AMSI
AUSTRALIAN MATHEMATICAL SCIENCES INSTITUTE

INTERNATIONAL CENTRE OF EXCELLENCE FOR EDUCATION IN MATHEMATICS

PLACE VALUE

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Place Value K to 4

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Place Value K - 4

Just as we need the alphabet to write down words and sentences, so we need a notation to write down numbers. We use a base-ten place-value notation developed over several centuries in India and the Arab world, so we call it Hindu-Arabic notation. The place-value nature of Hindu-Arabic notation enabled the development of highly efficient algorithms for arithmetic. Place value is core to our understanding of so many areas of mathematics. In this session we will look at the development of place value in a historical sense and make some parallels with the ways students come to know these concepts. Ideas for place value in the classroom will be discussed.

Place Value

Australian Curriculum - Kindergarten

1. **Counting** Say, understand and reason with number sequences, initially to and from 20, and then beyond, moving to any starting point (MKNA1)

2. **Numeration** Understand numbers to 10, including matching number names, numerals and quantities, and work fluently with small numbers including subitising and partitioning (MKNA2)

Place Value

Australian Curriculum - Year 1

1. **Counting** Say, understand and reason with number sequences to and from 100 by ones from any starting point, and say number sequences of twos, fives and tens starting from zero (M1NA1)

2. **Numeration** Recognise, model and represent numbers to 100, and read, write and order those numbers (M1NA2)

3. **Place value** Understand and work fluently with counting collections to 100 by grouping in tens, and counting the tens, and use place value to partition and regroup those numbers (M1NA3)

Place Value

Australian Curriculum - Year 2

1. **Counting** Say, understand and reason with number sequences increasing by twos, fives and tens from any starting point including using calculators (M2NA1)

2. **Numeration** Recognise, model and represent numbers to 130, and read, write and order those numbers (M2NA2)

3. **Place value** Work fluently with counting increasingly larger collections up to 1000, grouping in hundreds and tens and counting the tens and hundreds and use place value to partition and regroup these numbers (M2NA3)

Place Value

Australian Curriculum - Year 3

1. **Counting** Understand and reason with number sequences increasing and decreasing by twos, fives and tens from any starting point, moving to other sequences, emphasising patterns and explaining relationships (M3NA1)
2. **Numeration** Recognise, model, represent and visualise numbers initially to 1000 and then beyond, and read, write and order those numbers (M3NA2)
3. **Place value** Justify various uses of the place value system to describe numbers to 1000, using the hundreds and tens as units, and to partition...

Place Value

Australian Curriculum - Year 4

2. **Numeration** Recognise, represent, visualise and work fluently with reading, writing and ordering numbers to 1 million (M4NA2)

3. **Place value** Justify various uses of the place value system to describe large numbers, and to partition and regroup those numbers to assist calculation and solve problems (M4NA3)

Place Value

Australian Curriculum - Year 5

1. **Decimals** Recognise and represent numbers involving tenths and hundredths; read, write and order those numbers and connect them to fractions (M5NA1)

2. **Place value** Justify various uses of the place value system to describe decimal numbers, and to partition and regroup those numbers to assist calculations and solve problems (M5NA2)

3. **Fractions and decimals** Solve problems involving making comparisons using equivalent fractions and decimals and everyday uses of percentages, relating them to parts of 100 and hundredths (M5NA3)

Place Value

Australian Curriculum - Year 6

1. **Integers** Read, represent, write, interpret and order positive and negative integers (M6NA1)

2. **Decimals** Recognise and represent numbers involving thousandths, read, write and order those numbers, and connect them to fractions (M6NA2)

3. **Place value** Justify uses of the place value system to describe decimal numbers, and to partition and regroup those numbers to assist calculation and solve problems (M6NA3)

6. **Decimals** Understand and work fluently with decimal numbers to thousandths, and multiply and divide numbers including decimals by whole numbers to solve additive problems, including using technology (M6NA6)

Place Value

Prior knowledge

Children arriving for their first year of school may be able to:

- Recite the numbers up to 20 in order
- Write the numerals 0 to 9
- Grasp the connection between the

numeral ‘3’, the word ‘three’ and a picture such as

Place Value

Hindu Arabic Numerals

Hindu-Arabic numerals exhibit some of the qualities that make mathematics so powerful, namely

- they can be used by understanding a small number of ideas, and
- they can be generalized beyond the original setting for which they were devised.

