MENTAL COMPUTATION:
A STRATEGIES APPROACH

MODULE 4
two-digit whole numbers

Alistair McIntosh

Mental Computation: A strategies approach
Module 4 Two-digit whole numbers

Alistair McIntosh

This is one of a set of 6 modules providing a structured strategies approach to mental computation.

Module 1 Introduction
Module 2 Basic facts addition and subtraction
Module 3 Basic facts multiplication and division
Module 4 Two-digit whole numbers
Module 5 Fractions and decimals
Module 6 Ratio and percent

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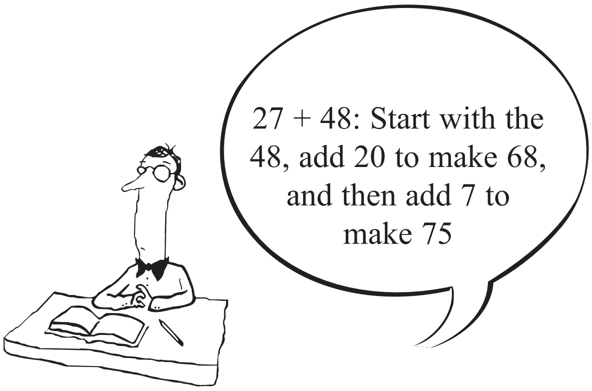
INTRODUCTION

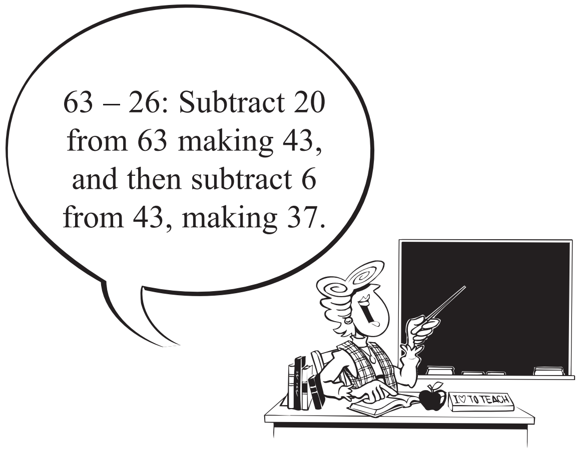
LEARNING ACTIVITIES

BLACKLINE MASTERS

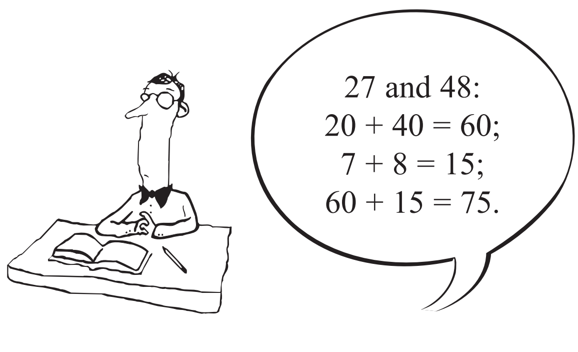
STRATEGIES USED FOR MENTAL ADDITION/SUBTRACTION OF LARGER NUMBERS

There are only two main strategies commonly used to add or subtract two- or three-digit numbers mentally. Using the first main strategy we start with one number and gradually add to it or subtract from it the other number.





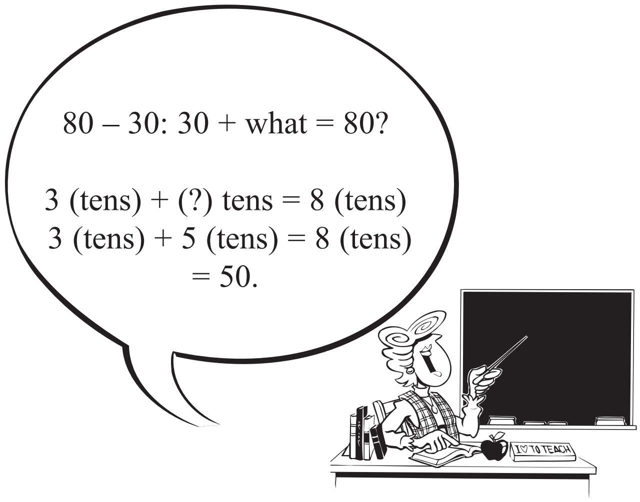
Using the second main strategy, we separate the tens and units and deal with them separately, and then combine the results.



We are less likely to use this second strategy for subtraction. For example to subtract 63 – 26, we would need to split the 63 as 50 + 13 (not 60 + 3), and then calculate 50 – 20 and 13 – 6, finally combining the results, 30 + 7, making 37.

Apart from these two main strategies, we often

- start by turning a subtraction into an addition, and,
- where both numbers are multiples of 10, we often treat the tens as though they were the units



As this 'taking off the noughts' strategy often leads to misunderstandings and errors by children (particularly when the same strategy is applied to multiplication and division), it is better to insist on the longer wording given above: that is '8 (tens) – 3 (tens)', not '8 – 3'.

SUMMARY OF STRATEGIES FOR ADDITION AND SUBTRACTION OF 2-DIGIT NUMBERS

Initial strategy

- **Change Subtraction into Addition:** 63 – 58: 58 + ? = 63

Start with One Number, Process the Other

- **Adding/ Subtracting the second number in parts:** 63 + 15: 63, 73, 78.
- **Bridging Tens/Hundreds:** 45 – 8: 45, 40, 37. 85 + 27: 85, 100, 112.

Split One or Both Numbers, Process and Reassemble

- **Working from the left** (tens first): 36 + 28: 50 + 14, 64.
- **Working from the right** (units first): 36 + 28: 14 + 50, 64.
- **Using a mental form of the written algorithm:** 36 + 28: 6 and 8 = 14, put down 4, carry the one, 3 and 2 and 1 make 6; 64.

Further Strategy

- **Using Tens as the Unit:** 120 – 50: 12 (tens) – 5 (tens) = 7 (tens), 70.

This strategy appears to work reliably with addition and subtraction but, unless it is accompanied by understanding, it causes considerable problems with multiplication and division.

SUBSKILLS FOR ADDING AND SUBTRACTING TWO-DIGIT NUMBERS

If children have developed automatic recall of basic addition and subtraction facts, and acquired Counting On and Back, Bridging Ten and Using Doubles strategies in the process, what additional skills do they need in order to calculate mentally addition and subtraction of two-digit numbers?

What follows is an attempt to identify specific necessary subskills, to place them in clusters which may be developed together, and to associate them with the development of specific strategies for addition and subtraction of two-digit numbers. The clusters of sub-skills have been designated Stages: this reflects the theoretical position that the Stages do indicate a general hierarchy of skills. However this is tentative at this stage; certainly some, probably many, children will have some subskills from a higher Stage while not having all the subskills from a lower Stage.

Summary of Stages

Stage 1 – Essential underpinning for all two-digit strategies
Understandings, skills, facts and strategies learned in Module 2. Basic addition and subtraction facts and strategies.

Stage 2 – Essential underpinning for all two-digit strategies
Basic numeration understandings and skills. Understanding the composition of two-digit numbers. Counting on and back in tens.

Stage 3 –Needed for specific strategies Adding and subtracting a one-digit to/from a two-digit number
Adding and subtracting multiples of ten.

Stage 4 – For developing flexible strategy use
Adding/subtracting a multiple of ten to/from a two-digit number.
Complements to 100.

STAGES AND SUBSKILLS FOR ADDING AND SUBTRACTING TWO-DIGIT NUMBERS

Stage 1	*Examples*
1 Instant recall of basic addition/subtraction facts	Response within 3 seconds
2. Facility with these mental computation strategies for basic +/- facts:	
• Counting On and Back	7 + 2: 7, 8, 9
• Bridging Ten	
• Using Doubles strategies for basic facts	
Stage 2	***Examples***
3. Splitting a two-digit number 10a + b into 10a and b	37 = 30 + 7
4. Adding any two-digit multiple of ten to any single-digit number	50 + 6
5. Subtracting units digit from a two-digit number.	67 - 7
6. Subtracting tens digit from a two-digit number.	67 - 60
7. Naming the next multiple of ten for any two-digit number	57? 60
8. Counting on in tens from a multiple of ten	30, 40, 50…
9. Counting back in tens from a multiple of ten	100, 90, 80…
10. Counting on in tens from any two-digit number	47, 57, 67…
11. Counting back in tens from any two-digit number	94, 84, 74…
Stage 3	***Examples***
12. Saying what needs to be added to any two-digit number to make the next multiple of ten	57 + ? = 60
13. Subtracting a single digit from any two-digit multiple of ten	80 - 6
14. Adding a single digit to any two-digit number	25 + 7
15. Subtracting any two-digit number from the next multiple of ten	60 - 57
16. Subtracting a single digit from any two-digit number	53 - 7
17. Doubling any two-digit multiple of ten	60 + 60
18. Adding any two two-digit multiples of ten	40 + 70
19. Subtracting any two-digit multiple of ten from any two-digit multiple of ten	90 - 50
20. Subtracting any two-digit multiple of ten from its double	120 - 60
21. Subtracting any two-digit multiple of ten from any multiple of ten less than two hundred	130 - 50
Stage 4	***Examples***
22. Adding a two-digit multiple of ten to any two-digit number	74 + 40
23. Splitting a two-digit number into any tens and ones	37 = 20 + 17
24. Subtracting a two-digit multiple of ten from any two-digit number	74 - 30
25. For any two digit number, give its complement to 100	73 + ? = 100

It is intended that this analysis is used

- as a check-list of things to teach individually or collectively before a strategy is introduced.
- as a basis for planning after diagnosis of individual needs and weaknesses.

