Mental Computation: A strategies approach

Module 2 Basic facts addition and subtraction

Shelley Dole
Alistair McIntosh

This is one of a set of 6 modules providing a structured strategies approach to mental computation.

Module 1 Introduction
Module 2 Basic facts addition and subtraction
Module 3 Basic facts multiplication and division
Module 4 Two-digit whole numbers
Module 5 Fractions and decimals
Module 6 Ratio and percent


University of Tasmania
Department of Education Tasmania
Catholic Education Office Hobart
Department of Education and Training, Australian Capital Territory
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OVERVIEW OF MODULE 2

INTRODUCTION
This module is the second in a series of six that comprise a resource of activities for developing students' mental computation. The focus of this module is on the basic facts of addition and subtraction.

BASIC FACTS OF ADDITION AND SUBTRACTION
The basic facts of addition are combinations of pairs of numbers 1 to 10. The basic facts of subtraction are the addition facts in reverse. There are 121 basic facts of addition that include facts involving zero, and 121 basic facts of subtraction. Automatic recall of the basic facts of addition and subtraction is the platform for successful mental computation of numbers of two or more digits. Basic fact recall is promoted through the use of thinking strategies and these strategies also assist in mental computation involving larger numbers.

Successful and efficient mental computation is greatly enhanced when basic facts can be retrieved relatively automatically. Promoting basic fact retrieval can be assisted through providing students with opportunities to visualise basic facts and to use thinking strategies to see patterns and connections between facts. Although there are 242 basic addition and subtraction facts to be mastered, the basic facts can be categorised by particular thinking strategies. Teaching thinking strategies for learning the basic facts of addition and subtraction reduces the number of facts to be learned, and the effort associated with memorising individual facts.

TEACHING SEQUENCE
The activities and ideas presented in this module describe specific strategies for thinking, to assist firstly, basic fact understanding, and then, through practice, basic fact retrieval. In some cases, more than one thinking strategy can be used to recall particular facts. When planning for instruction for basic fact development, at all times continually encourage students to develop and explain flexible use of strategies, so that they do not feel constrained to associate any particular calculation or fact with one ‘correct’ strategy. For example, the fact 9 + 8 is included in the ‘Bridging 10’ strategy: 9 + 1 = 10, + 7 = 17. But it is equally valid for a student to calculate this as a ‘Near Double’: 8 + 8 = 16, + 1 = 17. This section has been structured to ensure that the student is introduced to each strategy, as well as meeting at least one way of calculating each basic fact.

COMMUTATIVITY
When students understand commutativity, the number of addition facts to be learned is halved. Commutativity relates to addition and multiplication, and simply means that order does not matter. Commutativity means that 2 + 7 and 7 + 2 are the same. Being able to see this means students can see one fact and its ‘spin-around’.

INVERSE (‘THINK ADDITION’)
When students understand inverse, learning subtraction facts can be
relatively simple. Each fact can be considered as a set of four facts: two addition and two subtraction facts, generated through rearrangement of the three numbers in the fact. For example, the numbers 4, 6 and 10 link as follows: 6 + 4 = 10, 10 - 6 = 4, 4 + 6 = 10, 10 - 4 = 6. Using inverse for subtraction is the way to teach students to think addition. The think addition strategy is best applied once the addition facts have been fully consolidated; that is, when students can rapidly determine the solution to the addition fact. The think addition strategy requires familiarity and comfortableness with addition facts. Once students know that 6 + 4 = 10, (and therefore 4 + 6 = 10) when confronted with 10 - 6, they think of the number that adds to 6 to give 10.

THE SEQUENCES
In this module, the sequences for the Addition and Subtraction Basic Facts are separated and are shown side by side in the flow chart on page 6. You may decide to follow through the whole addition sequence first before starting the related subtraction sequence. On the other hand there may be benefits in following up each addition stage with its related subtraction set. Research is yet to indicate which approach is more effective.

EXTENSIONS
The Extensions at the end of each activity usually expand the principle of the strategy to larger numbers. Most of these extensions will be developed as activities in later modules. They are not included as teaching material at this stage but rather as an indication that if some students articulate such extensions, they should be encouraged to pursue the connections.
In this module, the addition facts have been categorised according to a particular thinking strategy that can be applied to attain the fact. These thinking strategies are also useful for mental computation of larger numbers (two-digits or more). A description of each of these thinking strategies and an example are briefly described below.

**COMMUTATIVITY (ADDITION)**
Reversing the order of the numbers so that the larger comes first. For example: 2 + 9; it is easier to start with 9 and add 2.
• For developing and encouraging this strategy, use activity 2.2.

**COUNTING ON AND BACK**
Efficient for adding 1, 2, 3, or 0 but increasingly untrustworthy for larger numbers. As this is the most common method clung to by weaker students, it is important to develop confidence with other strategies and discourage use of ‘Counting-On/Back’ for larger numbers.
• For developing this strategy, use activities 2.4 and 2.12.

**DOUBLES/ NEAR DOUBLES**
Most students acquire knowledge of ‘Doubles’ (3 + 3, 7 + 7) earlier than other facts and use these for related calculations. For example: 5 + 6; I know 5 + 5, so 5 + 6 is 1 more. 14 - 6; I know 7 + 7 = 14, so 14 - 7 = 7, so 14 - 6 = 8.
• For developing and encouraging this strategy, use activities 2.6, 2.8, 2.10.

**BRIDGING 10**
Knowledge of pairs of numbers whose sum is 10 is very valuable and students use this to derive other facts. For example, in addition: 8 + 5; 8 + 2 = 10, + 3 = 13. To use this strategy students need to have immediate recall of ‘10 Facts’ (in the example above, 8 + 2 = 10) and other basic facts (in the example above, 5 = 2 + 3). An example of ‘Bridging 10’ in subtraction: 14 – 5: 14 - 4 = 10, - 1 = 9.
• For developing and encouraging this strategy, use activities 2.5 and 2.11.

**CONVERTING SUBTRACTION TO ADDITION – INVERSE (‘THINK ADDITION’)**
Students are usually more confident with addition than with subtraction. Two examples of this strategy have been given above in combination with other strategies. For instance: 16 – 8; I know 8 + 8 = 16, so 16 - 8 = 8 or 9 - 7; I know 7 + 2 = 9, so 9 - 7 = 2 (much safer than counting back 7 from 9).
• For developing and encouraging this strategy, use activity 2.9 and also incorporate it in activities 2.3 to 2.7.