(The notation was developed to express whole numbers, but it extends to the representation of fractions and decimals.)

Place Value

Hindu Arabic Numerals

Hindu-Arabic numerals are a decimal, or base-ten, place-value number system with the ten digits 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9 as fundamental building blocks.

Place Value

The number system

Counting numbers or Natural numbers

{1, 2, 3, 4, 5, 6, 7, …}

Whenever you add or multiply two counting numbers, you get another counting number. This may not be true when you subtract or divide (e.g., 4-7 = -3, and –3 is not a counting number even though 4 and 7 are).

Place Value

The number system

Whole numbers

$\{0, 1, 2, 3, 4, 5, 6, \ldots\}$

These are the counting numbers, together with 0.

Place Value

The number system

Integers

$\{\ldots -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, \ldots\}$

Note that you can add, subtract and multiply two integers and you will always get an integer, but you may have trouble with division (e.g., $3 \div 6 = 1/2$).

Place Value

The number system

Rational numbers {m/n, where m and n are integers}

For example, 2/3, 7/11, -34/97, 7, 0, -1000, 0.15 = 15/100, 0.5555… = 5/9

Rational numbers are just fractions. In decimal form, the rationals are those numbers which terminate or those which have a recurring block. Division of a rational by a rational always gives a rational (except that you can't divide by 0).

Place Value

The number system

Irrational numbers

Cannot be written in the form m/n, i.e, not rational.

For example, π= 3.1415926535879…,

0.101001000100001000001…, e = 2.71828…,

0.1234567891011121314151617…$\sqrt{2}$ = 1.4142…

The irrationals are those numbers that cannot be written as a fraction. In decimal form, the irrational numbers are precisely the non-recurring non-terminating decimals. There are actually more irrational numbers than there are rational numbers.

Place Value

The number system

Real numbers The rational numbers together with the irrational numbers make up the real numbers. These are the numbers we use in everyday life.

Complex numbers

[Learned in late high school or university, if at all.]

There are numbers which help us deal with things like $\sqrt{-5}$ (i.e. square roots of negative numbers). The real numbers can't do this.

Place Value

Learning to count - history

Early counting was done via one-to-one correspondence.

The word calculus means stone. It is the root of the word

calculation because shepherds used pebbles to count their sheep.

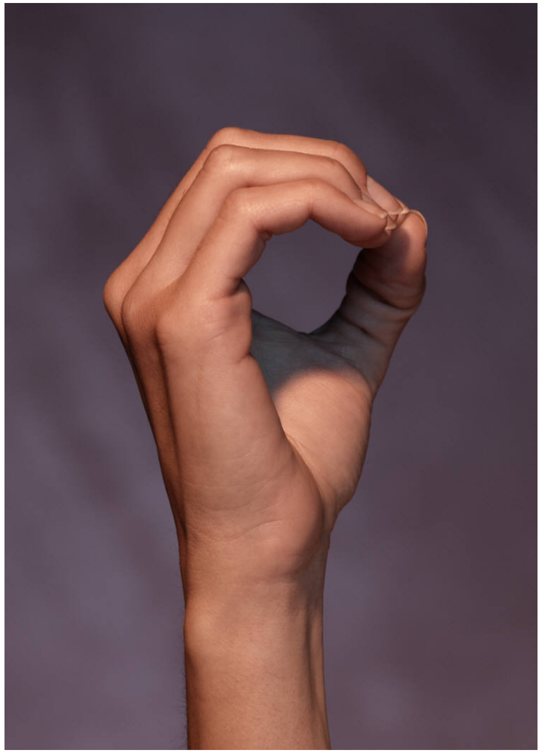
Place Value

Ten as the basis for our place value system

Hindu-Arabic notation is a place value system based on bundles of 10; so it is a decimal system.