Many children will already have acquired some of these subskills with or without explicit classroom teaching. However it is currently too common for children to be asked to learn formal written algorithms without understanding the underlying place value structure.

EXAMPLES FOR PRACTICE OR ASSESSMENT OF SUB-SKILLS

SUB-SKILLS

Stage 1

See Module 2 (Basic Facts Addition and Subtraction)

Stage 2

3. Splitting a two-digit number 10a + b into 10a and b

31 =	53 =	74 =	18 =	60 =

4. Adding any two-digit multiple of ten to any single-digit number

30 + 8 =	90 + 6 =	20 + 4 =	9 + 80 =	7 + 10 =

5. Subtracting units digit from a two-digit number.

57 - 7 =	26 - 6 =	61 - 1 =	22 - 2 =	19 – 9 =

6. Subtracting tens digit from a two-digit number.

43 – 40 =	75 – 70 =	37 – 30 =	66 – 60 =	15 – 10 =

7. Naming the next multiple of ten for any two-digit number

28?	83?	45?	61?	12?

8. Counting on in tens from a multiple of ten (Give the next five numbers)

20	40	30	50	80

9. Counting back in tens from a multiple of ten (Give the next five numbers)

70	90	80	100	130

10. Counting on in tens from any two-digit number (Give the next five numbers)

23	44	37	56	78

11. Counting back in tens from any two-digit number (Give the next five numbers)

75	92	81	124	137

Stage 3

12. Saying what needs to be added to any two-digit number to make the next multiple of ten

28	46	55	83	12

13. Subtracting a single digit from any two-digit multiple of ten

50 - 1	30 - 4	60 - 6	80 - 3	40 - 8

14. Adding a single digit to any two-digit number

27 + 4	59 + 3	36 + 7	65 + 9	87 + 8

15. Subtracting any two-digit number from the next multiple of ten

40 - 38	20 - 15	50 - 43	80 - 74	60 - 51

16. Subtracting a single digit from any two-digit number

41 - 3	34 - 5	26 - 9	64 - 8	81 - 6

Stage 4

17. Doubling any two-digit multiple of ten

2 x 20	2 x 50	2 x 70	2 x 60	2 x 90

18. Adding any two two-digit multiples of ten

10 + 30	40 + 30	50 + 30	70 + 60	40 + 90

19. Subtracting any two-digit multiple of ten from any two-digit multiple of ten

50 - 10	60 - 40	70 - 60	80 - 30	90 - 30

20. Subtracting any two-digit multiple of ten from its double

80 - 40	100 - 50	140 - 70	180 - 90	160 - 80

21. Subtracting any two-digit multiple of ten from any multiple of ten less than two hundred

120 - 50	150 - 80	170 - 90	140 - 60	110 - 30

Stage 5

22. Adding a two-digit multiple of ten to any two-digit number

30 + 23	52 + 40	50 + 67	83 + 20	80 + 76

23. Splitting a two-digit number into any tens and ones

34 = 20 +	53 = 10 +	72 = 40 +	87 = 30 +	91 = 30 +

24. Subtracting a two-digit multiple of ten from any two-digit number

43 - 10	52 - 40	78 - 50	65 - 20	94 - 40

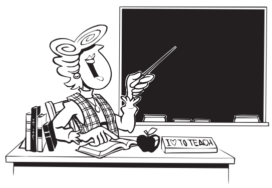
25. For any two digit number, give its complement to 100

89?	25?	73?	46?	61?

STRATEGIES USED FOR MENTAL MULTIPLICATION/DIVISION OF LARGER NUMBERS

There are four general strategies for multiplying mentally a two-digit by a one-digit number:

1. Use or relate to a known fact;
2. Use extension of 1-digit multiplication strategies;
3. Skip count; and
4. Use the distributive property



3 x 30: I just know that's 90

4 x 15: that must be the same as 2 x 30

4 x 25: I know that's 100.

7 x 25: that must be 100 + 75, 175.

1. Use or relate to a known fact

Either we know some results, or we can relate them to some we know, for example multiples of 25.

2. Use extension of 1-digit multiplication strategies

Module 2 introduced strategies for multiplying single digits by single digits. These strategies can be extended for use in multiplying larger numbers by a single digit. Here is an example of each:

Multiple	*Strategy*	*Example*
2 x	Double	2 x 24: Double 24 = 48
3 x	Double and one more	3 x 24: Double 24 + 24 = 48 + 24 = 72
4 x	Double twice	4 x 24: 2 x 24 = 48, 2 x 48 = 96
5 x	Half of 10 x	5 x 24: 10 x 24 = 240, half of 240 = 120
6 x	Five times and one more	6 x 24 = 120 + 24 = 144
7 x	Five times and two more	7 x 24 = 120 + 48 = 168
8 x	Double three times	8 x 24: 48, 96, 192
9 x	One less than ten x	9 x 24 = 240 – 24 = 216

3. Skip count

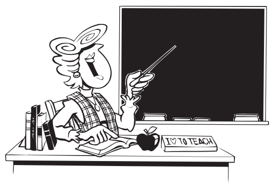
There are a few two-digit numbers for which we may be able to use skip-counting:

4 x 30: 30, 60, 90, 120

5 x 15: 15, 30, 45, 60, 75

3 x 25: 25, 50, 75

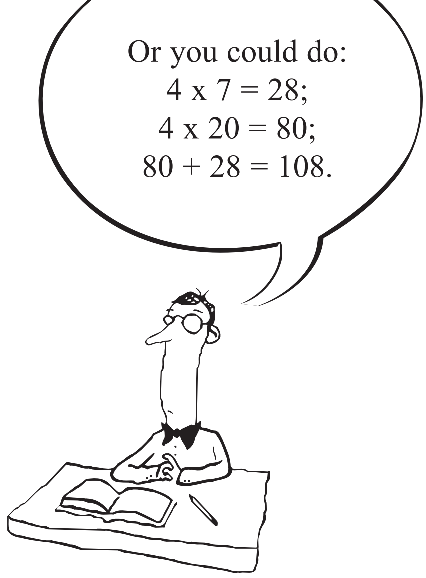
5 x 21: 21, 42, 63, 84, 105.

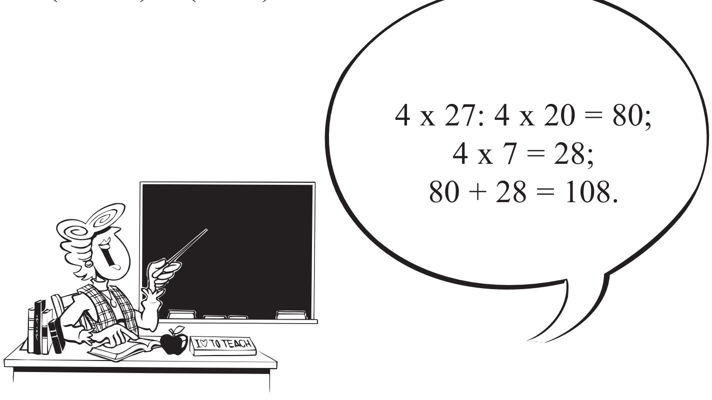


4. Using the distributive property

This is the mental equivalent of the normal written algorithm, in which we multiply separately the tens and the units digit, and then add the two results together. It does not matter whether we first multiply the tens or the units, but in practice mentally most people start with the tens digit.

The distributive law of multiplication over addition is: a(b + c) = (a x b) + (a x c), so that the multiplication is 'distributed' over the addition. In the case below we used 4 x 27 = (4 x 20) + (4 x 7).





Subskills for multiplying and dividing two-digit by one-digit whole numbers

There is not enough evidence at present to indicate a hierarchy of difficulty for these sub-skills.