**USING PLACE VALUE TO ADD OR SUBTRACT 10**
• For developing and encouraging this strategy, use activities 2.7 and 2.13.
Assess where the class or individual students should enter the sequence

*Use Activity 2.1 (p10) and oral or written Tests (e.g. BLM 2.1 and 2.2)*

**FLOW CHART OF THE TEACHING SEQUENCE**

**ADDITION SEQUENCE**

**GENERAL STRATEGIES**

*Use Activities 2.2(p11) and 2.3 (p13)*

**COUNT ON 1, 2, 3, 0**

*Activity 2.4 (p15)*

**TENS FACTS**

*Activity 2.5 (p17) and BLM 2.3/4*

**DOUBLES**

*Activity 2.6 (p19)*

**ADD 10**

*Activity 2.7 (p21)*

**BRIDGE 10**

*Activity 2.5 (p17)*

**NEAR DOUBLES**

*Activity 2.8 (p23)*

**SUBTRACTION SEQUENCE**

**GENERAL STRATEGY**

*Use Activity 2.9(p25)*

**COUNT BACK 1, 2, 3, 0**

*Activity 2.12 (p31)*

**TENS FACTS**

*Activity 2.11 (p29)*

**DOUBLES**

*Activity 2.10 (p27)*

**SUBTRACT 10**

*Activity 2.13 (p33)*

**BRIDGE 10**

*Activity 2.11 (p29)*

**NEAR DOUBLES**

*Activity 2.10 (p27)*
THE TEACHING SEQUENCE FOR THE BASIC ADDITION FACT STRATEGIES

The facts categorised according to thinking strategy are listed below. The fact categories are not necessarily hierarchical, but particular groups of facts must be in place before some other fact categories are learned. The sequence is a suggestion only. Introduction of groups of facts according to strategy will depend on the needs of the students and the teacher’s own professional judgement. Some facts fall into two or more thinking strategies. The teacher’s role is to encourage students to express their opinion of the new thinking strategy for previously learned facts.

SET       COUNT ON FACTS

A  Count On 1 (+1)                      2 + 1 3 + 1 4 + 1 5 + 1 6 + 1 7 + 1 8 + 1
     Spin-around (1+)                      1 + 2 1 + 3 1 + 4 1 + 5 1 + 6 1 + 7 1 + 8
  Count On 2 (+2)                      3 + 2 4 + 2 5 + 2 6 + 2 7 + 2
     Spin-around (2+)                      2 + 3 2 + 4 2 + 5 2 + 6 2 + 7
  Count On 3 (+3)                      4 + 3 5 + 3 6 + 3
     Spin-around (3+)                      3 + 4 3 + 5 3 + 6
  Count On 0 (+0)                      1 + 0 2 + 0 3 + 0 4 + 0 5 + 0 6 + 0 7 + 0 8 + 0 9 + 0
     Spin-around (0+)                      0 + 1 0 + 2 0 + 3 0 + 4 0 + 5 0 + 6 0 + 7 0 + 8 0 + 9

B  TENS FACTS
     Tens Facts                             10 + 0 9 + 1 8 + 2 7 + 3 6 + 4 5 + 5
     Spin-around                             0 + 10 1 + 9 2 + 8 3 + 7 4 + 6 5 + 5

C  DOUBLES
     Doubles                               0 + 0 1 + 1 2 + 2 3 + 3 4 + 4 5 + 5
     6 + 6 7 + 7 8 + 8 9 + 9 10 + 10

D  ADD 10
     Add 10 (10+)                           10 + 1 10 + 2 10 + 3 10 + 4 10 + 5 10 + 6 10 + 7 10 + 8 10 + 9
     Spin-around (+10)                      1 + 10 2 + 10 3 + 10 4 + 10 5 + 10 6 + 10 7 + 10 8 + 10 9 + 10

E  BRIDGE 10
     Bridge 10 (9+)                          9 + 2 9 + 3 9 + 4 9 + 5 9 + 6 9 + 7 9 + 8
     Spin-around (+9)                        2 + 9 3 + 9 4 + 9 5 + 9 6 + 9 7 + 9 8 + 9
     Bridge 10 (8+/7+)                       8 + 3 8 + 4 8 + 5 8 + 6 8 + 7 7 + 4 7 + 5
     Spin-around (+8+/7+)                    3 + 8 4 + 8 5 + 8 6 + 8 7 + 8 4 + 7 5 + 7

F  NEAR DOUBLES
     Doubles + 1/2                           4 + 5 5 + 6 6 + 7 5 + 7
     Spin-around                            5 + 4 6 + 5 7 + 6 7 + 5
### THE TEACHING SEQUENCE FOR THE RELATED SUBTRACTION FACT STRATEGIES

The subtraction facts categorised according to their related addition fact are displayed below. This display shows how subtraction facts are developed if the teaching approach is to link to addition facts (i.e. promote understanding of inverse and reinforce the ‘think addition’ strategy). In many cases, subtraction facts can be promoted by using a counting-back strategy, but students need to be reminded that this strategy is only efficient for counting back 3 or less.