The key to a place value system is the use of a place marker.

A place value system using 9 digits and a space or the word kha (for emptiness) as place marker was used in India the 6th century.



Place Value

Ten as the basis for our place value system

By the 9th century the system had made its way to the Arab world (including Persia and Al-Andalus in what is now Spain).

The digit 0 evolved from "·" and was used in both Madhya Pradesh (Northern India) and the Arab world by the 10th century.

Leonardo Fibonacci learned to use the notation from merchants in Africa when he was a boy and wrote a book, Liber Abaci, in 1202 which popularized the system.

Place Value

Learning to count

Number is an abstract concept. Three is the concept of threeness.

Numerals

A numeral is a symbolic representation of a number.

So 7, 5 + 2, and VII are all numerals for the number we call seven;

10, 23 − 13 and X are all numerals for the number we call ten.

Place Value

Learning to count

Digits

We call the symbols 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9 digits. They are numerals consisting of a single symbol.

(It is not a coincidence that the same word means fingers. Early counting was, and still is, done using fingers.)

Place Value

Learning to count

Recitation of a list of numbers is not necessarily counting.

Many children can recite the number names when they arrive at school. Some of them may be able to recite them in order up to 20 or more. It is likely, however, that some of them are only just beginning to understand that each numeral and its corresponding number word represent a quantity that is fixed.

For children to be considered as having the ability to count, there are certain behaviours that must be evident; each of these is essential to counting.

Place Value

Learning to count

One-to-one correspondence with number names and objects

The understanding that we use different number names for each object included in a count is a major milestone.

A child who counts a set of objects by saying "four, two, one" has understood the need for one-to-one correspondence, as has the child who correctly counts "one, two, three". A child who counts "three, two, three" has not used a unique number name for each object and neither has the child who uses more number names than objects and counts "one, three, six, seven, two, four" when counting these stars  .

Place Value

Learning to count

Stable Order

To count accurately and reliably, it is important to say the number names in the right order and without skipping any.

The ability to assign the number names in order to objects being counted and without skipping any numbers, is known as the stable-order principle.

Place Value

Learning to count

Counting objects in any order or arrangement

The number of elements in a set does not depend on the way the objects are presented or the order in which they are counted.

Knowing that the order in which objects are counted has no relevance to the actual number of objects in the group of objects is known as the order-irrelevance principle.

Place Value

Learning to count

Cardinality

One of the deep observations about counting is that when you set up a 1-1 correspondence between the number names in their correct order and the set of objects you are trying to count, then the last number name you say is the cardinality (or size) of the set.

We can help children develop the understanding of cardinality by involving them in activities where they answer questions about 'how many'.

Place Value

Learning to count

Ordering numbers

Ordering is the basis of our number system.

The ability to place quantities in order of increasing (or decreasing) size demonstrates a deep understanding of how the number system fits together.

Beginning with the idea of one more or one less, pre-school children become adept at moving around on a mental number line.

Classroom

Developing automaticity with number facts is the aim of many of the tasks we do with children.
The tasks themselves are not the aim.
To achieve this we need to help children pull apart numbers and put them back together again.
As children grow, tasks that require mental strategies need to become more cognitively demanding.
Children often develop these skills by themselves, other children need some gentle encouragement to do so.

Never say anything a child can say!

