM

To promote mental computation of fraction multiplication through visualisation of movements along a number line.

OVERVIEW

In this activity, students explore simple fraction multiplication through skip counting on a number line.

MATERIALS

BLM 5.5 (one copy per student)

TEACHING POINTS

- Encourage students to continue to display each situation on the number line until they can visualise the solution.
- Discuss the meaning of the symbolic representation as $4 \times \frac{1}{3}$ meaning 4 thirds.
- Make links to 5.1 where fraction pieces were used to explore wholes and parts, and discuss the circle as a visual image compared with the number line. Encourage students to make choices about the mental images that are most meaningful for them when performing mental calculations.

ASSESSING PROGRESS

- Students readily discuss their visual images of fraction multiplication.
- Students' mental computation becomes more accurate.
- Students can articulate that fraction multiplication does not result in a bigger solution (as is the case with whole numbers).

PRACTICE EXAMPLES

$$1 \times \frac{1}{2}$$

$$3 \times \frac{1}{2}$$

$$4 \times \frac{1}{3}$$

$$6 \times \frac{1}{2}$$

$$2 \times \frac{1}{4}$$

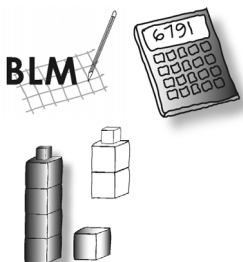
$$5 \times \frac{1}{3}$$

$$2 \times \frac{1}{5}$$

$$5 \times \frac{1}{5}$$

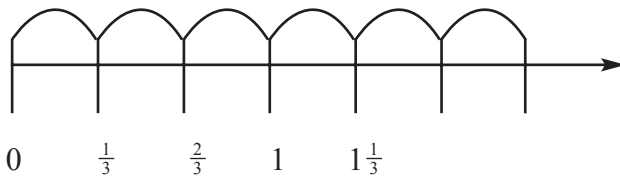
$$5 \times \frac{1}{4}$$

$$5 \times \frac{1}{3}$$



ACTIVITY OUTLINE

1. Practice skip counting orally in thirds: $\frac{1}{3}$, $\frac{2}{3}$, $\frac{3}{3}$ (1), $1\frac{1}{3}$, $1\frac{2}{3}$,...
2. Draw a number line on the board and mark in thirds, identifying the position of the numbers one and two. Demonstrate the skip counting pattern for counting in thirds by moving finger along the number line as each fraction is said.



3. Distribute **BLM 5.5**. Direct students to record the missing fractions on the number line as they repeat the skip counting sequence for thirds.
4. Skip count in quarters, fifths, halves, and complete the number lines on **BLM 5.5**. Encourage students to count aloud as they draw the sequence on the number line. Complete the blank number lines on 5.5 with fractions of your choice.
5. Using the halves number line, point-count along in halves, six times. Draw students' attention to the position they have reached on the number line (3).
6. Write the symbolic calculation that represents the action undertaken:
 e.g., $6 \times \frac{1}{2}$
 Explain that the action was to count in halves, six times, to give a solution of 3.
7. Practice other calculations in a similar fashion, using the completed number lines to track the action:
 e.g., $5 \times \frac{1}{3}$
 $6 \times \frac{1}{4}$
 $8 \times \frac{1}{2}$
 $7 \times \frac{1}{4}$
 $4 \times \frac{1}{3}$
8. Pose similar situations, but ask students to close their eyes and visualise the number line to reach the solution.
9. Discuss visual pictures and strategies students used to arrive at the answer.
10. Practice some mental calculations of fraction multiplication.

ACTIVITY 5.6 VISUALISING FRACTION DIVISION

AIM

To promote students' simple fraction division mental computation through visualisation of fractional positions on a number line.

OVERVIEW

In this activity, students explore the number of fractional parts within wholes through reference to number lines.

MATERIALS

BLM 5.6 (one copy per student)

TEACHING POINTS

- Encourage students to continue to display each situation on the number line until they can visualise the solution.
- Reinforce the language necessary to interpret the symbolic representation: $3 \div \frac{1}{2}$ is read as "How many halves in 3?"
- Contrast this to the language for fraction multiplication $4 \times \frac{1}{2}$ and compare the difference in solution size.
- Make links to the difference in use of the number line for fraction multiplication (see 5.5) as for fraction division.

ASSESSING PROGRESS:

- Students readily discuss their visual images of fraction division.
- Students can articulate that fraction division does not result in a smaller solution (as is the case with whole numbers).
- Students' mental computation becomes more accurate.

PRACTICE EXAMPLES:

$$2 \div \frac{1}{2}$$

$$2 \div \frac{1}{3}$$

$$1 \div \frac{1}{4}$$

$$3 \div \frac{1}{2}$$

$$1 \div \frac{1}{3}$$

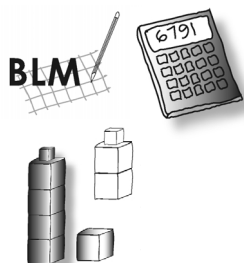
$$2 \div \frac{1}{5}$$

$$3 \div \frac{1}{4}$$

$$3 \div \frac{1}{5}$$

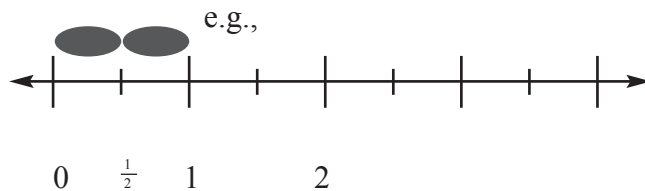
$$1 \div \frac{1}{10}$$

$$2 \div \frac{1}{4}$$



ACTIVITY OUTLINE

1. Ask students to close their eyes and visualise a number line for counting in halves. Ask them to mentally locate the position of the following numbers: $\frac{1}{2}$, $2\frac{1}{2}$, 1, 6, $3\frac{1}{2}$.
2. Distribute **BLM 5.6**. Direct students' attention to the halves number line, and ask them to visualise how many halves are in one, and how it might be drawn on that number line.
3. Draw a possible representation on the board to show how many halves are in one:



4. Invite students to share other possible representations on the number line. Discuss the merits of each representation. Have students show that there are 2 halves in one on the number line on **BLM 5.6**.
5. Ask students to add to their picture to show that there are four halves in two.
6. Ask students to fill in missing numbers from other number lines on **BLM 5.6**.
7. Ask students to use the number lines to determine the following:
 - how many thirds in 2
 - how many quarters (fourths) in 2
 - how many thirds in 4
 - how many fifths in 2
8. Ask students to consider how many halves in 3. Write the symbolic recording that represents the action undertaken:

$$3 \div \frac{1}{2}$$

Encourage students to read this as '*How many halves in 3?*'

9. Write similar equations on the board, and ask students to close their eyes and visualise the solution. Discuss whether the number line assisted them to attain the solution.
10. Practise some mental calculations of fraction division.

ACTIVITY 5.7 VISUALISING DECIMALS TENTHS

AIM

To promote visualisation of a number line when performing mental computation of addition and subtraction in tenths to one whole.

OVERVIEW

In this activity, students use a number line to explore decimal tenths.

MATERIALS

BLM 5.7 (one copy per student)

TEACHING POINTS

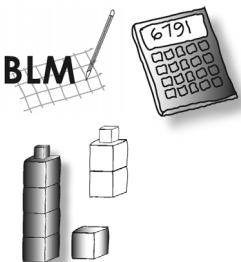
- Discuss the features of the number line for addition and subtraction. Link this action to similar counting activities using a number line.
- Link this activity to students' strategies for basic tens facts and discuss similarities (bonds) and differences (total of one, not 10).

ASSESSING PROGRESS

- Students readily discuss their visual images of decimal addition and subtraction to one.
- Students' mental computation becomes more accurate.

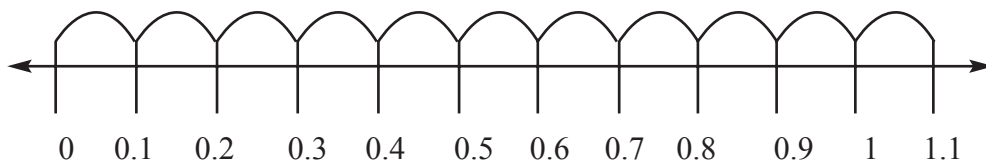
PRACTICE EXAMPLES

$0.5 + 0.5$	$1 - 0.7$
$0.2 + 0.8$	$1 - 0.2$
$0.6 + 0.4$	$1 - 0.3$
$0.3 + 0.7$	$1 - 0.9$
$0.4 + 0.6$	$1 - 0.5$



ACTIVITY OUTLINE

1. Practice skip counting in decimal tenths 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1, 1.1..., using the language point one, point two..., rather than its appropriate fraction name, to differentiate between fraction number lines.
2. Listen for any students saying point ten after point nine, and discuss this possible misconception. Make the link to fraction tenths to assist in developing meaning.
3. Distribute **BLM 5.7**. Direct students to fill in missing decimals on the number line as they repeat the skip counting sequence on the number line showing tenths.



4. Pose some addition and subtraction computations of tenths to one
e.g., $0.3 + 0.7$, $0.5 + 0.5$, $1 - 0.6$, $1 - 0.4$, $0.6 + 0.4$
5. Pose similar situations, but ask students to close their eyes and visualise the number line to reach the solution.
6. Discuss visual pictures and strategies students use to arrive at the solution.
7. Practice some mental calculations of decimal addition and subtraction to one.

ACTIVITY 5.8 DECIMAL MULTIPLICATION

AIM

To promote mental computation of decimal multiplication through visualisation of movements along a number line.

OVERVIEW

In this activity, students explore simple decimal multiplication through skip counting on a number line.

MATERIALS

BLM 5.8 (one copy per student), calculators

TEACHING POINTS

- Encourage students to continue to use the number line until they can visualise the solution.
- Discuss the language of the symbolic recording: 3×0.5 is counting in 0.5s.

ASSESSING PROGRESS

- Students readily discuss their visual images of decimal multiplication.
- Students can articulate that decimal multiplication does not result in a bigger solution (as in whole number multiplication).
- Students can link this activity with fraction multiplication.
- Students' mental computation becomes more accurate.