<table>
<thead>
<tr>
<th>SET</th>
<th>COUNT BACK FACTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Count Back 1 (-1)</td>
</tr>
<tr>
<td></td>
<td>3 - 1 4 - 1 5 - 1 6 - 1 7 - 1 8 - 1 9 - 1</td>
</tr>
<tr>
<td></td>
<td>Spin-around (=1)</td>
</tr>
<tr>
<td></td>
<td>3 - 2 4 - 3 5 - 4 6 - 5 7 - 6 8 - 7 9 - 8</td>
</tr>
<tr>
<td></td>
<td>Count Back 2 (-2)</td>
</tr>
<tr>
<td></td>
<td>5 - 2 6 - 2 7 - 2 8 - 2 9 - 2</td>
</tr>
<tr>
<td></td>
<td>Spin-around (=2)</td>
</tr>
<tr>
<td></td>
<td>5 - 3 6 - 4 7 - 5 8 - 6 9 - 7</td>
</tr>
<tr>
<td></td>
<td>Count Back 3 (-3)</td>
</tr>
<tr>
<td></td>
<td>7 - 3 8 - 3 9 - 3</td>
</tr>
<tr>
<td></td>
<td>Spin-around (=3)</td>
</tr>
<tr>
<td></td>
<td>7 - 4 8 - 5 9 - 6</td>
</tr>
<tr>
<td></td>
<td>Count Back 0 (-0)</td>
</tr>
<tr>
<td></td>
<td>1 - 0 2 - 0 3 - 0 4 - 0 5 - 0 6 - 0 7 - 0 8 - 0 9 - 0</td>
</tr>
<tr>
<td></td>
<td>Spin-around (=0)</td>
</tr>
<tr>
<td></td>
<td>1 - 1 2 - 2 3 - 3 4 - 4 5 - 5 6 - 6 7 - 7 8 - 8 9 - 9</td>
</tr>
<tr>
<td>B</td>
<td>TENS FACTS</td>
</tr>
<tr>
<td></td>
<td>Tens Facts</td>
</tr>
<tr>
<td></td>
<td>10 - 0 10 - 1 10 - 2 10 - 3 10 - 4 10 - 5 10 - 6 10 - 7 10 - 8 10 - 9 10 - 10</td>
</tr>
<tr>
<td>C</td>
<td>DOUBLES</td>
</tr>
<tr>
<td></td>
<td>0 - 0 2 - 1 4 - 2 6 - 3 8 - 4 10 - 5 12 - 6 14 - 7 16 - 8 18 - 9 20 - 10</td>
</tr>
<tr>
<td>D</td>
<td>SUBTRACT 10</td>
</tr>
<tr>
<td></td>
<td>Subtract to 10 (=10)</td>
</tr>
<tr>
<td></td>
<td>11 - 1 12 - 2 13 - 3 14 - 4 15 - 5 16 - 6 17 - 7 18 - 8 19 - 9 11 - 10</td>
</tr>
<tr>
<td></td>
<td>Spin-around (-10)</td>
</tr>
<tr>
<td></td>
<td>11 - 10 12 - 10 13 - 10 14 - 10 15 - 10 16 - 10 17 - 10 18 - 10 19 - 10 10 - 10</td>
</tr>
<tr>
<td>E</td>
<td>BRIDGE 10</td>
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<tr>
<td></td>
<td>Bridge 10 (=9)</td>
</tr>
<tr>
<td></td>
<td>11 - 2 12 - 3 13 - 4 14 - 5 15 - 6 16 - 7 17 - 8 11 - 9 12 - 9 13 - 9 14 - 9 15 - 9 16 - 9 17 - 9</td>
</tr>
<tr>
<td></td>
<td>Spin-around (=9)</td>
</tr>
<tr>
<td></td>
<td>11 - 9 12 - 9 13 - 9 14 - 9 15 - 9 16 - 9 17 - 9</td>
</tr>
<tr>
<td></td>
<td>Bridge 10 (=8=7)</td>
</tr>
<tr>
<td></td>
<td>11 - 3 12 - 4 13 - 5 14 - 6 15 - 7 11 - 4 12 - 5</td>
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<tr>
<td></td>
<td>Spin-around (=8=7)</td>
</tr>
<tr>
<td></td>
<td>11 - 8 12 - 8 13 - 8 14 - 8 15 - 8 11 - 7 12 - 7</td>
</tr>
<tr>
<td>F</td>
<td>NEAR DOUBLES</td>
</tr>
<tr>
<td></td>
<td>Doubles + 1</td>
</tr>
<tr>
<td></td>
<td>9 - 5 11 - 6 13 - 7</td>
</tr>
<tr>
<td></td>
<td>Spin-around -1</td>
</tr>
<tr>
<td></td>
<td>9 - 4 11 - 5 13 - 6</td>
</tr>
</tbody>
</table>
**ACTIVITY 2.1 ASSESSING BASIC FACT KNOWLEDGE**

**AIM**
To assess individual student’s knowledge of basic facts and to determine which strategies to focus on with the class and with individuals.

**SUGGESTED MATERIALS**
Counters, linking cubes, dominoes, calculators, base 10 blocks, BLM 2.1, BLM 2.2.

**ACTIVITIES**
- Use class or individual assessments to find out those addition and subtraction basic facts for which students do not have instant recall (cannot answer within 3 seconds maximum).
- Look for patterns. For example, a student may make errors when the total is more than 10, or when adding zero, or where the first number is the smaller.
- Ask individual students to explain their strategies for calculating some answers. Include items to which they gave correct as well as incorrect answers. Note the following:
  - Did they almost always count forward or back in ones?
  - Did they rely on use of fingers?
  - What other strategies did they use? (For descriptions see page 5).
- Select activities appropriate to individual students’ strengths/weaknesses. Particular errors that are common to several students can be adapted as class activities.
- Encourage students to keep a list of ‘difficult’ facts and to practice them.

**COMMENTS ON THE ACTIVITY**
This process is an on-going one. The initial tasks are (a) to identify students whose only strategy is to count on and back in ones and to begin to expose them to alternative and more reliable strategies, and (b) to select specific strategies to concentrate on with groups of students.
ACTIVITY 2.2 SPIN-AROUNDS

AIM
To help students develop understanding of commutativity as a mental strategy for deriving basic facts that they can not yet retrieve automatically.

SUGGESTED MATERIALS
Counters, dominoes, paper, calculator.

TEACHING POINTS
• The activities are intended as suggestions of the kind of activities (both organized and incidental) that can help students see commutativity as a useful strategy when calculating mentally. It is not necessarily intended that they be carried out as a connected sequence.

• Activities selected are likely to be most effective if concentrated on for short periods (10 - 15 minutes), and are repeated for consolidation.

• It is essential that students are encouraged to give explanations and reasons and to articulate their thinking.

• Encourage students to discuss when, how and why the strategy is useful.

ASSESSING PROGRESS
• Students are acquiring confidence and competence in basic addition/ subtraction fact.
• They can use and explain the use of commutativity as a strategy for calculating a given basic addition/ subtraction fact.
• They are moving away from reliance on ‘Counting-On and Back in Ones’ strategies to more efficient strategies.

PRACTICE EXAMPLES

1. 1 + 4  6. 2 + 9
2. 2 + 5  7. 3 + 6
3. 1 + 8  8. 2 + 13
4. 2 + 7  9. 4 + 9
5  3 + 8  10. 5 + 22
Activities

- Two students sit opposite each other with a domino between them. Pairs of students can compete in giving the total number of pips on a domino placed between them.

  Jane, what addition sum do you see?
  \[5 + 3\]

  Alex, what addition sum do you see?
  \[3 + 5\]

  Ranni, what addition sum do you see?
  \[3 + 5\]

  What is the answer?
  Will this always happen?

- Explore the number facts on dominoes.

  \[6 + 2 \text{ is the same as } 2 + 6\]

- Place some counters on a piece of paper with a fold line down the centre. By turning the paper, show that \[3 + 8\] is the same as \[8 + 3\].

- Add pairs of numbers on a calculator twice, entering each number first in turn.

  \[6 + 8 = 14\]
  \[8 + 6 = 14\]

  Does this always work?

- Calculate mentally \[2 + 9\]. Now calculate mentally \[9 + 2\]. What do you notice? Which is easier to calculate mentally? Why?
ACTIVITY 2.3 USING RELATED FACTS

AIM
To promote students’ use of related facts to derive facts they cannot yet retrieve automatically.

SUGGESTED MATERIALS
Counters, linking cubes, dominoes, calculators, base 10 blocks.

TEACHING POINTS
- The activities are intended as suggestions of the kind of activities (both organized and incidental) that can help students use related facts when calculating mentally. It is not necessarily intended that they be carried out as a connected sequence.

- Activities selected are likely to be most effective if concentrated on for short periods (10 – 15 minutes), and are repeated for consolidation.

- It is essential that students are encouraged to give explanations and reasons and to articulate their thinking.