Classroom

Counting

- Forwards and backwards
- By ones
- Skip counting
- By fraction and decimal amounts
- Different start points
- Over hurdles

Classroom

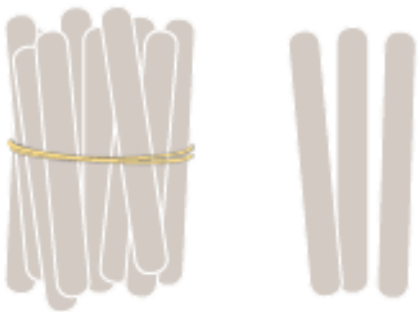
- Number Ladder
- Look at place value
- Highlight patterns
- Forward and backwards counting
- Skip counting
- Predict where you will be…

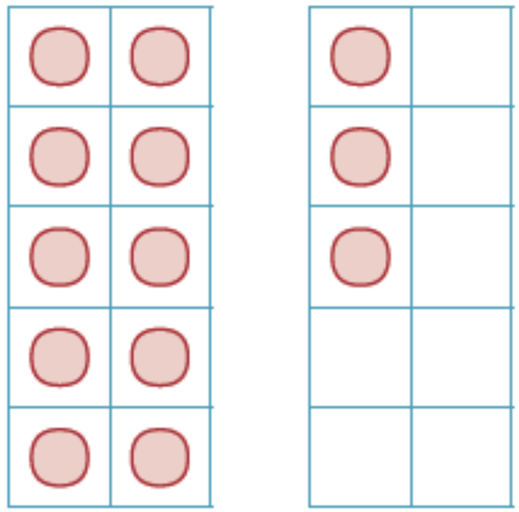
Classroom

Ten as the basis for our place value system

Once the numbers below ten are established, the next goal is to look at the numbers from ten to twenty.

We want students to



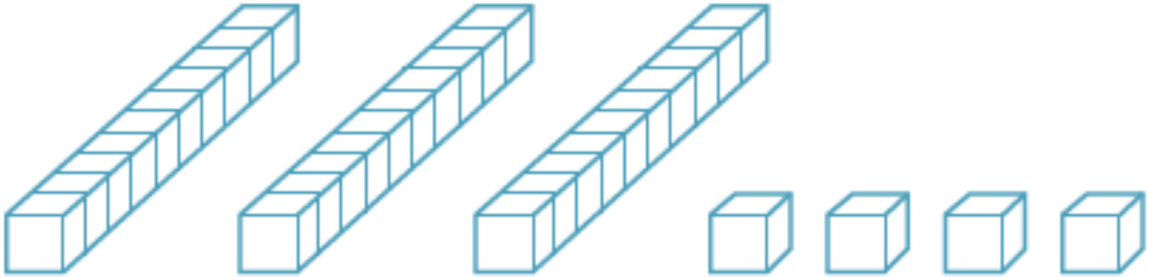


- see the importance of ten
- start to use ten as a countable unit.

Classroom

Ten as the basis for our place value system

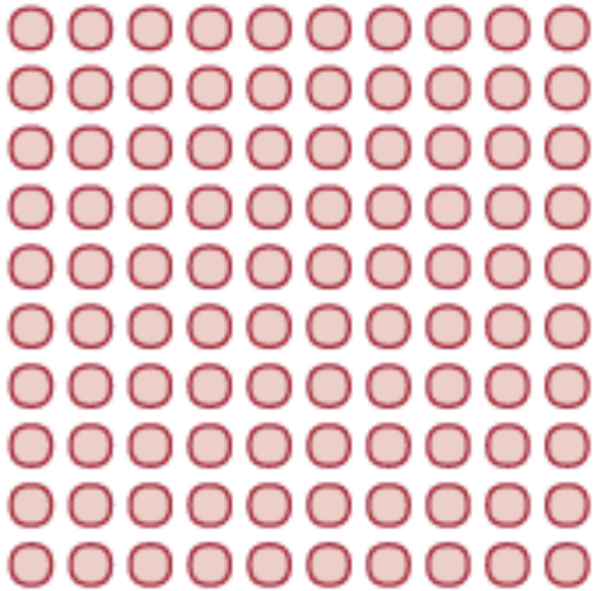
Use a variety of materials



Classroom

Ten as the basis for our place value system

- What does one hundred look like?