PRACTICE EXAMPLES

$$2 \times 0.5$$

$$3 \times 0.5$$

$$6 \times 0.5$$

$$5 \times 0.2$$

$$8 \times 0.2$$

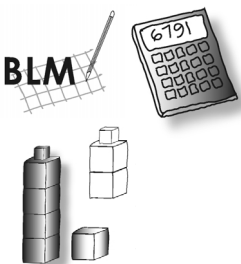
$$7 \times 0.2$$

$$3 \times 0.6$$

$$4 \times 0.7$$

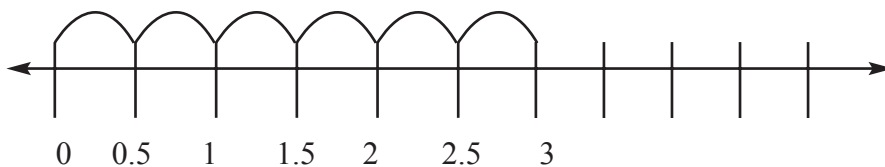
$$5 \times 0.7$$

$$5 \times 0.4$$



ACTIVITY OUTLINE

1. Practise skip counting in decimal fifths: 0.5, 1, 1.5, 2, 2.5...
2. Draw a number line on the board and mark in decimal fifths. Demonstrate the skip pattern by marking in the 'skips' along the number line as each decimal is said.
3. Distribute **BLM 5.8**. Direct students to fill in the missing numbers on the number line.



4. Have students observe the skip counting pattern on the calculator:
Clear calculator, enter 0.5, then press '+' button twice, and then the '=' button. Check that the '1' is displayed. Ask students to press the '=' button again to see if the calculator is performing a counting action. Ask students to continue to press the '=' button and observe the counting sequence.
Discuss the movement of the decimal point on the screen.
5. Using the 0.5 number line, ask students to count in 0.5s, six times, tracking the sequence with their fingers. Draw students' attention to the position they have reached on the number line (3).
6. Write the symbolic calculation that represents the action undertaken:
e.g., 6×0.5
Explain the action was to count in 0.5, six times, to give a solution of 3.
Draw students' attention to strategies they used to find fraction multiplication.
7. Practise other calculations in a similar fashion, using the number line to track the action:
e.g. 5×0.5
 4×0.2
 8×0.5
 5×0.4
 2×0.6
8. Pose similar situations, but ask students to close their eyes and visualise the number line to reach the solution.
9. Discuss visual pictures and strategies students used to arrive at the answer.
10. Practise some mental calculations of decimal multiplication.

ACTIVITY 5.9 VISUALISING DECIMAL DIVISION

AIM

To promote students' simple decimal division mental computation through visualisation of decimal positions on a number line.

OVERVIEW

In this activity, students explore the number of decimal parts within a whole through reference to a number line.

MATERIALS

BLM 5.9 (one copy per student), calculators

TEACHING POINTS

- Encourage students to continue to use the number line until they can visualise the solution.
- Reinforce the language necessary to interpret the symbolic representation: $3 \div 0.5$ is read as *How many 0.5s in 3?*
- Encourage students to verbalise the strategy of thinking of decimal sections within 1 as a reference point for thinking of numbers greater than one.

ASSESSING PROGRESS

- Students readily discuss their visual images of decimal division.
- Students can articulate that decimal division does not result in a smaller solution (as in whole number division).
- Students can link this activity with fraction division.
- Students' mental computation becomes more accurate.

PRACTICE EXAMPLES

$2 \div 0.5$

$1.5 \div 0.5$

Extension

$1.5 \div 0.5$

$3 \div 0.5$

$2.5 \div 0.5$

$2.5 \div 0.5$

$1 \div 0.2$

$1.2 \div 0.2$

$2 \div 0.4$

$2 \div 0.2$

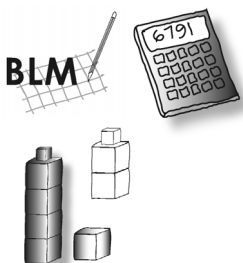
$1.6 \div 0.2$

$1.8 \div 0.3$

$4 \div 0.2$

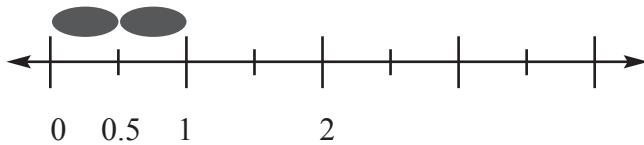
$0.8 \div 0.2$

$1.2 \div 0.3$



ACTIVITY OUTLINE

1. Ask students to close their eyes and visualise a number line marked in 0.5 sections. Ask them to mentally locate the following points on the number line:
0.5, 2.5, 1, 0, 6, 3.5.
2. Distribute **BLM 5.9**. Direct students' attention to the number line marked in 0.5 sections, and ask them to state how many 0.5 sections are in one. Discuss how this might be drawn on a number line.
3. Draw a possible representation on the board:



4. Have students record this (or their own) representation on the number line on **BLM 5.9**.
5. Ask students to add to their picture to show that there are four 0.5s in two.
6. Ask students to use the number lines to determine solutions to the following:
 - how many 0.5s in 2
 - how many 0.5s in 5
 - how many 0.2s in 1
 - how many 0.2s in 2
 - how many 0.2s in 6
7. Ask students to consider how many 0.5s in 2 and write the symbolic representation on the board:

$$2 \div 0.5$$

Encourage students to read this as *How many 0.5s in 2...*

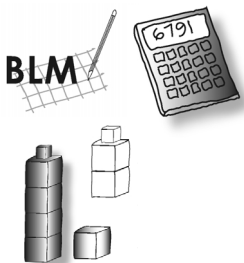
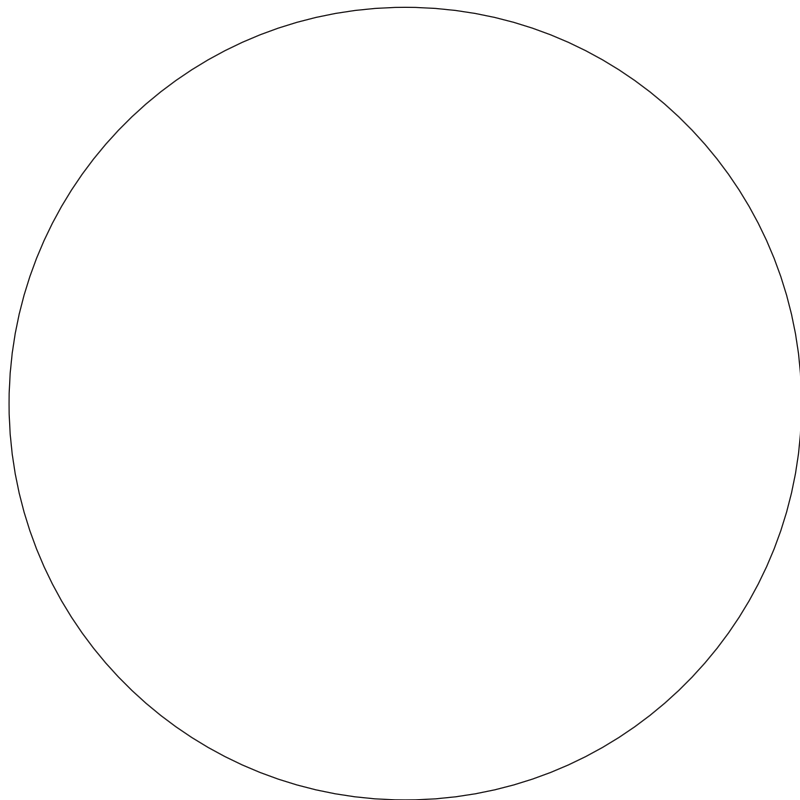
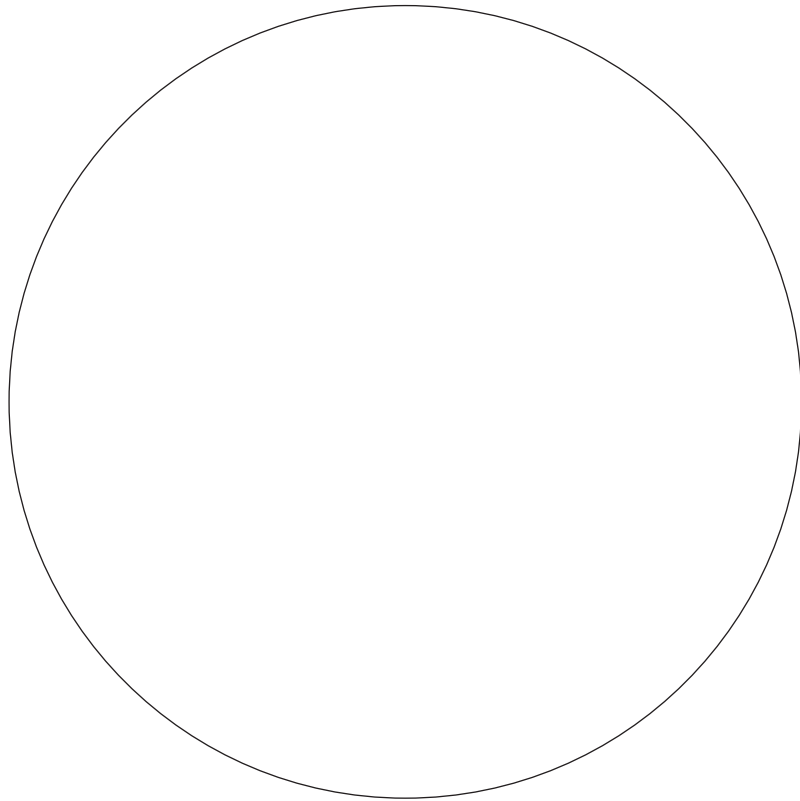
8. Write similar situations on the board, but ask students to close their eyes and visualise the solution. Discuss whether the number line assisted them to attain the solution and what thinking they engaged in.



$3 \div 0.5$
 means *how many 0.5s in 3...*
 Locate 3 on the number line
 There are six 0.5s in three.