- Encourage students to discuss when, how and why the strategy is useful.

ASSESSING PROGRESS
- Students are acquiring confidence and competence in basic fact addition/subtraction.

- They can use and explain the use of related facts as a strategy for calculating a given basic addition/subtraction fact.

- They are moving away from reliance on ‘Counting-On and Back in Ones’ strategies to more efficient strategies.

PRACTICE EXAMPLES
1. 3 + 4
2. 6 + 5
3. 10 - 5
4. 9 + 4
5. Explain two ways of calculating 10 - 8
6. 2 + 9
7. 10 - 8
8. 16 - 7
9. 13 + 37
10. Explain two ways of calculating 14 - 7
Basic Facts Addition and Subtraction

Module 2

Activities

- Place a pencil down the centre of a piece of paper. Place some counters on each side. State the fact. Add or remove up to 3 counters on one side, one at a time and, without counting, determine solution.

How many counters altogether?
$5 + 2 = 7$

Add one counter
Without counting, what is $6 + 2$?

Add one counter
Without counting, what is $7 + 2$?

- Use connecting blocks

Show $7 + 4$

How many altogether?

- Use a calculator
What is $10 + 8$?
So, what is $9 + 8$?

- Use a calculator
calculate
2+3
12+3
23+3
33+3

• Write the other addition and two subtraction facts that relate to the basic fact: $7 + 2$

$7 + 2 = 9$
$2 + 7 = 9$
$9 - 2 = 7$
$9 - 7 = 2$
**ACTIVITY 2.4  COUNTING ON 1, 2, 3, 0 (ADDITION)**

<table>
<thead>
<tr>
<th>SET</th>
<th>COUNT ON FACTS</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A</strong></td>
<td>Count On 1 (+1)</td>
</tr>
<tr>
<td></td>
<td>2 + 1  3 + 1  4 + 1  5 + 1  6 + 1  7 + 1  8 + 1</td>
</tr>
<tr>
<td></td>
<td>Spin-around 1 (1+)</td>
</tr>
<tr>
<td></td>
<td>1 + 2  1 + 3  1 + 4  1 + 5  1 + 6  1 + 7  1 + 8</td>
</tr>
<tr>
<td></td>
<td>Count On 2 (+2)</td>
</tr>
<tr>
<td></td>
<td>3 + 2  4 + 2  5 + 2  6 + 2  7 + 2</td>
</tr>
<tr>
<td></td>
<td>Spin-around 2 (2+)</td>
</tr>
<tr>
<td></td>
<td>2 + 3  2 + 4  2 + 5  2 + 6  2 + 7</td>
</tr>
<tr>
<td></td>
<td>Count On 3 (+3)</td>
</tr>
<tr>
<td></td>
<td>4 + 3  5 + 3  6 + 3</td>
</tr>
<tr>
<td></td>
<td>Spin-around 3 (3+)</td>
</tr>
<tr>
<td></td>
<td>3 + 4  3 + 5  3 + 6</td>
</tr>
<tr>
<td></td>
<td>Count On 0 (+0)</td>
</tr>
<tr>
<td></td>
<td>1 + 0  2 + 0  3 + 0  4 + 0  5 + 0  6 + 0  7 + 0  8 + 0  9 + 0</td>
</tr>
<tr>
<td></td>
<td>Spin-around 0 (0+)</td>
</tr>
<tr>
<td></td>
<td>0 + 1  0 + 2  0 + 3  0 + 4  0 + 5  0 + 6  0 + 7  0 + 8  0 + 9</td>
</tr>
</tbody>
</table>

**AIM**
To promote counting-on as a strategy for adding on amounts up to 3.

**SUGGESTED MATERIALS**
Counters, marbles and a tin, overhead projector.

**TEACHING POINTS**
- Begin with counting-on 1 until this is consolidated. Then introduce counting-on 2, then 3. Treat counting-on zero separately as ‘adding’ usually means an increase.
- Check that students can count fluently and quickly to 20, starting from any number.
- Check that students can explain commutativity (that it doesn’t matter which number you start with).
- Note whether any students count the first set. For example if calculating 4 + 2, do they count 1, 2, 3, 4, 5, 6. If so, discuss why this is not necessary.
- Discuss what ‘adding zero’ means in practical situations. I have four dollars and nobody gives me any more. How much do I have? or I had no lollies but Sam gave me three. How many do I have now? Check that students can translate these situations into symbols; 4 + 0 = 4, 0 + 3 = 3

**PRACTICE**
- Students can practice Set A on their own or with a partner, using a calculator to check their answers. Test Sets A on BLM 2.1 can be used as a test or given as cards for students to practice.

**EXTENSION**
- Some students can be encouraged to find connections between these ‘compatible numbers’ and other calculations: for example
- Subtractions: 4 - 1, 7 - 3.
- Counting on from larger numbers: 14 + 3, 2 + 23, 27 + 0.

**ASSESSING PROGRESS**
- Students should either have instant recall of the above facts or should be able to calculate them quickly and fluently.
Activities

• Place 5 marbles in a tin
  Add 3 more
  Count-on from 5 as each marble is added
  ...6, 7, 8

• Flash collections of counters on an overhead projector. Add 1, 2, 3, or 0, then flash again.

• Circle the biggest number, then count-on
  1+7= 7, count-on 1
  8+3= 8, count-on 3
  0+5= 5, count-on 0

• Show the largest amount with counters. Cover, then count-on smaller amount.

• Model real situations for counting on zero and relate to symbolic form
  Three lollies. My brother gave me no more.
  I had no lollies. My friend gave me 7.
Mental Computation  
A Strategies Approach

ACTIVITY 2.5 TENS FACTS AND BRIDGING TEN (ADDITION)

| B | Tens Facts | 10 + 0 | 9 + 1 | 8 + 2 | 7 + 3 | 6 + 4 |
|   |            | 0 + 10 | 1 + 9 | 2 + 8 | 3 + 7 | 4 + 6 |
| E | Bridge 10 (9+) | 9 + 2 | 9 + 3 | 9 + 4 | 9 + 5 | 9 + 6 | 9 + 7 | 9 + 8 |
|   | Spin-around (+9) | 2 + 9 | 3 + 9 | 4 + 9 | 5 + 9 | 6 + 9 | 7 + 9 | 8 + 9 |
| E | Bridge 10 (8+/7+) | 8 + 3 | 8 + 4 | 8 + 5 | 8 + 6 | 8 + 7 | 7 + 4 | 7 + 5 |
|   | Spin-around(+8/+7) | 3 + 8 | 4 + 8 | 5 + 8 | 6 + 8 | 7 + 8 | 4 + 7 | 5 + 7 |

AIM
To promote the knowledge that pairs of numbers which make 10 (sometimes called ‘compatible numbers’), is not only useful in itself, but can be used for other valuable mental strategies, for example ‘Bridging Ten’.