Classroom

Number charts

100 charts

Make tables in word

Number Ladder

Make puzzles out of them

Start at different numbers

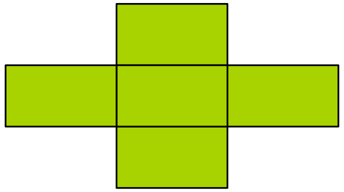
Go over the hundred

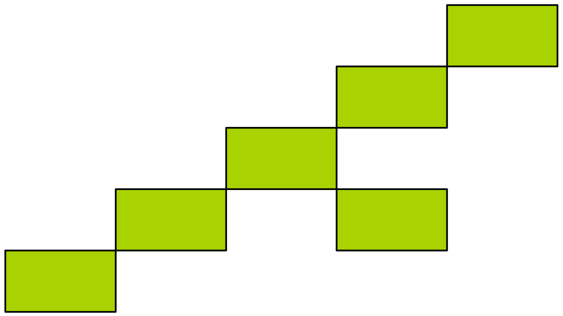
Fill in the blanks

Extend to thousands

Classroom

Number charts

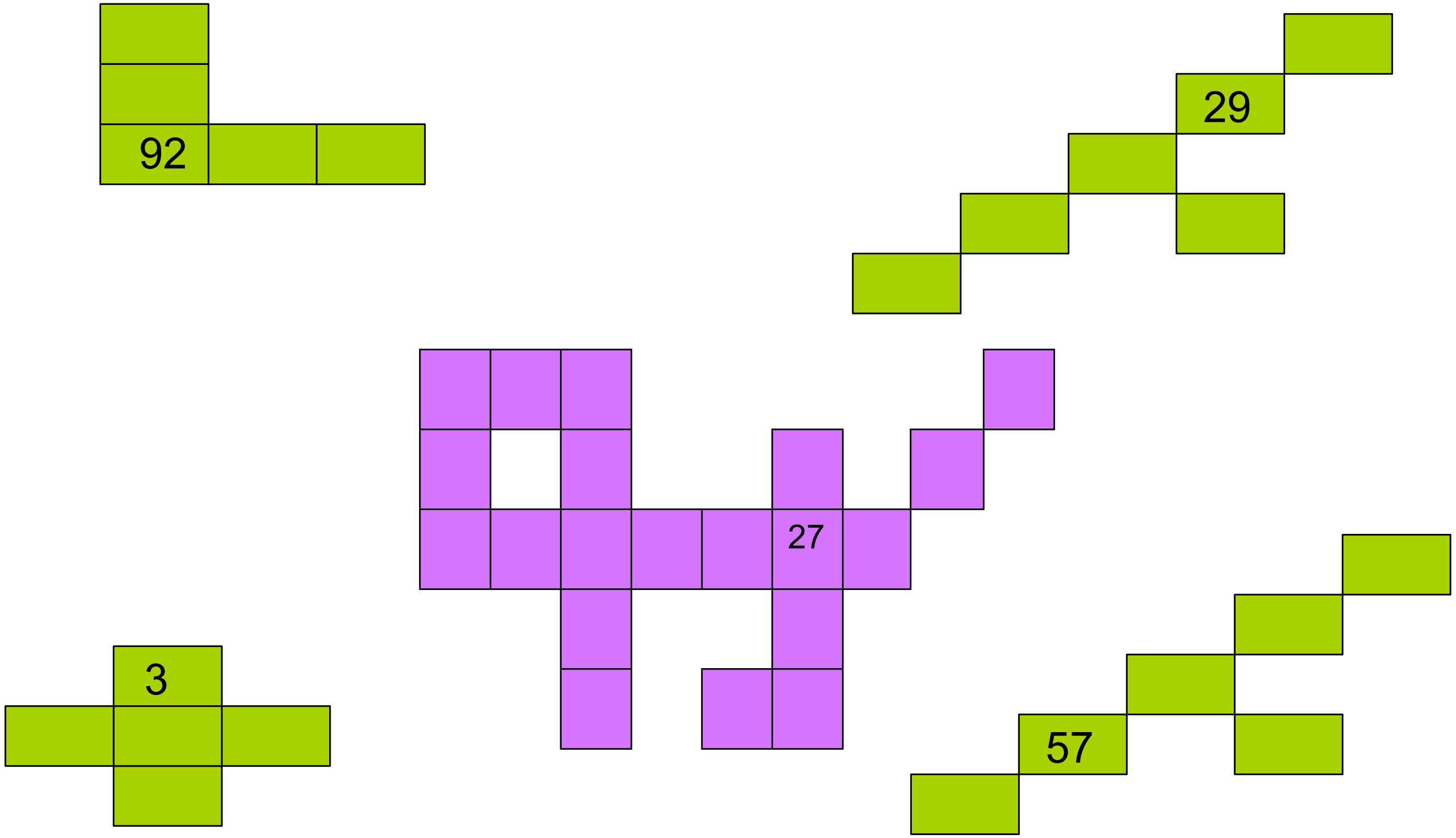




1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Classroom