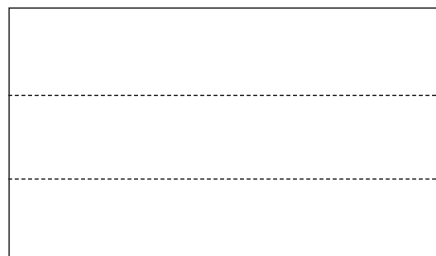
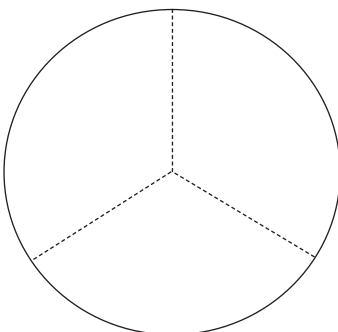
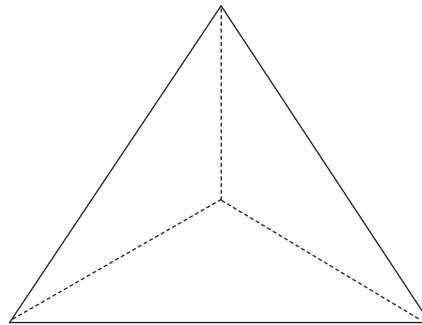
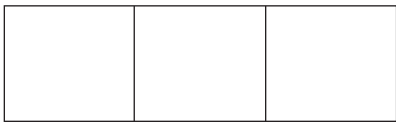
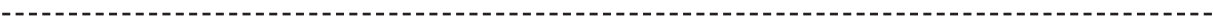
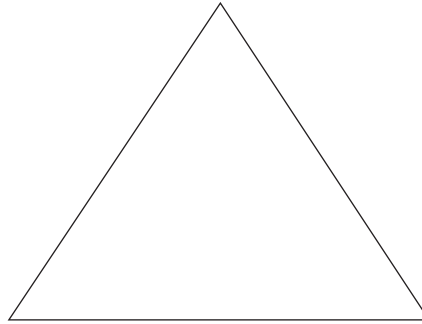
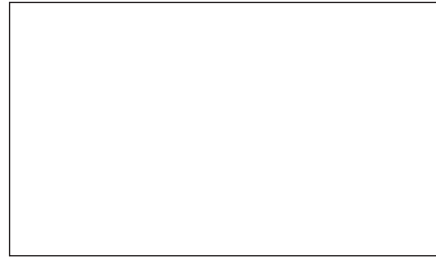
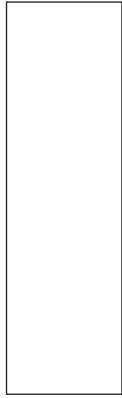
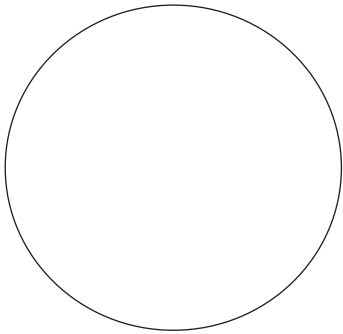
9. Practise some mental calculations of decimal division.

CIRCLES



ONE THIRD

Cut out the shapes at the bottom of the page.
Write the fraction name on each shape.
Paste the thirds on the shapes below to make whole shapes.



FRACTION PARTS AND WHOLES

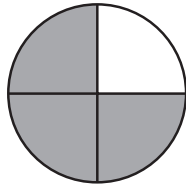
Look at each shape or number line.

1) Write how many parts in each shape.

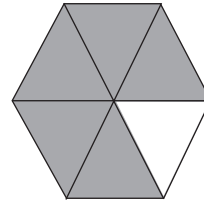
2) Write the fraction name for the number of parts that are shaded.

3) Write the fraction name for the number of parts that are unshaded.

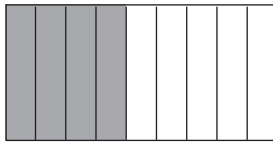
1. Number of parts in total = _____
2. Fraction shaded = _____
3. Fraction unshaded = _____



1. Number of parts in total = _____
2. Fraction shaded = _____
3. Fraction unshaded = _____



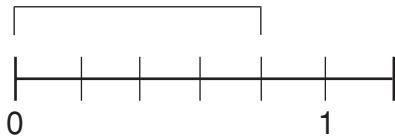
1. Number of parts in total = _____
2. Fraction shaded = _____
3. Fraction unshaded = _____



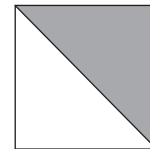
1. Number of parts in total = _____
2. Fraction shaded = _____
3. Fraction unshaded = _____



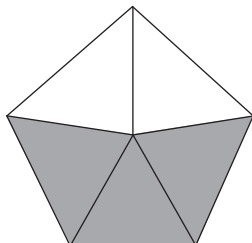
1. Number of parts in total = _____
2. Fraction marked = _____
3. Fraction unmarked = _____



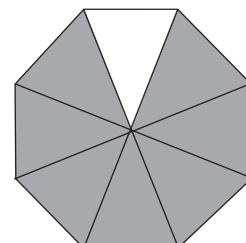
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2. Fraction shaded = _____
3. Fraction unshaded = _____