1. All single-digit addition and subtraction skills covered in Section 2	
2. All single-digit multiplication and division skills covered in Section 3	
3. All two-digit addition and subtraction skills covered in Section 4	
4. Halving and doubling any 2- or 3-digit number	
5. Extension of table facts to multiples of 10	3 x 20 = 6 x 10 = 60 4 x 30 = 12 x 10 = 120
6. Adding a 2-digit number to a 2-digit multiple of 10	70 + 14 = 84 80 + 32 = 112
7. Adding a 2-digit number to a 3-digit multiple of 10	180 + 42 = 222
8. Extension of the Section 3 strategies to 2 x 1 digits. • 2 x = Doubles • 3 x = 2 x + 1 multiple • 4 x = 2 x 2 x • 5 x = half of 10 x • 6 x = 5 x + one multiple OR 2 x 3 x • 7 x = 5 x + 2 x • 8 x = 2 x 2 x 2 x • 9 x = 10 x – 1 multiple	 2 x 23 = double 23 3 x 14 = 28 + 14 4 x 17 = 2 x 34 5 x 28 = half of 280 6 x 28 = 140 + 28 7 x 28 = 140 + 56 8 x 35 x 4 x 70 = 2 x 140 9 x 56 = 560 - 56
9. Multiplying tens and units separately, then adding	3 x 14 = 30 + 12
10. Use of subtraction where helpful	3 x 29 = (3 x 30) – (3 x 1) 6 x 28 = 180 - 12
11. Use of known facts, for example 4 x 25 = 100	9 x 25 = 200 + 25
12. Skip counting in multiples of 10.	20, 40, 60, 80…200 30, 60, 90, 120…300
13. Subtracting a multiple of 10 from any two-digit number	72 - 60
14. Subtracting a multiple of 10 from any three-digit number	136 – 120
15. Splitting a two- or three-digit number into the nearest appropriate multiple of 10 and the remainder.	132 ÷ 4: (40 80,120) 120 + 12.

ACTIVITY 4.1 TWO-DIGIT ADDITION: BRIDGING MULTIPLES OF TEN

OVERVIEW In this strategy part of the second number is used to make the first number up to the next multiple of ten, then the remainder of the second number is added: for example 37 + 8; 37 + 3 = 40, + 5 = 45.

MATERIALS MAB (or adapt using popsticks in bundles of 10 and singles)

THE ACTIVITY

1. Check that students can use the ‘Bridging Ten’ strategy with single digit numbers (for example 8 + 6: 8 + 2 = 10, + 4 = 14). This strategy is introduced in Module 2.
2. Model the strategy using 36 + 7 as an example. Using MAB have student(s) display separately 36 and 7, lining up the 36 so that the 6 units lie alongside the 3 tens, showing the gap of 4 to make 40. *How many to make 40? 4. Move 4 units to complete the 40. How many left? 3. What is the total? 43.*
3. Model the language: 36 and 4 make 40 and 3 more makes 43.
4. Repeat using other 2-digit plus 1-digit combinations, with the students using MAB or not as they prefer, but always modelling the language. Examples: 26 + 5, 48 + 7, 39 + 3, 57 + 6, 15 + 8.

COMMENTS ON THE ACTIVITY

- This strategy is mainly used when adding a single-digit to a 2-digit number, for example 36 + 7.

ASSESSING PROGRESS

- Students can add a 2-digit and 1-digit number mentally, using the Bridging Ten strategy.
- Students can explain their strategy.

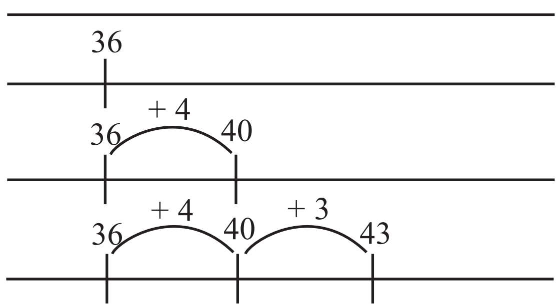
PRACTICE EXAMPLES

Students should give the answer and then explain the Bridging Multiples of Ten strategy for each of the following:

1.	18 + 3	6.	57 + 6
2.	27 + 5	7.	38 + 5
3.	49 + 4	8.	45 + 7
4.	76 + 7	9.	19 + 8
5.	65 + 8	10.	26 + 9

VARIATIONS /EXTENSIONS

- Have the students model their strategy using the blank number line. This prepares its use as a model and as a mental image in other strategies. For example 36 + 7.
- Blank Number Line:
- Mark 36
- Mark + 4
- Mark + 3



ACTIVITY 4.2 TWO-DIGIT ADDITION: ADDING PARTS OF THE SECOND NUMBER (0-99 BOARD)

OVERVIEW In this strategy the second number is added to the first in parts, usually based on place value: for example 37 + 26 is calculated as 37 + 20 + 6.

MATERIALS Counters, **BLM 4.1** (or **4.2**), **BLM 4.3** (or **4.4**)

THE ACTIVITY

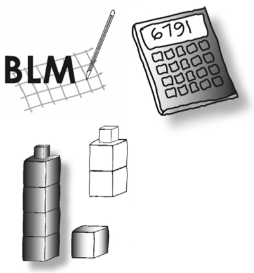
1. Use this preliminary activity for students who are unfamiliar with the positions of numbers on the board, or who need consolidation with breaking a 2-digit number into tens and units. *Place a counter on zero. Now move it onto 27 in this way: down 10, down 10 and right 1, 2, 3, 4, 5, 6, 7. Check the counter is on 27.*
2. The student now practises moving onto other 2-digit numbers, always starting with the counter on zero and always first counting down the tens and then along the row for the units.
3. As a challenge the student may try to locate a given number by placing the counter on zero and then, with eyes closed, moving the counter.
4. Adding 24 + 32. *Place the counter on zero and move to 24. How can we add 32? First add 30 (down three places), then add 2 (to the right two places). Check that the counter is on 56.*
5. The student now practises other additions with sums less than 100 and with the sum of the units digits not exceeding 9: for example 31 + 43, 62 + 27, 54 + 34.
6. Adding 36 + 29. *Place the counter on zero and move to 36. Add 20. How can we add 9? (there are not enough squares to the right)* Allow the student to explore and suggest a strategy. There are two alternatives: either count on 9, moving down to the next row; or move down an additional 10 and then move 1 square back (to the left). It is important that the student makes these suggestions and that neither is imposed as THE correct strategy.
7. It is important that the student now verbalizes the actions, in order to make the link with mental computation. *To add 36 and 29, first I added 36 and 30 (66) and then I subtracted 1 (65).*
8. Using the board the student practises other additions with sums less than 100 and with the sum of the units digits exceeding 9: for example 43 + 39, 26 + 18, 38 + 59.
9. The student now applies the strategy mentally without the board and explains the strategy used.

COMMENTS ON THE ACTIVITY

- This activity is written using the 0 – 99 board (**BLM 4.1**) as it has advantages over the 1 – 100 board (**BLM 4.2**) for this activity.

ASSESSING PROGRESS

- Students can add two 2-digit numbers using the board and explain what is happening.
- Students can add two 2-digit numbers mentally.
- Students can apply mentally and explain the Adding Parts of the Second Number strategy.



PRACTICE EXAMPLES

These should be done with or without the board as appropriate.

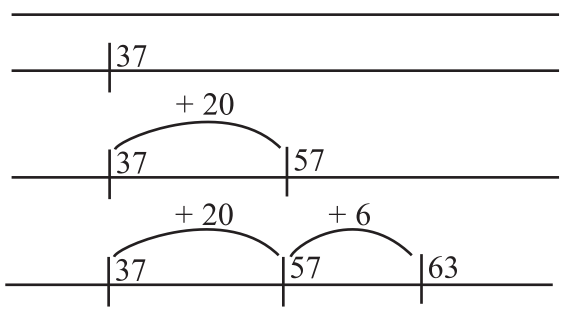
1.	34 + 25	6.	12 + 73
2.	72 + 24	7.	56 + 32
3.	27 + 18	8.	28 + 57
4.	55 + 35	9.	47 + 26
5.	49 + 46	10.	36 + 54

VARIATIONS / EXTENSIONS

- Have the students model their strategy using the blank number line. The blank number line is here shown horizontal: some prefer to use it in the vertical form.

For example 37 + 26.

- Blank Number Line:
- Mark 37
- Mark + 20
- Mark + 6.



This is not the only way this calculation can be shown using the blank number line: for example some students will prefer to use two jumps of 10 (to 47 then 57) rather than one jump of 20. Students should be encouraged to use the blank number line flexibly to explain their thinking, provided that they always mark the starting number, the direction of the 'jump' and its size (+ 20, + 6) and the landing point.

ACTIVITY 4.3 TWO-DIGIT ADDITION: WORKING FROM THE LEFT (MAB)

OVERVIEW In this strategy the tens digits are added together, then the units digits, and then these two are added: for example 37 + 26 is calculated as (30 + 20) + (7 + 6) = 50 + 13 = 63.

MATERIALS MAB (or adapt using popsticks in bundles of 10 and singles)

THE ACTIVITY

1. Check that students can represent 2-digit numbers using MAB with understanding; for example represent 35 as three rods and five units, explaining that 35 represents 3 tens and 5 ones.
2. Adding 37 + 26: *Put out as separate piles MAB representing 37 and 26. Collect the rods/tens together. How much is this? 50. Collect the units together. How many? 13. How many together? 63*. (If the student cannot instantly add 50 and 13, practise this using MAB and modelling 50 add 10 add 3.)
3. Using MAB the student now practises other additions with sums less than 100 and with the sum of the units digits exceeding 9: for example 36 + 17, 48 + 36, 65 + 28.
4. When confident the student now applies the strategy mentally without using MAB and explains the strategy used.