SUGGESTED MATERIALS
Counters, linking cubes, calculators, BLMs 2.3 to 2.7.

TEACHING POINTS
- For ‘Tens Facts’, ensure students can determine the number of counters displayed on a 10-Frame before expecting them to connect pairs of numbers to make 10.
- For ‘Bridging Tens’ facts, display each number in a different colour to promote visualisation and tracking of the movement to the top frame.
- Ensure the frames are oriented horizontally at all times.

PRACTICE
- In pairs, students can take in turns to place counters on a 10-Frame (for ‘Tens Facts’) or the 20-Frame (for ‘Bridging Ten’), show it to their partner who must instantly say (not count) how many counters are on the board and how many blanks. The board is shown for one or two seconds before it is covered again.
- Play ‘Make Compatible’ game using BLM 2.5 and BLM 2.6 (see instructions on page 37). Encourage students to make up more boards using BLM 2.7.
- Test Set B and Set E(9) and Set E(8,7) on BLM 2.1 can be used as a test written on cards for students to practice. Allow checking of answers with a calculator if students are unsure.

EXTENSION
- Some students can be encouraged to find connections between these ‘compatible numbers’ and other calculations. For example:
  - Numbers that make 20: 12 + 8, 7 + 13.
  - Numbers that make 100: 30 + 70, 80 + 20.
  - Subtractions: 10 - 4, 10 - 6.
- Some students can be encouraged to explore bridging of other multiples of 10. For example:
  - 38 + 6 (think 38 + 2 = 40, + 4 = 44)
  - 53 – 5 (think 53 – 3 = 50, – 2 = 48)

ASSESSING PROGRESS
- Given any number from 1 to 10, students should give the number that makes 10 without hesitating or counting.
- Students can use ‘Bridging Ten’ to calculate any of the items in Set E above.
Activities

- Use a 10-Frame (BLM 2.3) and counters to promote a visual image of combinations to 10. Establish recognition of numbers 0-10 on the 10-Frame first.

- Then put matching combinations together to connect the number pairs that make 10. When consolidated, link to spin-arounds.

- Use a 20-Frame (BLM 2.4) to model ‘Bridging 10’.

- Use the ‘Make 10’ boards (BLMs 2.5 to 2.7) for practice. Each student needs a board and 24 counters. Students cover pairs of numbers that make 10 on any one of the boards until only one number is left uncovered. If this is the correct number then the student has covered the board correctly. A blank ‘Make 10’ board (BLM 2.7) is included so that students can make up their own for others to try.
ACTIVITY 2.6 DOUBLES

\[ \begin{array}{c|c|c|c|c|c|c|c|c} 
  \text{C} & \text{Doubles} & 0 + 0 & 1 + 1 & 2 + 2 & 3 + 3 & 4 + 4 & 5 + 5 \\
  \hline 
  & 6 + 6 & 7 + 7 & 8 + 8 & 9 + 9 & 10 + 10 \\
\end{array} \]

AIM
To promote use of mental images of ‘Doubles’ to encourage recall and build on students’ intuitive knowledge of ‘Doubles’ facts.

SUGGESTED MATERIALS
Counters, linking cubes, calculators, grid paper.

TEACHING POINTS
• Check understanding. ‘Double’ means ‘two lots of’. ‘Double 3’ means the same as ‘3 + 3’.
• Check understanding of 0 + 0.
• When students have difficulty in recalling a double, link to a real object (e.g. 8 + 8; think of a spider - how many legs if there are two spiders?)
• Challenge students to memorise the doubles from 0 + 0 to 10 + 10.

PRACTICE
• Students can practice, checking their answers themselves with a calculator if necessary.

EXTENSION
• Some students can be encouraged to find connections between these doubles and other calculations. For example:
  • Subtractions: 6 – 3, 4 – 2, 16 - 8;
  • Adding tens: 20 + 20, 50 + 50.
  • Adding larger numbers: 25 + 25, 62 + 62.

ASSESSING PROGRESS
• Check that students can double any number from 0 to 10 without hesitating or counting.
**Activities**

• Brainstorm a list of ‘things’ that relate to the numbers 1-10. Visualise each ‘thing’ and then double it.

1. person, stop sign, sun, nose  
   1+1=2
2. drum sticks, eyes, ears, bicycle wheels  
   2+2=4
3. clover leaf, cricket stumps, tricycle wheels  
   3+3=6
4. car tyres  
   4+4=8
5. fingers  
   5+5=10
6. legs of insects  
   6+6=12
7. calendar (2 weeks)  
   7+7=14
8. octopus, spider  
   8+8=16
9. channel 9  
   9+9=18
10. fingers, toes  
   10+10=20

• Model with material to show mirror image.

![Mirror Image Example](image)

• Link to multiplication, check with a calculator

4 + 4 = 8
2 groups of 4 = 8
2 x 4 = 8

Does 9 + 9 = 2 x 9?
### Activity 2.7 Using Place Value to Add Ten

<table>
<thead>
<tr>
<th>D</th>
<th>ADD 10 (10+)</th>
<th>Spin-around (+10)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>10 + 1</td>
<td>1 + 10</td>
</tr>
<tr>
<td></td>
<td>10 + 2</td>
<td>2 + 10</td>
</tr>
<tr>
<td></td>
<td>10 + 3</td>
<td>3 + 10</td>
</tr>
<tr>
<td></td>
<td>10 + 4</td>
<td>4 + 10</td>
</tr>
<tr>
<td></td>
<td>10 + 5</td>
<td>5 + 10</td>
</tr>
<tr>
<td></td>
<td>10 + 6</td>
<td>6 + 10</td>
</tr>
<tr>
<td></td>
<td>10 + 7</td>
<td>7 + 10</td>
</tr>
<tr>
<td></td>
<td>10 + 8</td>
<td>8 + 10</td>
</tr>
<tr>
<td></td>
<td>10 + 9</td>
<td>9 + 10</td>
</tr>
</tbody>
</table>

**AIM**

To assist the development of understanding that our place value and notation system makes ‘Adding 10’ to a number a simple process, which does not require the need to count-on in ones.

**Suggested Materials**

Base 10 ones and tens blocks, bundling sticks, calculators.

**Teaching Points**

- Check that students understand the relationship between base 10 blocks tens and ones and the recording: 2 tens and 3 ones is recorded as 23. They should instantly be able to record the value of a collection of tens and ones, and also use base 10 blocks to match a given number.
- Ensure that when students hear the instruction ‘Add 10’, they do not automatically start counting-on ten in ones.

**Practice**

- Test Sets D on BLM 2.1 can be used as a test or given as cards for students to practice.

**Extension**

- Some students can be encouraged to extend the idea to related computations. They can use a calculator to discover the pattern and check their calculations.
- Adding larger multiples of 10: 40 + 7, 2 + 50.
- Adding 10 to larger numbers: 26 +10, 10 + 42.