Number charts



Classroom

Ten as the basis for our place value system

Paper bag activities:

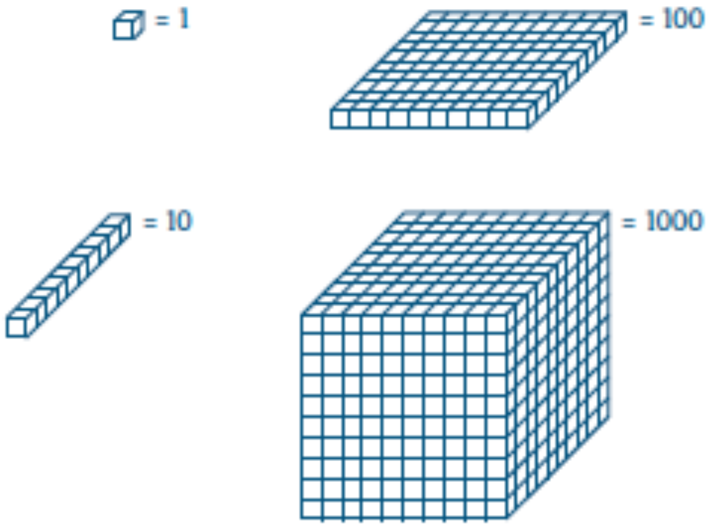
- sort numbers bigger than ten into one
- bag and smaller than ten into another

Beach Ball

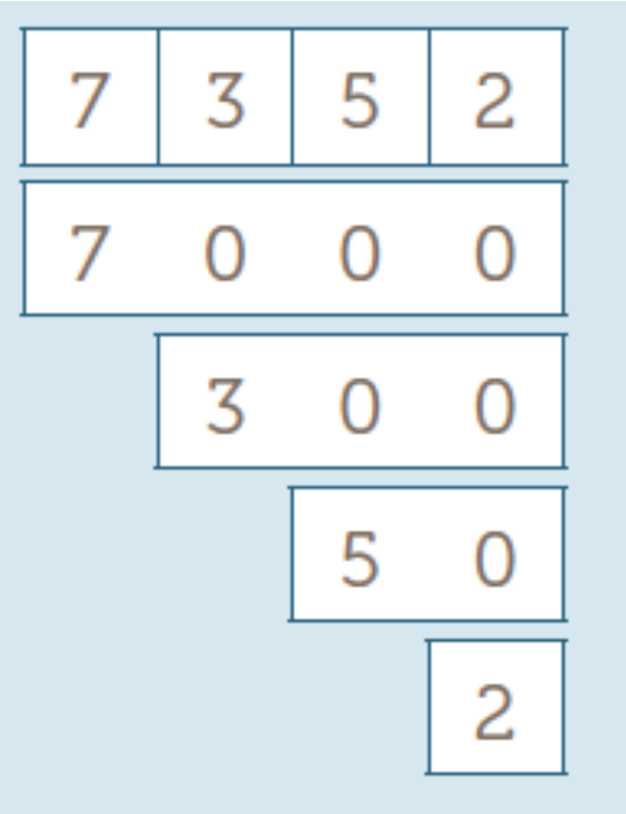
- Place the digits 0 - 9 on stickers on the ball - the person who catches the ball adds 10, 100 or 1000 to the number