COMMENTS ON THE ACTIVITY

- This strategy and Adding Parts of the Second Number are the most commonly used strategies for adding 2-digit numbers and all students should develop confidence with them.

ASSESSING PROGRESS

- Students can add two 2-digit numbers using MAB and explaining what is happening.
- Students can add two 2-digit numbers mentally.
- Students can apply mentally and explain the Working from the Left strategy.

PRACTICE EXAMPLES

1.	24 + 35	6.	13 + 72
2.	62 + 34	7.	46 + 42
3.	47 + 18	8.	38 + 47
4.	25 + 45	9.	36 + 26
5.	39 + 56	10.	56 + 34

ACTIVITY 4.4 TWO-DIGIT ADDITION: WORKING FROM THE RIGHT

OVERVIEW In this strategy the units digits are added together, then the tens digits, and then these two are added: for example 37 + 26 is calculated as (7 + 6) + (30 + 20) = 13 + 50 = 63.

MATERIALS MAB (or adapt using popsticks in bundles of 10 and singles)

THE ACTIVITY

1. Check that students can represent 2-digit numbers using MAB with understanding; for example represent 43 as four rods and three units, explaining that 43 represents 4 tens and 3 ones.
2. Adding 37 + 26: *Put out as separate piles MAB representing 37 and 26. Collect the units together. How many? 13. Collect the rods/tens together. How much is this? 50. How many together? 63*. (If the student cannot instantly add 13 and 50, practise this using MAB and modelling 50 add 10 add 3.)
3. Using MAB the student now practises other additions with sums less than 100 and with the sum of the units digits exceeding 9: for example 54 + 37, 76 + 15, 28 + 56.
4. When confident the student now applies the strategy mentally without using MAB and explains the strategy used.

COMMENTS ON THE ACTIVITY

- This strategy is very similar in appearance to the mental form of the standard written algorithm. However students using this strategy can talk of adding the ones, then the tens and then adding the two sums: students who use the mental form of the standard written algorithm often close their eyes and/or move their fingers as though writing, and talk of ‘putting down a 3’ and ‘carrying a one to the tens column’. This strategy is therefore included, though not so commonly used, in order to draw this distinction between a flexible strategy (Working from the Right) and a procedural inflexible strategy (picturing the written form).

ASSESSING PROGRESS

- The student can add two 2-digit numbers using MAB, and explain what is happening
- The student can add two 2-digit numbers mentally
- The student can apply mentally and explain the Working from the Right strategy, clearly not picturing the written algorithm

PRACTICE EXAMPLES

1.	34 + 52	6.	62 + 24
2.	13 + 54	7.	54 + 25
3.	49 + 32	8.	19 + 74
4.	56 + 38	9.	43 + 37
5.	37 + 43	10.	38 + 45

ACTIVITY 4.5 TWO-DIGIT ADDITION: WORKING FROM THE LEFT (PLACE VALUE BOARD)

OVERVIEW In this strategy the tens digits are added together, then the units digits, and then these two are added: for example 37 + 26 is calculated as (30 + 20) + (7 + 6) = 50 + 13 = 63.

MATERIALS Counters, **BLM 4.11, BLM 4.12**

THE ACTIVITY

1. Each student will need at least 6 counters and **BLM 4.11** (and **BLM 4.12** for preliminary work - see Comments on the Activity below).
2. Place two counters to represent 48 and two more counters to represent 25 on the Place Value Board (**BLM 4.11**).
3. Remind students of Rules 1 to 3 on **BLM 4.12:**
    1. A number is represented by not more than 1 counter in any row.
    2. You can replace any two counters with an equivalent counter (for example you can replace 3 and 4 with 7, or 20 and 40 with 60.
    3. You can replace any two counters with two equivalent counters (for example you can replace 7 and 9 with 10 and 6, or 30 and 40 with 50 and 10).
4. Ask for suggestions as to how the four counters can be combined through addition, following the rules. Discuss suggestions. One possible sequence is:
    2. *Replace 40 and 20 with a counter on 60.*
    3. *Replace 8 and 5 with a counter on 10 and a counter on 3.*
    4. *Replace 60 and 10 with a counter on 70. The total is 75.*
5. Students should now verbalise these actions, in order to make the link with mental computation. *48 + 25: 40 and 20 make 60; 8 and 5 make 13; 60 and 13 make 73.*

COMMENTS ON THE ACTIVITY

6. The class or individual students should work items 1 to 23 of the activities on BLM 4.12 if they have not used the board before. Items 24 to 29 provide a summary of the addition steps for students who are revising this activity.

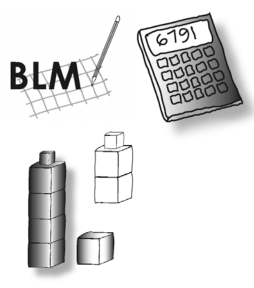
ASSESSING PROGRESS

- Students can add 2-digit numbers using the board, and explain what is happening.
- Students can add two 2-digit numbers mentally.
- Students can apply mentally and explain the Working from the Left strategy.

PRACTICE EXAMPLES

These should be done with or without the board as appropriate.

1.	42 + 35	6.	59 + 36
2.	63 + 16	7.	47 + 44
3.	33 + 37	8.	34 + 58
4.	54 + 18	9.	87 + 46
5.	17 + 26	10.	45 + 45 + 45



ACTIVITY 4.6 COMPLEMENTS TO 100

OVERVIEW This activity provides the basis for more competent students to add for example 89 + 25 by saying '89 + 11 = 100, and 14 more makes 114'.

MATERIALS **BLM 4.2 or 4.5**

THE ACTIVITY

1. Show students a partially numbered 100 board and ask: This board is numbered to 37. *How many more to make 100? (or How many squares are un-numbered?)*

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37			

2. Encourage discussion of strategies. *I saw three blank squares after 37, and then 6 more rows of 10, so 67. OR I counted down the board from 37 in tens then added three more. OR I added 3 to 37 to make 40 and 60 more to make 100.*

3. Give pairs of students a blank or a numbered 100 square **BLM 4.2 or 4.5** and get them to challenge each other with similar problems. With numbered squares, cover up all numbers and rows after a given number (a piece of card can be cut for this purpose). With unnumbered squares, point to a square and ask: *How many squares after this one?*

4. As an extension, get students to write pairs of numbers that total 100 and ask them if they can see any pattern or general rule. For example:

25	38	42	56	77	84
75	62	58	44	23	16.

The tens digits always total 9 and the units digits always total 10.

ASSESSING PROGRESS

The student can give the complement to 100 of any number and give reasons.

ACTIVITY 4.7 TWO-DIGIT SUBTRACTION: BRIDGING MULTIPLES OF TEN

OVERVIEW This strategy is mainly used when subtracting a single-digit from a 2-digit number, for example 43 - 7.

MATERIALS MAB

THE ACTIVITY

1. Check that students can use the Bridging Ten strategy for subtraction with single digit numbers (for example, 13 – 4: 13 – 3 = 10, - 1 = 9). This strategy is introduced in Module 2.
2. Model the strategy using 43 - 7 as an example. Using MAB have the student(s) display 43. *Remove the 3 units. 40 left. How many more to remove? 4. Can you see how many would be left? 36.* (It is preferable for the student to visualise mentally removing 4 from the 4 tens rather than physically make the exchange).
3. Model the language: *43 take 3 leaves 40, take 4 more leaves 36.*
4. Repeat using other 2-digit subtract 1-digit combinations, with the students using MAB or not as they prefer, but always modelling the language. Examples: 31 - 5, 55 - 7, 42 - 3, 63 - 6, 23 - 8.

COMMENTS ON THE ACTIVITY

- Bundles of 10 are not as suitable as MAB for this activity as the visual imagery is not so clear, for example in ‘seeing’ that 40 - 4 = 36

ASSESSING PROGRESS

- Students can subtract a 1-digit from a 2-digit number, using the Bridging Ten strategy.
- Students can explain their strategy.

PRACTICE EXAMPLES

Students should give the answer and then explain the ‘Bridging Multiples of Ten’ strategy for each of the following:

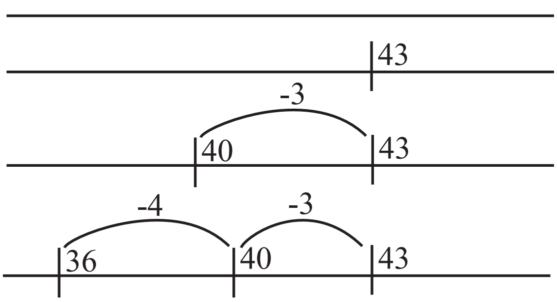
1.	21 - 3	6.	61 - 6
2.	32 - 5	7.	43 - 5
3.	53 - 4	8.	52 - 7
4.	85 - 7	9.	27 - 9
5.	73 - 8	10.	35 - 9

VARIATIONS /EXTENSIONS: (Optional)

- Have the students model their strategy using the blank number line.