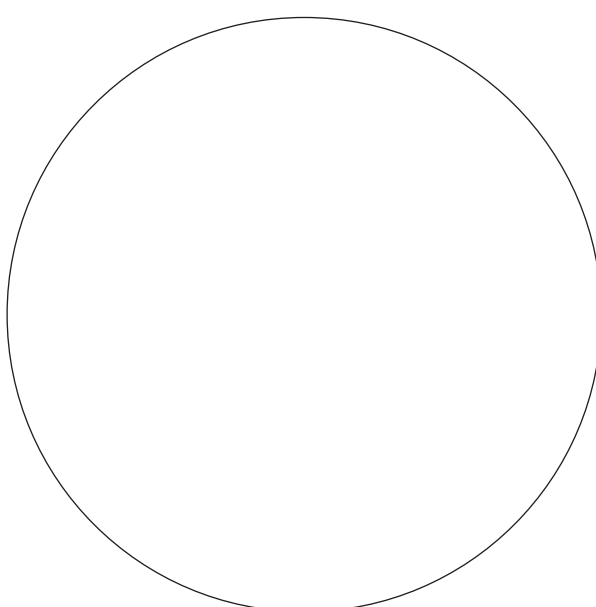
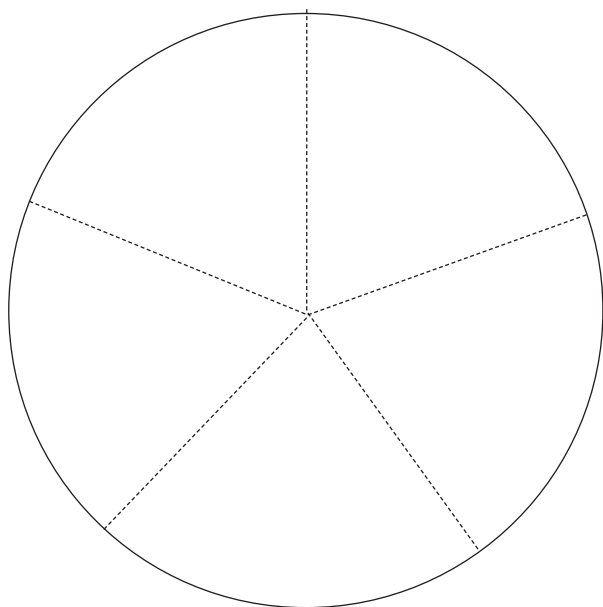
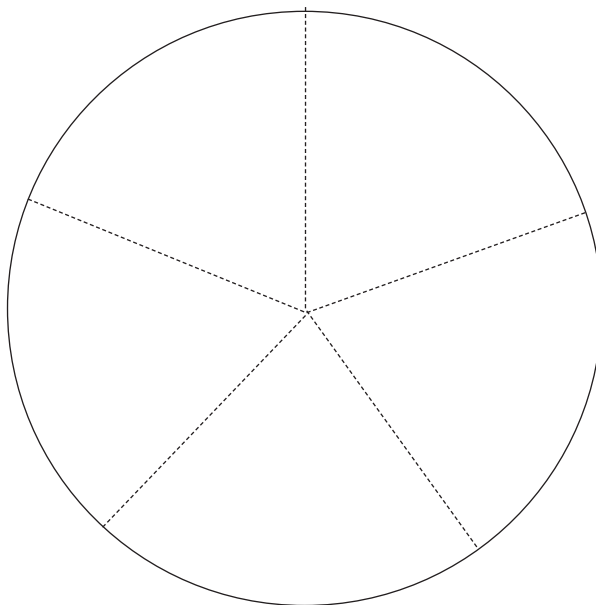
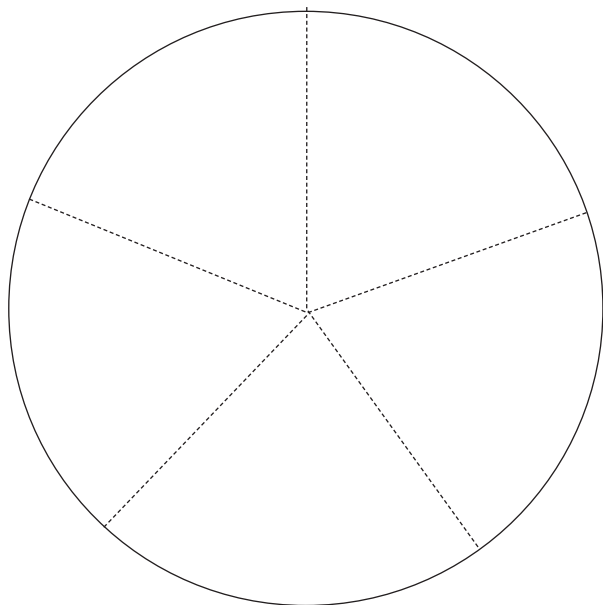
1. Number of parts in total = _____
2. Fraction of parts shaded = _____
3. Fraction of parts unshaded = _____



1. Number of parts in total = _____
1. Fraction of parts shaded = _____
2. Fraction of parts unshaded = _____

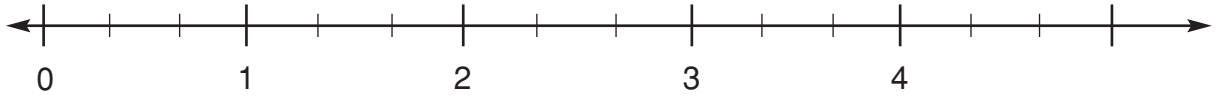


FRACTION FIFTHS



SKIP COUNTING NUMBER LINES

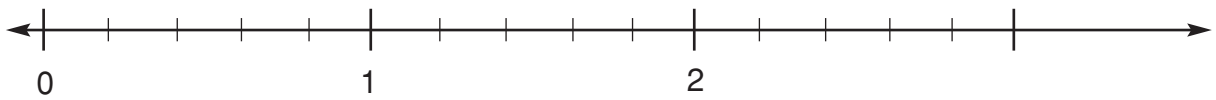
THIRDS



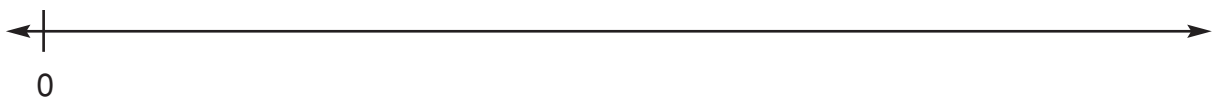
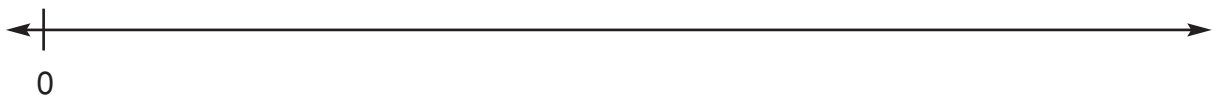
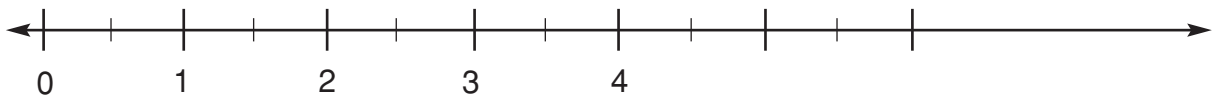
QUARTERS