**Assessing Progress**

- Students respond immediately to any of the items in Set D.
Activities

- Use bundles of 10 paddle pop sticks to count in tens from various starting points

- Use base 10 blocks

- Repeatedly add 10 to any number on a calculator and observe the pattern.

Enter 34
Press + 10 =
What is the answer? 44
Press = again Answer = 54
Press = again Answer = 64
Press = again Answer = 74

What digits change?
What digits stay the same?
What happens when you add 10 to a number?
**ACTIVITY 2.8 NEAR DOUBLES**

<table>
<thead>
<tr>
<th>NEAR DOUBLES</th>
</tr>
</thead>
<tbody>
<tr>
<td>Doubles + 1/2</td>
</tr>
<tr>
<td>Spin-around</td>
</tr>
</tbody>
</table>

**AIM**
To encourage students to draw upon their knowledge of ‘Doubles’ facts to derive solutions to facts that are almost doubles.

**SUGGESTED MATERIALS**
Counters, linking cubes, calculators, grid paper.

**TEACHING POINTS**
- Check recall of doubles. If students experience difficulty with doubles, they may find this strategy confusing.
- When students display the ‘Doubles +1’ and ‘Doubles +2’ facts using materials, ensure that the fact is displayed in symbolic form.
- For some students, these facts may be more appropriately recognized as ‘Doubles less one or two’. That is, $5 + 6$ may be seen more easily as $6 + 6 - 1$; $6 + 8$ may be $8 + 8 - 2$. Promote this type of discussion around the visual images provoked by the concrete representations and encourage students to use the strategy that appeals to them most. It is better if students select one strategy until particular facts become consolidated, otherwise confusion may occur and the strategy may be of little use.
- Some students may suggest adding 2 or 3 to a double, and using doubles such as $20 + 20$ to calculate $20 + 21$ or $20 + 19$. Encourage this exploration if suggested by students.

**PRACTICE**
- Students can practice, checking their answers with a calculator if necessary.

**EXTENSION**
- Some students can be encouraged to find connections between these ‘Near Doubles’ and other calculations: for example.
  - Subtractions: $13 - 6$, $5 - 2$, $17 - 9$.
  - Adding tens: $29 + 30$, $51 + 52$.
  - Adding larger numbers: $25 + 26$, $60 + 59$.

**ASSESSING PROGRESS**
- Check that students can double any number from zero to ten without hesitating or counting.
- Check that students can calculate ‘Near Doubles’ by adding and subtracting from ‘Doubles’.
Activities

- Linking to known ‘Doubles’ using counters and grids.

\[ 6 + 6 = 12 \]

\[ \begin{array}{cccc}
  & & & \\
  & & & \\
  & & & \\
  & & & \\
  & & & \\
  & & & \\
 \end{array} \]

\[ 6 + 7 = 6 + 6 + 1 \]

\[ \begin{array}{cccc}
  & & & \\
  & & & \\
  & & & \\
  & & & \\
  & & & \\
  & & & \\
 \end{array} \]

- Display ‘Near Doubles’ and ask students to describe how they can use their knowledge of ‘Doubles’ to assist in determining the answer. Continue this discussion with ‘Near Doubles’ presented in symbolic form.

How would you calculate?

\[ \begin{array}{cccc}
  & & & \\
  & & & \\
  & & & \\
  & & & \\
  & & & \\
  & & & \\
 \end{array} \]

8 + 6

How would you calculate?

\[ 8 + 6 \]

What do you see in your mind?
**ACTIVITY 2.9 THINK ADDITION**

**AIM**
To promote the use of converting subtraction to addition as a mental strategy appropriate to specific individual or sets of basic facts for which student do not yet have automatic recall.

**SUGGESTED MATERIALS**
Counters, linking cubes, dominoes, calculators, MAB.

**TEACHING POINTS**
- The activities are intended as suggestions of the kind of activities (both organized and incidental) that can help students see converting subtraction to addition as a useful strategy when calculating mentally. It is not necessarily intended that they be carried out as a connected sequence.
- Activities selected are likely to be most effective if concentrated on for short periods (10 – 15 minutes), and are repeated for consolidation.
- It is essential that students are encouraged to give explanations and reasons and to articulate their thinking.

**ASSESSING PROGRESS**
- Students are acquiring confidence and competence in basic fact additions/subtractions.
- They can implement and explain the use of converting subtraction to addition as a strategy for calculating a given basic subtraction fact.
- They are moving away from reliance on ‘Counting On and Back in Ones’ strategies to more efficient strategies.

**PRACTICE EXAMPLES**
1. 5 – 3
2. 8 – 5
3. 12 – 9
4. 11 – 7
5. Explain two ways of calculating 9 - 7
6. 7 – 6
7. 9 – 8
8. 16 – 7
9. 12 – 6
10. Explain two ways of calculating 14 – 9
Activities

Place a pencil down the centre of a piece of paper. Use counters to represent a particular fact. Put some counters on each side (for example 6 on the left, 3 on the right). How many counters in all? (9) Remove the 3 counters. How many left? (6). What is 9 - 3? (6). What is 6 + 3? (9). Write these. Show me another like this.

• Use connecting blocks

How many white?
How many black?
Remove the white. How many left?
14 - 8 = ?
Remove the black. How many left?
14 - 6 = ?

• Make up number sentences using sets of 3 related numbers: 17, 9, 8.

9 + 8 = 17  17 - 9 = 8
8 + 9 = 17  17 - 8 = 9
ACTIVITY 2.10 DOUBLES AND NEAR DOUBLES (SUBTRACTION)

<table>
<thead>
<tr>
<th>C Doubles + 1</th>
<th>2 - 1</th>
<th>4 - 2</th>
<th>6 - 3</th>
<th>8 - 4</th>
<th>10 - 5</th>
<th>0 - 0</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>12 - 6</td>
<td>14 - 7</td>
<td>16 - 8</td>
<td>18 - 9</td>
<td>20 - 10</td>
<td></td>
</tr>
<tr>
<td>F Doubles - 1</td>
<td>9 - 4</td>
<td>11 - 5</td>
<td>13 - 6</td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

AIM
To promote recognition of a total amount as a combination of ‘Doubles’ (eg. 12 = 6 + 6), or ‘Near Doubles’ (13 = 6 + 7).

SUGGESTED MATERIALS
Counters, linking cubes, calculators.

TEACHING NOTES
- Using ‘Doubles’ for subtraction depends on knowing the ‘Doubles’ instantly. For example, upon seeing 12 – 6, the student recognizes the ‘Doubles’ relationship between 12 and 6 and changes 12 – 6 = ? into 6 + ? = 12, knowing that 6 + 6 = 12.

- Consolidate activities with materials by encouraging the writing of addition equations for ‘Doubles’ and the related subtractions: 7 + 7 = 14; 14 – 7 = 7.