For example 43 - 7.

- Blank Number Line:
- Mark 43
- Mark - 3
- Mark - 4.



ACTIVITY 4.8 TWO-DIGIT SUBTRACTION: SUBTRACTING PARTS OF THE SECOND NUMBER (0-99 BOARD)

OVERVIEW In this strategy the second number is subtracted from the first in parts, usually based on place value: for example 63 - 26 is calculated as 63 - 20 - 6. It is essential that Activity 4.2 is covered before this activity.

MATERIALS Counters, **BLM 4.1** (or **BLM 4.2**; see comments on Activity 4.2)

THE ACTIVITY

1. Remind students of the skills learned in Activity 4.2.
2. Calculating 56 - 32. *Place the counter on 56. How can we subtract 32? First subtract 30 (up three places), then subtract 2 (to the left two places). Check that the counter is on 24.*
3. The student now practises other 2-digit subtractions where no ‘exchanges’ are needed: for example 64 - 43, 89 - 27, 78 - 34.
4. Calculating 63 - 29. *Place the counter on 63. Now subtract 20. How can we subtract 9? (there are not enough squares to the left)* Allow the student to explore and suggest a strategy. There are two alternatives: either count back 9, moving up to the previous row; or move up an additional 10 and then move 1 squares to the right. It is important that the student makes and can explain these suggestions and that neither is imposed as THE correct strategy.
5. Using the board the student practises other subtractions: for example 91 - 39, 53 - 27, 87 - 48.
6. The student now applies the strategy mentally without the board and explains the strategy used.

COMMENTS ON THE ACTIVITY

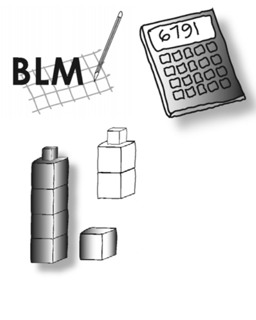
This activity is written using the 0 – 99 board **(BLM 4.1).**

ASSESSING PROGRESS
- Students can subtract 2-digit numbers using the board, and explain what is happening.
- Students can subtract two 2-digit numbers mentally.
- Students can apply mentally and explain the Subtracting Parts of the Second Number strategy.

PRACTICE EXAMPLES

These should be done with or without the board as appropriate.

1.	59 - 25	6.	85 - 73
2.	96 - 24	7.	88 - 32
3.	45 - 18	8.	85 - 57
4.	90 - 35	9.	73 - 26
5.	95 - 46	10.	90 - 54



ACTIVITY 4.9 TWO-DIGIT MULTIPLICATION: RELATING TO A KNOWN FACT

OVERVIEW In this activity students use multiplication facts that they know to help them multiply other numbers. Students can use basic multiplication facts and understanding of place value; for example 3 x 20 = 3 x 2 tens = 6 tens = 60. Or a student may know that 4 x 25 = 100 and use this to calculate 5 x 25.

MATERIALS Calculators can be used for checking.

THE ACTIVITY

1. Write the calculation 9 x 25 on the board. Invite students to calculate this mentally and to explain their strategies. Emphasise the facts they knew that they used to assist them.
2. Write some responses on the board, for example:
   - Known facts: 4 x 25 = 100; 9 = 2 x 4 + 1.
   - Known facts: 9 x 5 = 45, 9 x 20 = 180.
   - Known facts: 10 x 25 = 250, 9 = 10 – 1.
3. Write several calculations on the board, for example: 30 x 6, 5 x 25, 15 x 8, 19 x 6, 50 x 9, 13 x 7.
4. Invite students to calculate one or more of them and to record the known facts that they used. Students can use calculators to help check their calculations.

COMMENTS ON THE ACTIVITY

The main purpose is to show that one uses multiplication facts one knows in order to do calculations, and that these facts, and therefore one's mental strategies, vary from person to person. A secondary purpose is for some students to acquire strategies that they may not have thought of previously.

ASSESSING PROGRESS

- Students can calculate some multiplications by using known facts.
- Students can identify the known facts that they use.
- Students widen the range of strategies open to them when calculating mentally.

PRACTICE EXAMPLE

1.	40 x 5	6.	30 x 7
2.	15 x 6	7.	8 x 19
3.	5 x 36	8.	48 x 5
4.	23 x 8	9.	4 x 35
5.	7 x 50	10.	25 x 7

ACTIVITY 4.10 TWO-DIGIT MULTIPLICATION: USE EXTENSION OF ONE-DIGIT STRATEGIES

OVERVIEW Module 3 introduced strategies for multiplying single-digit by single-digit numbers. These strategies are also equally appropriate for multiplying two-digit by single-digit numbers. Each single digit has a strategy associated with it.

MATERIALS Calculators can be used for checking.

THE ACTIVITY

1. Before extending each of these strategies, check that students can use them with single digit calculations. Then invite students to extend their use to two-digit examples. Calculators can be used to check results. For example:
   - Strategy: 5 x = half of 10 x
   - Single-digit example: 5 x 7 = half of 10 x 7 = half of 70 = 35.
   - Two-digit example: 5 x 46 = half of 10 x 46 = half of 460 = 230.
   - Check result with calculator

This table gives an example of each strategy with a single-digit example, followed by some two-digit calculations which students can be asked to perform using the same strategy.

Strategy	***Single-digit by single-digit***	***Two-digit by single-digit***
2 x = Doubles	2 x 4	2 x 33, 2 x 42, 2 x 57
3 x = 2 x + 1 multiple	3 x 6 = 12 + 6	3 x 45, 3 x 52, 3 x 26
4 x = 2 x	4 x 2 x 6: 12, 24	4 x 15, 4 x 32, 4 x 54
5 x = half of 10 x	5 x 9: 90, 45	5 x 26, 5 x 42, 5 x 63
6 x = 5 x + one multiple	6 x 7: 35 + 7	6 x 70, 6 x 28, 6 x 46
7 x = 5 x + 2 x	7 x 4: 20 + 8	7 x 18, 7 x 28, 7 x 80
8 x = 2 x 2 x 2 x	8 x 7: 14, 28, 56	8 x 35, 8 x 32, 8 x 16
9 x = 10 x – 1 multiple	9 x 6: 60 - 6	9 x 16, 9 x 34, 9 x 47

2. Invite students to write other two-digit calculations with explanations of how these can be done using these strategies.

COMMENTS ON THE ACTIVITY

This activity assumes that students can already use each of the strategies with single digit numbers, and they now need practice in extending their use to multiplying two-digit numbers. For practice with single digits, use Module 3.

ASSESSING PROGRESS

- Students can use each strategy to perform a two-digit calculation.
- Students can explain their use of each strategy.
- Students can calculate two-digit by one-digit calculations using the appropriate strategies

ACTIVITY 4.11 TWO-DIGIT MULTIPLICATION: SKIP COUNTING

OVERVIEW Students have practiced skip counting with single digits in Module 3. This activity extends this to some two-digit numbers, for which skip counting is relatively simple, for example 15, 20, 25, 40.

MATERIALS Calculators can be used for checking.

THE ACTIVITY

1. Establish a sequential order from student to student round the class and back to the first student. Choose a simple two-digit number, such as 10, and have the class count up in tens in the established order until 100 is reached. Continue round, now starting at 20 and counting by 20s until 200 is reached.
2. Continue similarly with 30, 40…
3. Now count up in 15s, 25s.
4. Show students how to use a calculator to check skip counting. For example, to skip count in 15s, enter 0 + 15 =. Now each time the '=' button is pressed the calculator adds 15: 15, 30, 45, 60…
5. Students can now attempt to skip count with a number chosen by themselves or allocated by the teacher, using the calculator to check each multiple.

COMMENTS ON THE ACTIVITY

Students should explore those numbers which they can individually skip count, rather than setting the same goals for all students. However all students should appreciate the connection between, for example, skip counting in 3s and in 30s.

ASSESSING PROGRESS

- Students can skip count in a variety of 2-digit numbers.
- Students can distinguish 2-digit numbers which they can or cannot easily use to skip count.

PRACTICE EXAMPLES

These are suggestions only. Numbers should be chosen to suit the individual students.
Skip count a sequence of at least 5, and preferably 10 numbers.

1.	10	6.	12
2.	50	7.	20
3.	60	8.	30
4.	15	9.	25
5.	40	10.	21

ACTIVITY 4.12 TWO-DIGIT MULTIPLICATION: USE THE DISTRIBUTIVE PROPERTY

OVERVIEW Use of the distributive law forms the basis of the standard written algorithm for 2-digit multiplication: for example 4 x 27 = (4 x 7) + (4 x 20) = 28 + 80 = 108.