Classroom

Ten as the basis for our place value system



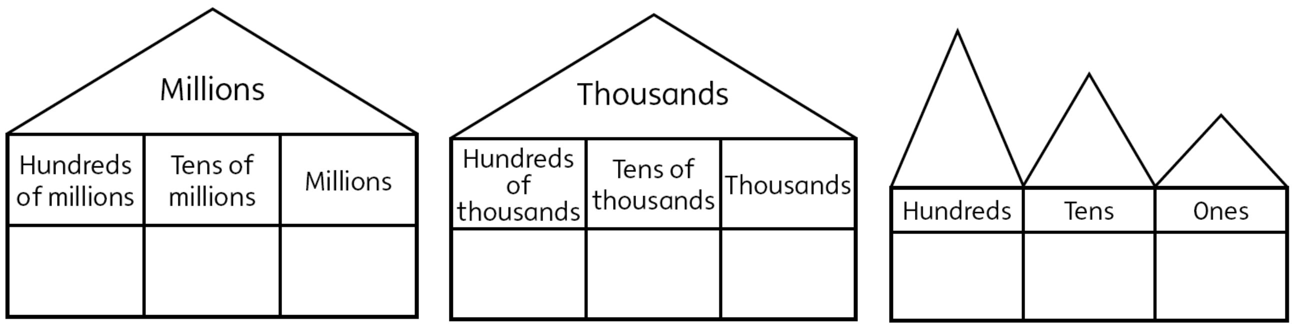
- What does one thousand look like?



Classroom

Ten as the basis for our place value system

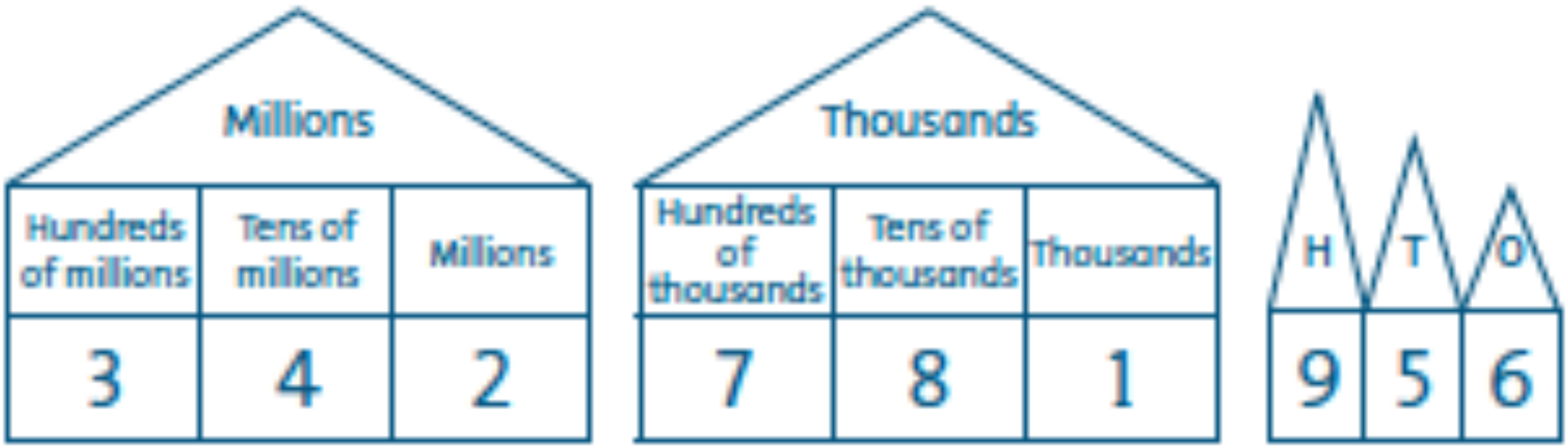
Place Value houses



Classroom

Ten as the basis for our place value system

Place Value houses



Classroom

Today's number

15

Tools session idea:
Today's number is… from MCTP

Classroom

Today's number

Use often to

- Reinforce number facts
- Engage students in thinking about strategies
- Allow students to pull apart and then reconstruct numbers

Use it to

- Settle and focus students
- Allow all students time to respond.
- Pre-test and post-test by counting number of facts in a minute with extra points for **FAT** facts.
- Insist on certain strategies. For example, students must include multiplying with fractions.