- You may deal with ‘Near Doubles’ at the same time as the ‘Doubles’ themselves. Check whether students can relate the answers to the related ‘Doubles’. If students do not understand this strategy for ‘Near Doubles’ without using objects, it is better to leave it for later. In most cases there is another strategy for these calculations.

- As this is a relatively unimportant strategy, do not try to force understanding.

- Check that students can calculate the examples in Set F either by using ‘Near Doubles’ or by another strategy.

PRACTICE
- Students can use the C and F test on BLM 2.2 to practice checking their answers with a calculator if necessary.

EXTENSION
- Some students can be encouraged to find connections between these doubles and other calculations. For example:
Activities

ASSESSING PROGRESS
• Check that students can subtract any number from 0 to 10 from its double and can explain their strategy.

• Use counters or blocks to emphasise the ‘doubles’ and the total. Link to addition knowledge of addition doubles.

• How many in each pile?

  How many altogether?

  Cover them all

  If I take away one pile, how many left?

  8 – 4 = 4

Have students close their eyes and visualize amounts as two groups of equal size. Use questioning to promote the strategy of linking to the known addition double.

Visualise two piles of 6 counters

How many counters altogether?

If I take away one pile, how many left?

12 – 6 = 6

12 counters altogether. 6 counters in each pile. Take away one pile, must have 6 counters left. 12 – 6 = 6
### AIM
Knowing the pairs of numbers which make 10 provides a useful subtraction strategy, not only for subtracting from ten, but also for ‘Bridging Ten’ when subtracting from other numbers.

### SUGGESTED MATERIALS
Counters, linking cubes, calculators, BLMs 2.3 and 2.4.

### TEACHING POINTS
- Students should know the pairs of numbers that make up ten when starting this activity (see Activity 2.5). They should also be familiar with the 10-Frame (BLM 2.3) and the 20-Frame (BLM 2.4). Activity 2.5 is repeated but with the subtraction aspect emphasised.
- Place 10 counters on the 10-Frame (BLM 2.3). Take away 3 counters. How many left? Seven. How many were taken? Three.
- If students do not immediately see the relationship, repeat the activity removing other counters of other amounts.
- Ask students some of the subtractions from Set E (9) and E (8,7), without using objects.

### THE TEACHING SEQUENCE Set E (Bridging Ten):
- Use the 20-Frame (BLM 2.4). Fill the first frame and placing any number in the second frame, ask questions such as: If I remove 5 counters how many left? Encourage explanations such as: There are 13 counters. If I remove 3 that leaves 10. 2 more from these 10 will leave 8. Later: 13 - 3 = 10, - 2 = 8.
- Practice, using any examples from Sets B and E above. In each case ask students for explanations of their strategy.

### ASSESSING PROGRESS
- Students can mentally subtract any number from 10 without hesitating or counting.
- Students can use ‘Bridging Ten’ to calculate any of the subtractions in Sets B and E above.

---

**ACTIVITY 2.11 TENS FACTS AND BRIDGING TEN (SUBTRACTION)**

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<thead>
<tr>
<th>B</th>
<th>Tens Facts</th>
</tr>
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<tr>
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<td>10 - 0</td>
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<tr>
<td></td>
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<td>10 - 9</td>
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<td>10 - 10</td>
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</table>

<table>
<thead>
<tr>
<th>E</th>
<th>Bridge 10 (=9)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>11 - 2</td>
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<tr>
<td></td>
<td>12 - 3</td>
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<td></td>
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<td>16 - 7</td>
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<td>17 - 8</td>
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<table>
<thead>
<tr>
<th>E</th>
<th>Spin-around (-9)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>11 - 9</td>
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<td></td>
<td>12 - 9</td>
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<td></td>
<td>13 - 9</td>
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<td>17 - 9</td>
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<table>
<thead>
<tr>
<th>E</th>
<th>Bridge 10 (=8=/7)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>11 - 3</td>
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<tr>
<td></td>
<td>12 - 4</td>
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<td>13 - 5</td>
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<td>11 - 4</td>
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<td>12 - 5</td>
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<table>
<thead>
<tr>
<th>E</th>
<th>Spin-around (-8/-7)</th>
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<tbody>
<tr>
<td></td>
<td>11 - 8</td>
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<td>12 - 8</td>
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<td>11 - 7</td>
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<td>12 - 7</td>
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Activities

Use a 10-Frame and counters to promote a visual image of 10 and the amount left when some are removed.

Fill the 10-Frame with counters

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Remove 3

<p>| | | | | |</p>
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</tbody>
</table>

How many left? How many removed?

10 – 7  think  7 plus what makes 10?
10 – 9  think  9 plus what makes 10?
10 – 4  think  4 plus what makes 10?

Use a 20-Frame to model subtraction of Bridging Ten Facts

12 – 5 = ?

12 counters

<p>| | | | | |</p>
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</table>

Take away 5

<p>| | | | |</p>
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<tbody>
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</tbody>
</table>

Remove 2 from second frame first

12 – 5 is the same as 12 - 2 - 3
ACTIVITY 2.12 COUNTING BACK 1, 2, 3, 0 (SUBTRACTION)

A  Count Back 1 (-1)  
    Spin-around (=1)  
    Count Back 2 (-2)  
    Spin-around (=2)  
    Count Back 3 (-3)  
    Spin-around (=3)  
    Count Back 0 (-0)  
    Spin-around (=0)  

<table>
<thead>
<tr>
<th></th>
<th>3 - 1</th>
<th>4 - 1</th>
<th>5 - 1</th>
<th>6 - 1</th>
<th>7 - 1</th>
<th>8 - 1</th>
<th>9 - 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spin-around (=1)</td>
<td>3 - 2</td>
<td>4 - 3</td>
<td>5 - 4</td>
<td>6 - 5</td>
<td>7 - 6</td>
<td>8 - 7</td>
<td>9 - 8</td>
</tr>
<tr>
<td>Count Back 2 (-2)</td>
<td>5 - 2</td>
<td>6 - 2</td>
<td>7 - 2</td>
<td>8 - 2</td>
<td>9 - 2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Spin-around (=2)</td>
<td>5 - 3</td>
<td>6 - 4</td>
<td>7 - 5</td>
<td>8 - 6</td>
<td>9 - 7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Count Back 3 (-3)</td>
<td>7 - 3</td>
<td>8 - 3</td>
<td>9 - 3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Spin-around (=3)</td>
<td>7 - 4</td>
<td>8 - 5</td>
<td>9 - 6</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>Count Back 0 (-0)</td>
<td>1 - 0</td>
<td>2 - 0</td>
<td>3 - 0</td>
<td>4 - 0</td>
<td>5 - 0</td>
<td>6 - 0</td>
<td>7 - 0</td>
</tr>
<tr>
<td>Spin-around (=0)</td>
<td>1 - 1</td>
<td>2 - 2</td>
<td>3 - 3</td>
<td>4 - 4</td>
<td>5 - 5</td>
<td>6 - 6</td>
<td>7 - 7</td>
</tr>
</tbody>
</table>

AIM
To promote counting back and counting-up from are useful strategies for related subtraction ‘count-on’ facts.