MATERIALS Rectangular arrays (**BLMs 4.9** and **4.10**).

THE ACTIVITY

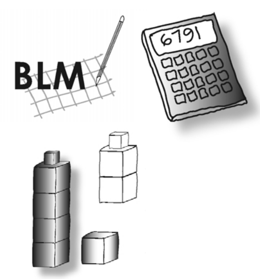
1. Give each student a copy of **BLM 4.9** and have them draw round a rectangle 4 (rows) by 13. Establish that the number of spots in this rectangle can be ascertained by calculating 4 x 13.
2. Have each student draw a vertical line to subdivide the rectangle into two rectangles : (4 x 10) and (4 x 3). Record the number of spots in each of these rectangles, and add them: 40 + 12 = 52.
3. If appropriate, relate this to the number of number cards and court cards in a pack of cards: 4 suits, each having 10 numbered cards (A – 10) and 3 court cards (J, Q, K).
4. Have individual students place a vertical line between other columns, so as to subdivide the rectangle into other pairs of rectangles, for example (4 x 8) and (4 x 5), or (4 x 7) and (4 x 6). Check that in each case the sum of columns (8 + 5, 7 + 6) is 13 and that the total number of spots in the two rectangles is 52.
5. Discuss why the split into (10 + 3) is more convenient than other splits (because 10 x 4 is easy to calculate).
6. Challenge students to use this method, but without drawing the rectangles), to calculate the number of spots in other sizes of rectangles, for example 4 x 16, 5 x 14, 7 x 18…
7. Extend the method to numbers above 20, for example 2 x 26, 5 x 33, 6 x 25.

COMMENTS ON THE ACTIVITY

The rectangular array is used in order to give a visual model which justifies the mental process. While some students may not need this visual image and can spontaneously understand the mental method, it is essential that students can justify their method (multiply the tens, multiply the units, then add) using visual or symbolic connections. It should also be clear that it does not matter whether the student prefers to multiply the tens or the units first.

ASSESSING PROGRESS

- Students can use the distributive property (but not necessarily use that name) to multiply a 2-digit by a single digit number.
- Students can explain the method in terms of the rectangular array.
- Students can explain why the tens or the units digit can be multiplied first.



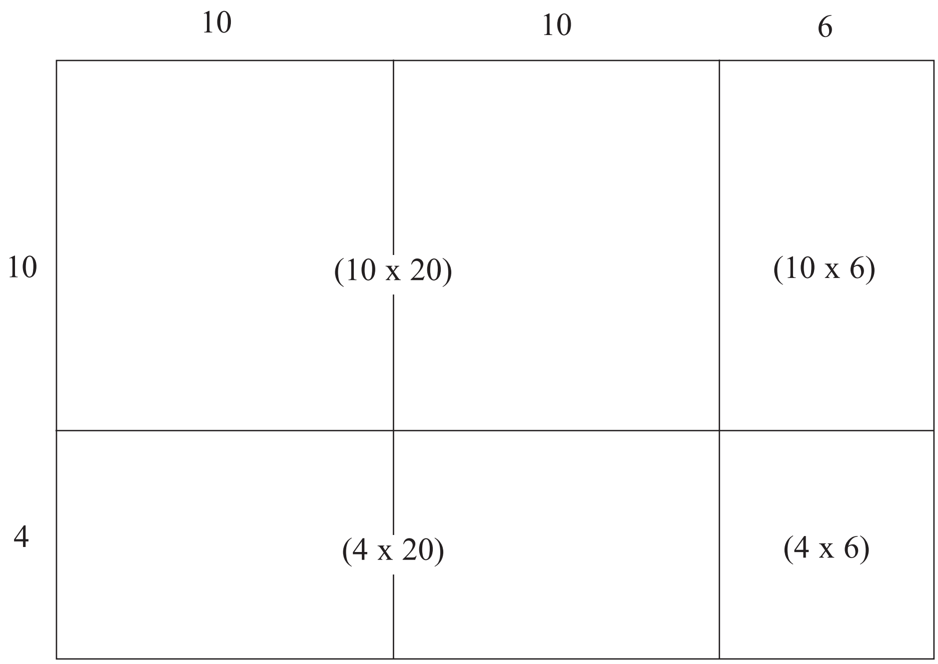
PRACTICE EXAMPLES

Students can be asked to draw a rectangular array to illustrate one of these examples.

1.	2 x 18	6.	2 x 19
2.	3 x 24	7.	3 x 35
3.	4 x 17	8.	5 x 47
4.	6 x 35	9.	9 x 26
5.	8 x 44	10.	7 x 54

EXTENSION

Some students can be challenged to extend the method to multiplying a 2-digit by a 2-digit number, by extending the idea of the rectangular array, for example 26 x 14



Note that this explains the need for four multiplications, and can be used to explain the standard written algorithm: however few students will choose to perform a 2-digit by 2-digit calculation mentally.

ACTIVITY 4.13 TWO-DIGIT DIVISION: MAKE IT MULTIPLICATION

OVERVIEW Most students are more comfortable with multiplication than with division, and will therefore often solve a division by turning it into a multiplication, for example: *120 divided by 4: 4 times what equals 120*. This can then be solved by trial and error or as in this case, by relating it to the known fact 4 x 3 = 12.

MATERIALS Rectangular arrays (**BLM 4.9**) may be needed for explanations. Calculators can be used for checking.

THE ACTIVITY

- For students who are unfamiliar or not yet secure with the relationship between multiplication and division in relation to basic facts, establish the sets of four related facts for some basic multiplication facts, for example 4 x 6 = 24: 4 x 6 = 24, 6 x 4 = 24, 24 div 4 = 6, 24 div 6 = 4. If necessary, justify these relationships by reference to a 4 x 6 rectangular array.

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- Give students some 2-digit by single digit divisions and ask them (a) to restate the calculation as a multiplication, and (b) give an answer and describe their solution strategy. For example, 68 ÷ 4: *4* x *what equals 68? I know 4* x *10 = 40, that leaves 28. 4* x *7 = 28. so 4* x *17 = 68* and 68 ÷ 4 = 17.

COMMENTS ON THE ACTIVITY

In the activity, some students will restate the division as a multiplication correctly, but not use this to solve the problem. For example for 68 ÷ 4, a student may correctly restate this as 4 x what = 68?, but may solve this by halving twice: half 68 = 34, half 34 = 17. In this case acknowledge a correct solution strategy, and then ask the student to solve the problem a different way, involving 4 x what = 68.

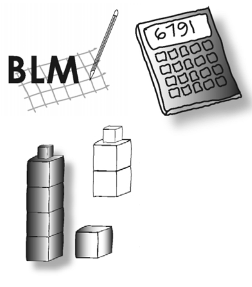
ASSESSING PROGRESS

- Students can divide 2-digit (and some 3-digit) numbers by a single digit number by rewording the problem in terms of multiplication.
- Students can justify the method by reference to visual or other representations.
- Students can explain why this method is or is not suitable for specific mental calculations.

PRACTICE EXAMPLES

These should be done with or without the board as appropriate.

1.	32 ÷ 2	6.	56 ÷ 2
2.	42 ÷ 3	7.	78 ÷ 3
3.	91 ÷ 7	8.	65 ÷ 5
4.	136 ÷ 4	9.	84 ÷ 6
5.	108 ÷ 9	10.	96 ÷ 6



ACTIVITY 4.14 TWO-DIGIT DIVISION: USE THE DISTRIBUTIVE PROPERTY

OVERVIEW One can use the distributive property to divide, by splitting the dividend into two convenient parts, dividing each by the divisor, and then adding the quotients, for example: 78 ÷ 6: 78 = 60 + 18; (60 ÷ 6) = 10; (18 ÷ 6) = 3; 10 + 3 = 13.

MATERIALS Calculators can be used for checking.

THE ACTIVITY

1. Show this diagram relating to the calculation 32 div 2.

   20 + 12

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   ☆ ☆ ☆ ☆ ☆ ☆ ☆ ☆ ☆ ☆   ☆ ☆ ☆ ☆ ☆ ☆

   (20 ÷ 2) = 10   (12 ÷ 2) = 6

2. Relate this to the method for dividing 32 by 2. Split 32 into 20 + 12 , divide each by 2, and add.
3. Give another example: 64 ÷ 4: 64 = 40 + 24 [*Ask: Why choose 40 + 24 rather than 30 + 34?]*. So 64 ÷ 4 = (10 + 6). Ask for explanations
4. Give further examples for students to calculate and explain the method. If students use other methods, acknowledge these but ask for them to also use this method.

COMMENTS ON THE ACTIVITY

A brief justification of the method is given. Students who do not understand should refer to Activity 4.12.

ASSESSING PROGRESS

- Students can use the distributive property to divide 2-digit (and some 3-digit) numbers by a single digit.
- Students can explain or justify the method.
- Students can explain why this method is or is not suitable for specific mental calculations.

PRACTICE EXAMPLES

These should be done with or without the board as appropriate.