FIFTHS

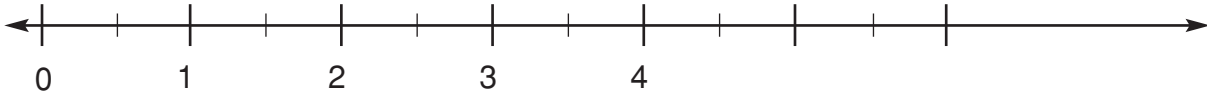


HALVES

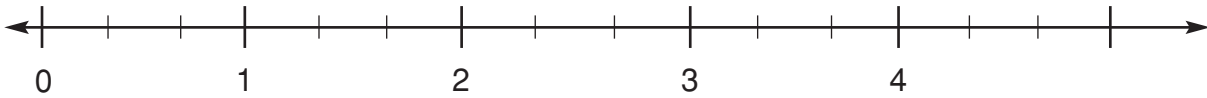


PICTURES FOR FRACTION DIVISION

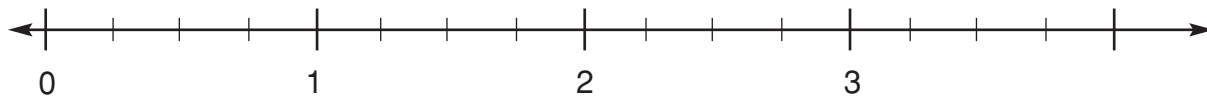
HALVES



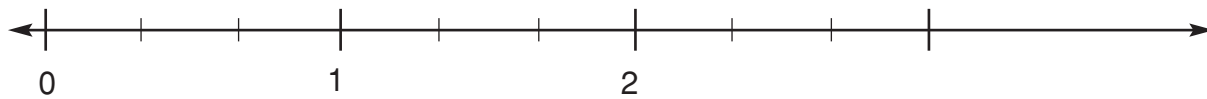
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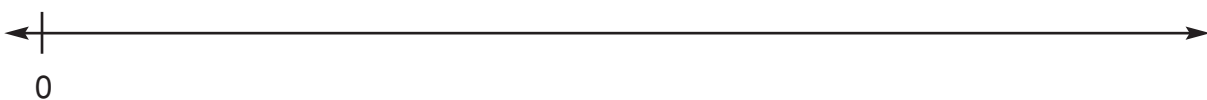
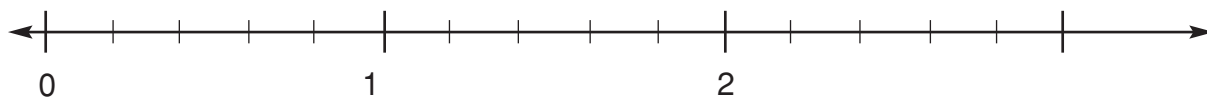
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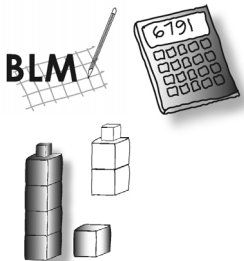
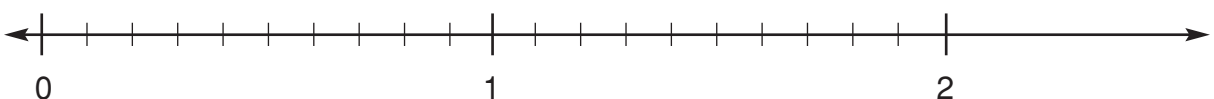
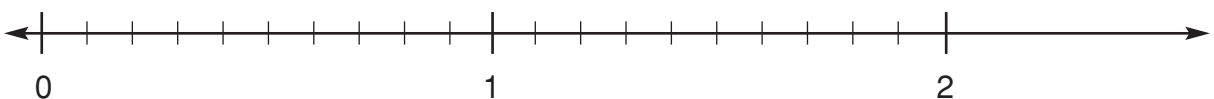
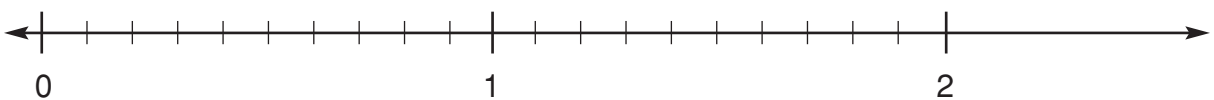
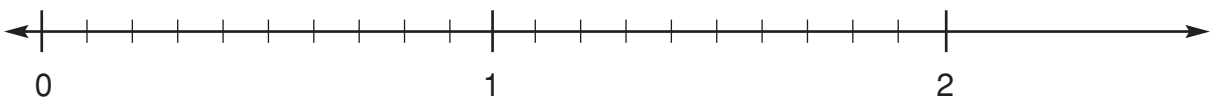
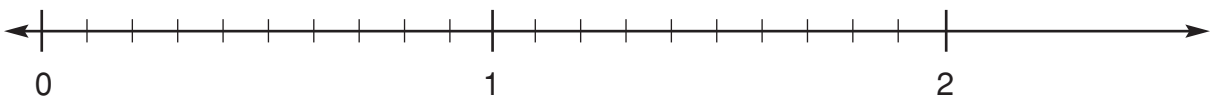
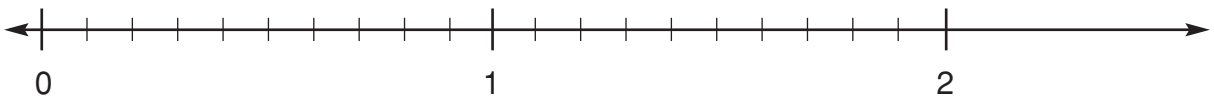
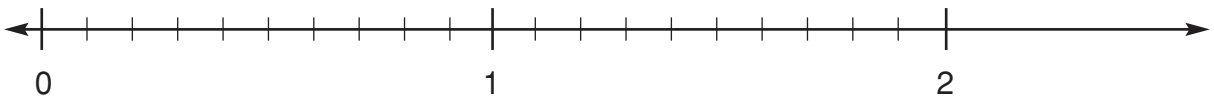
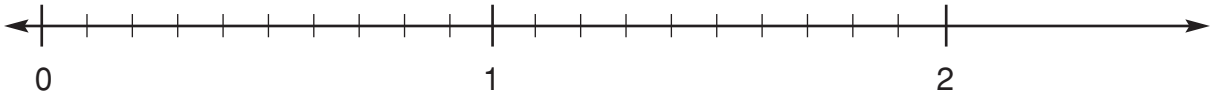
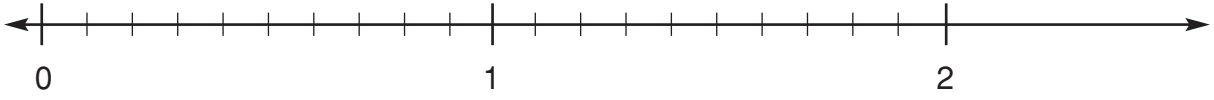
THIRDS