SUGGESTED MATERIALS
Counters, linking cubes, calculators.

TEACHING POINTS
- ‘Counting-back’ 1, 2 or 3 is safe and unlikely to lead to many errors. The most common error is for students to start the count on the wrong number. For example, when calculating 8 - 3, to say ‘8, 7, 6’. To avoid this, students should first model the calculation with objects.
- Begin with ‘Counting-back 1’, until this is consolidated. Then introduce ‘Counting-back 2’, then 3. Treat ‘Counting-back 0’ separately as ‘taking away’ usually means a decrease.
- Check that students can count backwards fluently and quickly from 20, starting from any number.
- Discuss what ‘taking away (subtracting) zero’ means in practical situations. I have four dollars and nobody takes any from me. How much do I have? Check that students can translate these situations into symbols: 4 - 0 = 4.

PRACTICE
- Students can practice Set A on their own or with a partner, using a calculator to check their answers. The relevant tests on BLM 2.2 can be used as a test or given as cards for students to practice.

EXTENSION
- Some students can be encouraged to make connections to this strategy for other calculations. For example:

ASSESSING PROGRESS
- Students count back fluently 1, 2 or 3 to solve subtractions.
- Where two numbers are close to each other, students change the subtraction to an addition and use count-up to find the answer.
Activities

- Promote the strategy of counting-back from when the number to be subtracted is less than 3.

  7 – 2

  Think 7
  Count back 2
  ..6...5

  Answer = 5

- Promote the strategy of counting up to when the difference is less than 3.

  8 – 5

  Think 5
  Count on from 5
  6…7…8

  Answer is 3

Use real situations to explore taking-away zero

I have $4. I spend none. I have $4 left

4 – 0 = 4

I have 6 lollies. I eat 6. I have no lollies left

6 – 6 = 0
ACTIVITY 2.13 USING PLACE VALUE TO SUBTRACT TEN

<table>
<thead>
<tr>
<th>D</th>
<th>Subtract to 10 (=10)</th>
<th>11 - 1</th>
<th>12 - 2</th>
<th>13 - 3</th>
<th>14 - 4</th>
<th>15 - 5</th>
<th>16 - 6</th>
<th>17 - 7</th>
<th>18 - 8</th>
<th>19 - 9</th>
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<tbody>
<tr>
<td>Subtract 10 (-10)</td>
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<td>13 - 10</td>
<td>14 - 10</td>
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<td>16 - 10</td>
<td>17 - 10</td>
<td>18 - 10</td>
<td>19 - 10</td>
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</tbody>
</table>

AIM
The aim is to promote students’ understanding that our place value and notation system makes subtracting 10 from a number (or subtracting, for example, 3 from 13), almost automatic. Particularly, that it does not need to involve counting- back in ones.

SUGGESTED MATERIALS
Base 10 ones and tens blocks, bundling sticks, calculators.

TEACHING POINTS
• Check that students understand the relationship between base 10 blocks, tens and ones and the recording: 2 tens and 3 ones is recorded as 23. They should instantly be able to calculate the value of a collection of tens and ones and also use base 10 blocks to match a given number.
• Ensure that when students hear the instruction ‘Take 10’, they do not automatically start counting-back ten in ones.

PRACTICE
• Test Set D on BLM 2.2 can be used as a test or given as cards to students to practice.

EXTENSION
• Some students can be encouraged to extend the idea to related computations. They can use a calculator to discover the pattern and check their calculations.
• Subtracting or leaving larger multiples of 10: 47 - 40, 52 - 2.
• Subtracting 10 from larger numbers: 36 - 10, 52 - 10, 124 - 10.

ASSESSING PROGRESS
• Students respond immediately to any of the items in Set D.
Activities

Use base 10 blocks.

Repeatedly subtract 10 from any number on a calculator and observe the pattern.

Enter 67

Press - 10 =

What is the answer? 57

Press = again Answer = 47

Press = again Answer = 37

Press = again Answer = 27

What digits change?
What digits stay the same?
What happens when you add 10 to a number?
### Practice Tests (Addition)

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<thead>
<tr>
<th>Set A (1)</th>
<th>Set A (2,3)</th>
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<tbody>
<tr>
<td>1. 2 + 1</td>
<td>1. 3 + 2</td>
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<td>2. 2 + 6</td>
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<td>6. 5 + 2</td>
<td>6. 10 + 8</td>
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<td>8. 1 + 9</td>
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<td>10. 1 + 8</td>
<td>10. 4 + 5</td>
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<th>Set C</th>
<th>Set D</th>
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<td>1. 7 + □ = 10</td>
<td>1. 2 + 2</td>
<td>1. 10 + 7</td>
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<tr>
<td>2. 4 + □ = 10</td>
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<td>3. 1 + □ = 10</td>
<td>3. 8 + 8</td>
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<td>4. 5 + □ = 10</td>
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<td>7. 6 + □ = 10</td>
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<td>8. 2 + □ = 10</td>
<td>8. 10 + 10</td>
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<td>9. 9 + □ = 10</td>
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<td>10. 10 + □ = 10</td>
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<tr>
<td>1. 9 + 4</td>
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## Practice Tests (Subtraction)

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<td>10. 15 - 9</td>
<td>10. 12 - 5</td>
<td>10. 15 - 8</td>
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**RULES OF THE ‘MAKE COMPATIBLES’ GAMES (BLMS 2.5 TO 2.7)**

**PURPOSE**
These activities are designed to give students practice in finding pairs of numbers that add up to a multiple of 10.

**INSTRUCTIONS**
Individual students need one 5 x 5 grid and 24 counters. They locate pairs of numbers on the board that add up to 10 and cover them with counters, until only one number is left uncovered. This is the key number that they give you. If they have the correct key number, then they have covered up the compatible numbers correctly. Alternately the key number can be written on the back of the board.

The blank board (BLM 2.7) is provided so students can make up their own boards and give them to friends to try.

By making a 5 x 5 table and using 48 point for the numbers the boards can be easily made on the computer.

Two boards fit on an A4 page.

Key Numbers (i.e. the number that should remain uncovered when all pairs of ‘compatible’ numbers have been covered with counters).

Make 10 (BLM 2.5 Board A): 4
Make 10 (BLM 2.5 Board B): 8
Make 10 (BLM 2.6 Board C): 7
Make 10 (BLM 2.6 Board D): 3
## Make 10 (A)

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## Make 10 (B)

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### Make 10 (C)

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### Make 10 (D)

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