1.	36 ÷ 2	6.	65 ÷ 5
2.	51 ÷ 3	7.	87 ÷ 3
3.	84 ÷ 7	8.	68 ÷ 4
4.	144 ÷ 6	9.	132 ÷ 6
5.	198 ÷ 9	10.	184 ÷ 8

BLM 4.1

0 – 99 SQUARE

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

BLM 4.2

1 – 100 SQUARE

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

BLM 4.3

0 – 99 SQUARES (SMALL)

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

BLM 4.4

1 – 100 SQUARES (SMALL)

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

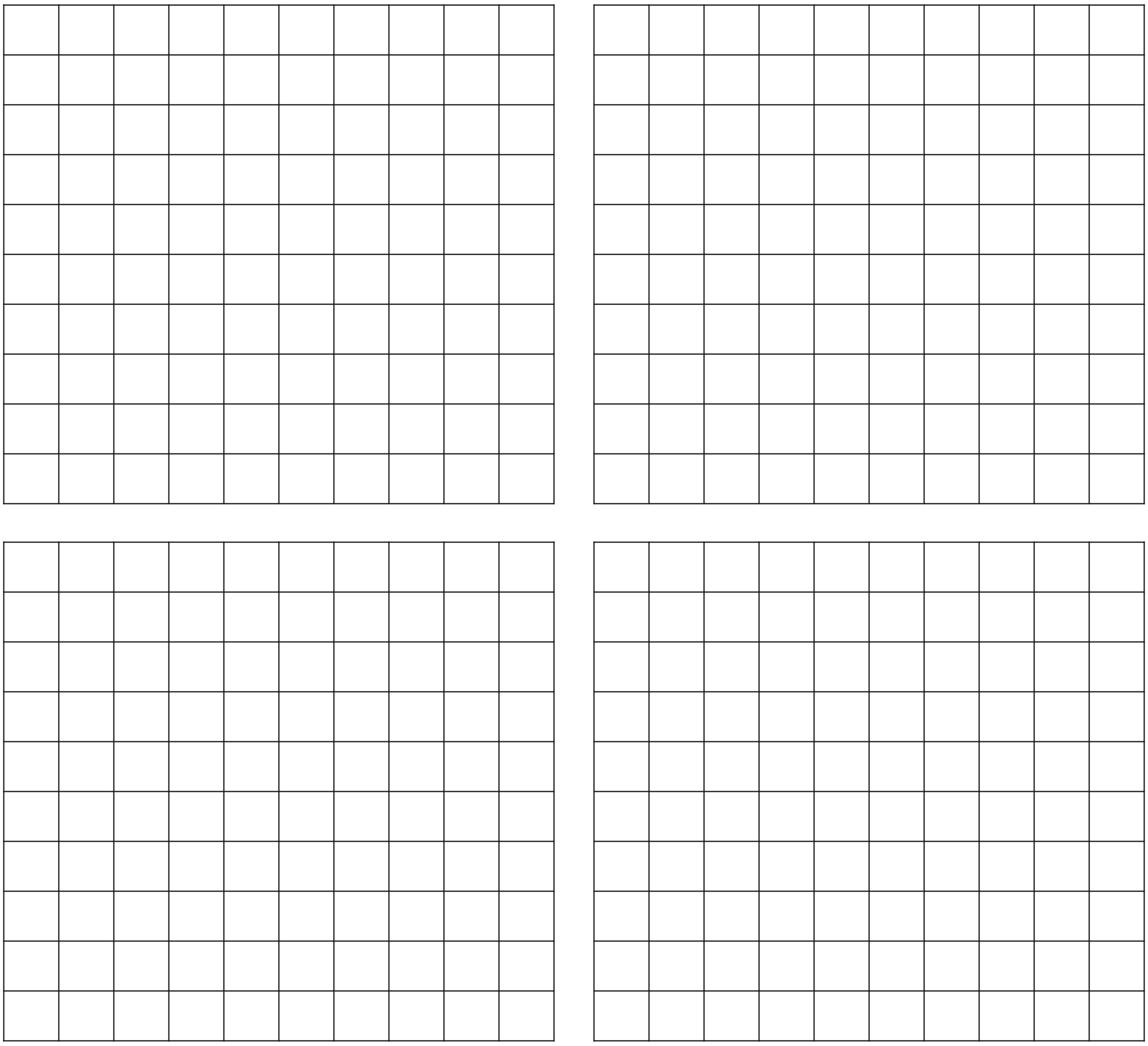
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

BLM 4.5

BLANK 100 SQUARE

BLM 4.6

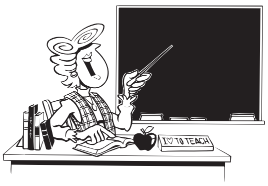
BLANK 100 SQUARES (SMALL)



BLM 4.7

HOW DID YOU DO IT?

NAME ....



BLM 4.8

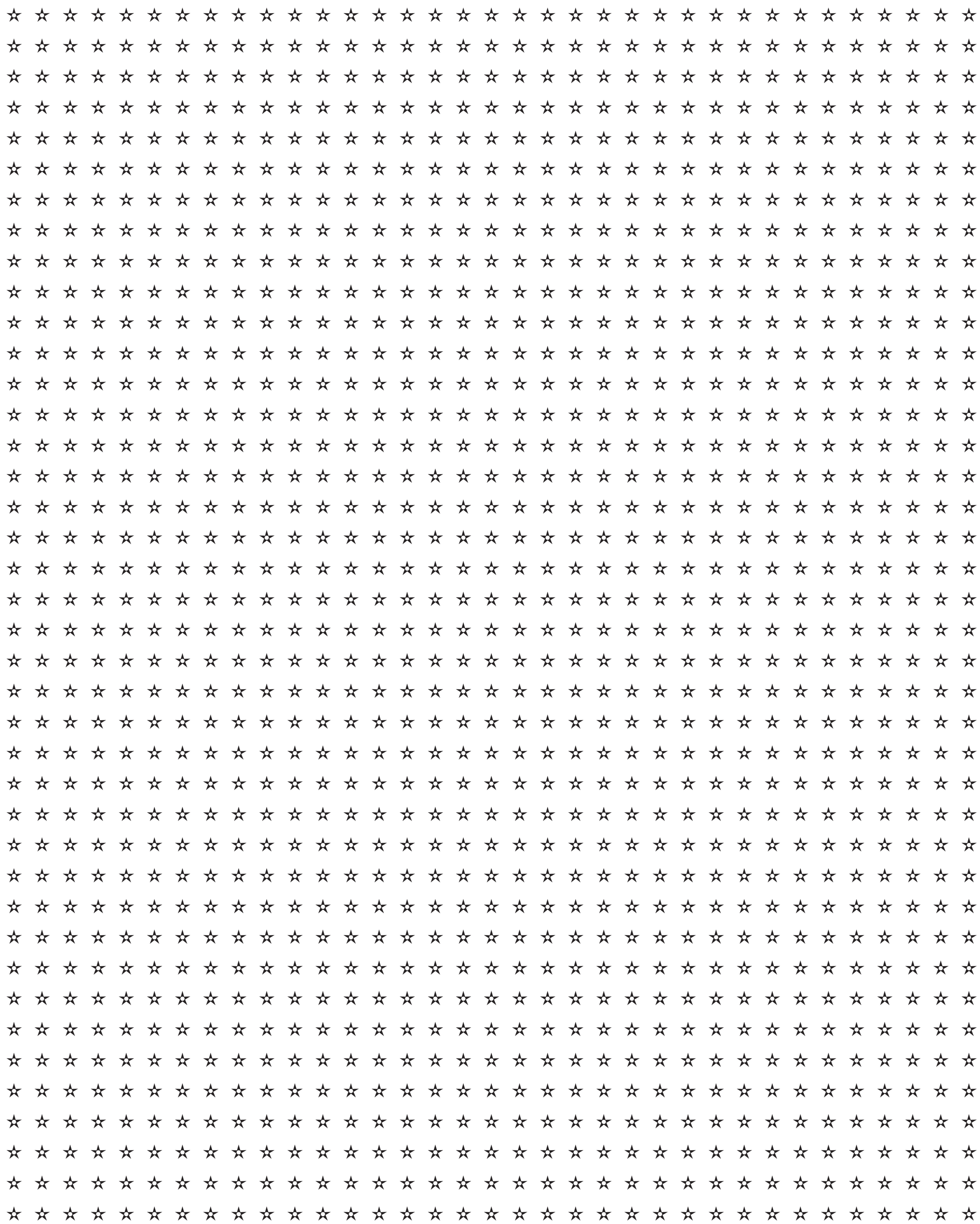
ONE ANSWER, MANY CALCULATIONS

NAME ....