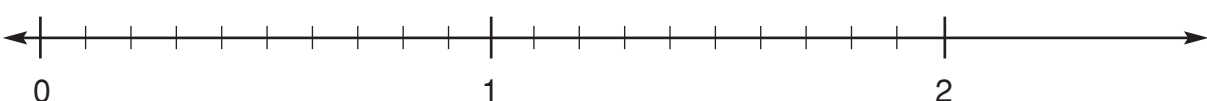
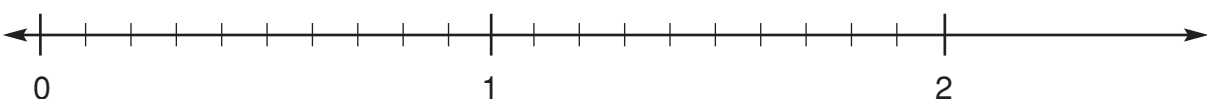
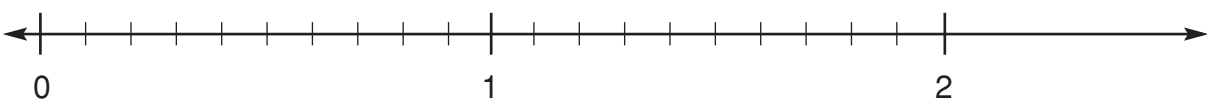
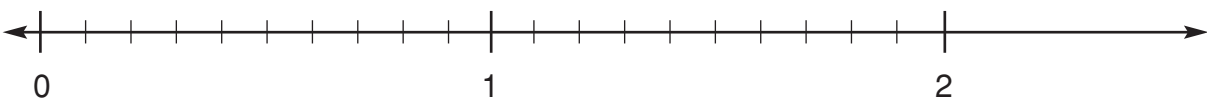
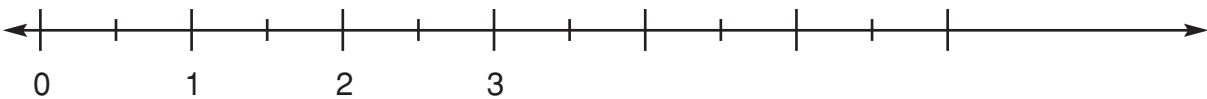
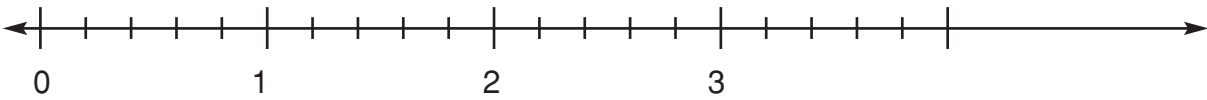
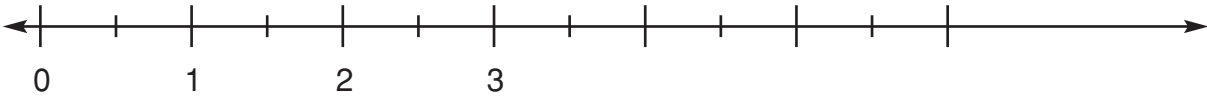
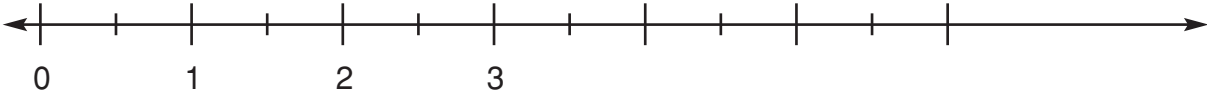
FIFTHS



TENTHS NUMBER LINES



DECIMAL FRACTION NUMBER LINES



PICTURES FOR DECIMAL FRACTION DIVISION