NUMBER OF THE DAY $\boxed{}$

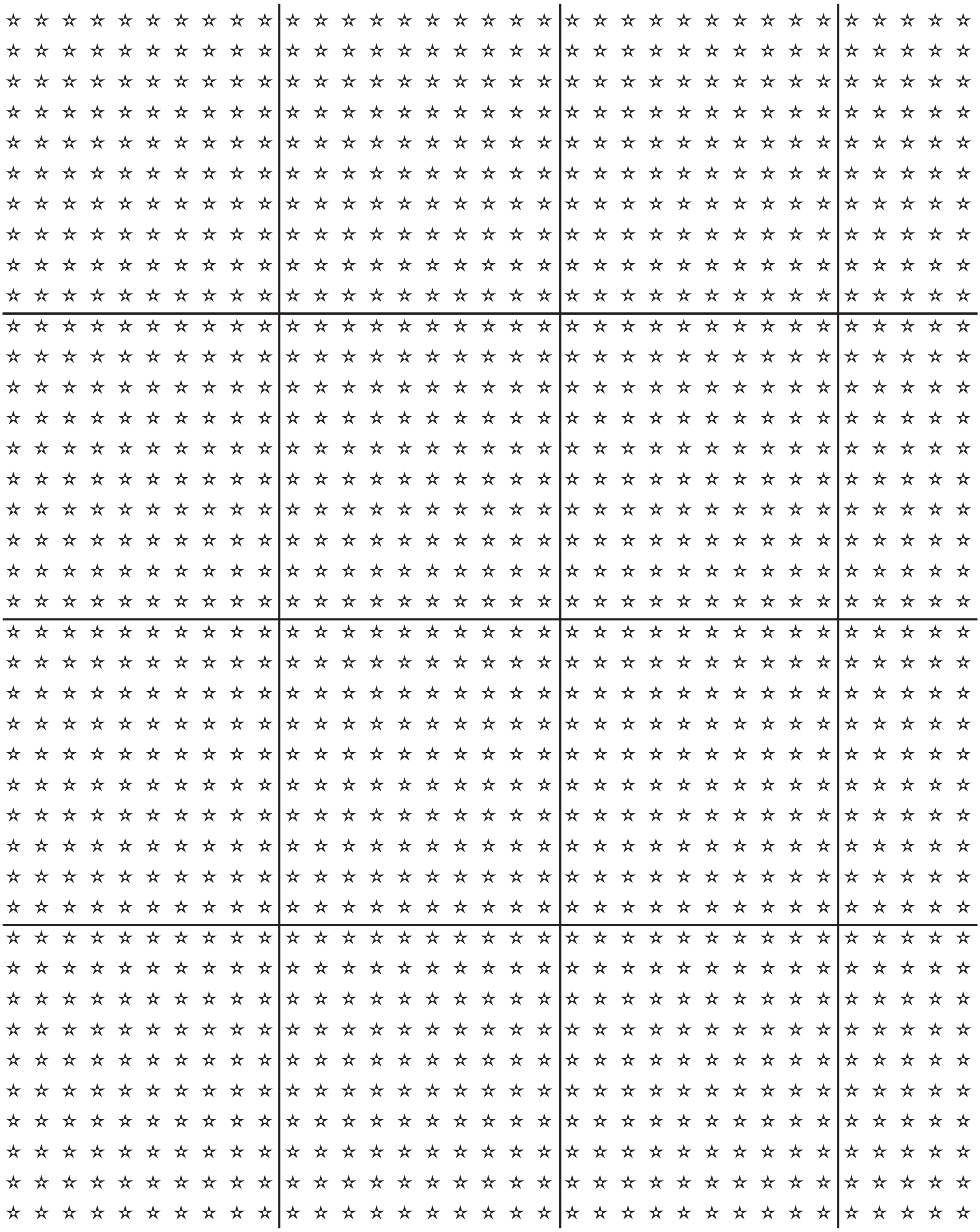
BLM 4.9

RECTANGULAR ARRAY



BLM 4.10

RECTANGULAR ARRAY (10 x 10 GRIDS)



BLM 4.11

PLACE VALUE NUMBER BOARD

	0		0		0
	1		10		100
	2		20		200
	3		30		300
	4		40		400
	5		50		500
	6		60		600
	7		70		700
	8		80		800
	9		90		900
	10		100		1000

BLM 4.12

PLACE VALUE NUMBER BOARD ACTIVITIES

MAKING ONE-DIGIT NUMBERS

1 Make 3. Make 5. Make 8. Make 0
2 Make 3 with 2 counters. Make 5 with 2 counters. Make 8 with 2 counters.
3 Make 9 with 2 counters in different ways.
4 In how many different ways can you make 4 with 2 counters?

MAKING TWO-DIGIT NUMBERS

5 Make 30. Make 50. Make 80.
6 Make 30 with 2 counters. Make 50 with 2 counters. Make 80 with 2 counters.
7 Make 90 with 2 counters in different ways.
8 In how many different ways can you make 60 with 2 counters?

MAKING THREE-DIGIT NUMBERS

9 Make 300. Make 500. Make 800.
10 Make 300 with 2 counters. Make 500 with 2 counters. Make 800 with 2 counters.
11 Make 900 with 2 counters in different ways.
12 In how many different ways can you make 700 with 2 counters?

MAKING ANY NUMBERS

13 Make 23 with 2 counters. Make 47 with 2 counters. Make 50 with 1 counter…with 2 counters
14 Make 35 with 2 counters…with 3 counters.
15 How many ways can you make 42 with 3 counters?
16 What is the least number of counters to represent 3…7… any 1-digit number?
17 What is the least number of counters to represent 25…83… any 2-digit number?
18 What is the least number of counters to represent 247…906… any 3-digit number?

CHANGING NUMBERS

19 Make 14 with 2 counters. What would you move to make it 15?…18?…12?
20 Make 47 with 2 counters. What would you move to make it 67?…97?… 17?
21 Make 38 with 2 counters. Add 1. Subtract 7. Add 6.
22 Make 38 with 2 counters. Add 20. Add 40. Subtract 30.
23 Make 32 with 2 counters. Add 11. Add 35.

ADDITION

Rules.

1. A number is represented by not more than 1 counter in any row.
2. You can replace any two counters with an equivalent counter (for example you can replace 3 and 4 with 7, or 20 and 40 with 60.
3. You can replace any two counters with two equivalent counters (for example you can replace 7 and 9 with 10 and 6, or 30 and 40 with 50 and 10).

24 Place one counter to make 8 and one counter to make 6. ‘Add’ them: that is, use rules 2 and 3 so that the final result satisfies rule 1. Explain what you did.
25 Place two counters to make 32 and two more counters to make 25. ‘Add’ them: that is, use rules 2 and 3 so that the final result satisfies rule 1. Explain what you did.
26 Place two counters to make 49 and two more counters to make 36. ‘Add’ them: that is, use rules 2 and 3 so that the final result satisfies rule 1. Explain what you did.
27 Place two counters to make 87 and two more counters to make 54. ‘Add’ them: that is, use rules 2 and 3 so that the final result satisfies rule 1. Explain what you did.
28 Add 267 and 428. Add 173 and 457. Add 876 and 654. Add 348 and 652.
29 Use the board to help you find pairs of numbers with a sum of 100…a sum of 1000

BLM 4.13 - 4.16

RULES FOR MAKE 50 AND MAKE 100

MAKE 50 AND MAKE 100

These activities are to practise finding pairs of numbers whose sum is 50 or 100.

Four boards (A, B, C and D) are provided. Students take a board and cover with counters any pair of numbers whose sum is 50 (or 100). Further pairs totalling 50 (or 100) are then covered until only one number is left uncovered. If this has been done correctly, the numbers left uncovered are:

BLM 4.13 and 4.14

Board A:19 Board B:34 Board C:41 Board D: 26

BLM 4.15 and 4.16

Board A: 27 Board B: 46 Board C: 67 Board D: 38

BLM 4.13

MAKE 50 (A)

4	13	44	15	21
32	36	25	7	41
43	19	33	23	18
29	27	37	25	14
9	35	17	46	6

MAKE 50 (B)

25	48	5	28	3
12	24	33	34	40
45	8	10	38	17
39	47	16	2	42
34	26	25	22	11

BLM 4.14

MAKE 50 (C)

26	39	17	28	41
25	44	34	1	36
16	20	11	41	19
30	31	22	33	6
14	49	9	25	24

MAKE 50 (D)

5	16	35	7	10
38	14	28	25	0
21	20	22	45	30
25	43	26	29	12
15	50	40	34	36

BLM 4.15

MAKE 100 (A)

53	82	46	29	87
16	35	79	30	68
24	27	13	42	54
70	58	32	76	18
21	71	47	84	65

MAKE 100 (B)

11	26	86	94	59
35	18	51	38	49
71	23	89	46	77
43	62	41	29	82
74	6	65	14	57

BLM 4.16

MAKE 100 (C)

22	61	13	66	34
44	90	37	71	58
84	25	48	78	10
42	56	52	16	63
39	67	87	29	75

MAKE 100 (D)

41	83	59	36	8
22	28	53	61	32
80	67	38	20	72
47	92	78	59	41
64	39	17	33	68