Classroom

The number line

The number line can be used as a model for number from the first years of primary school through to the early years of secondary schooling. It is a model that can be consistently applied in most areas of number.

- Comparing
- Modelling
- Ordering
- Operations

Classroom

The number line

Introducing the number line

- Mark in zero and one other reference point
- Convention of negative numbers to the left, zero in the middle and positive numbers to the right
- Move towards children drawing their own

Step through the introduction of the number line very slowly

Do not assume this has been done before

Remind the children all the time, where is the one?

Classroom

The number line

Use
- Masking tape on the floor
- String across the room
- Chalk in the playground
- Magnetized numbers on a blackboard or whiteboard
- Cash-register rolls
- Number ladder

Classroom

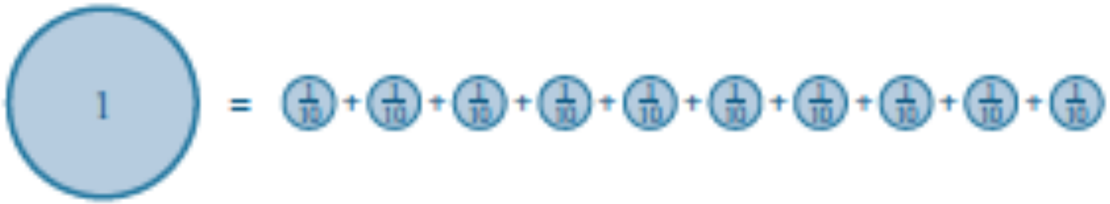
The number line

Put a cross where one hundred would be on this number line.

Classroom

Ten as the basis for our place value system

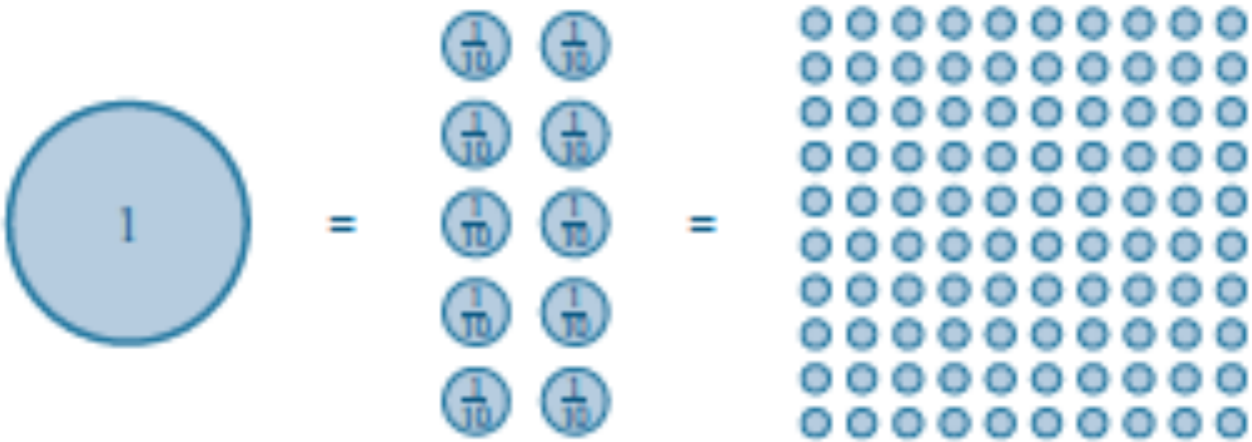
- Play dough
- What does one *tenth* look like?



Classroom

Ten as the basis for our place value system

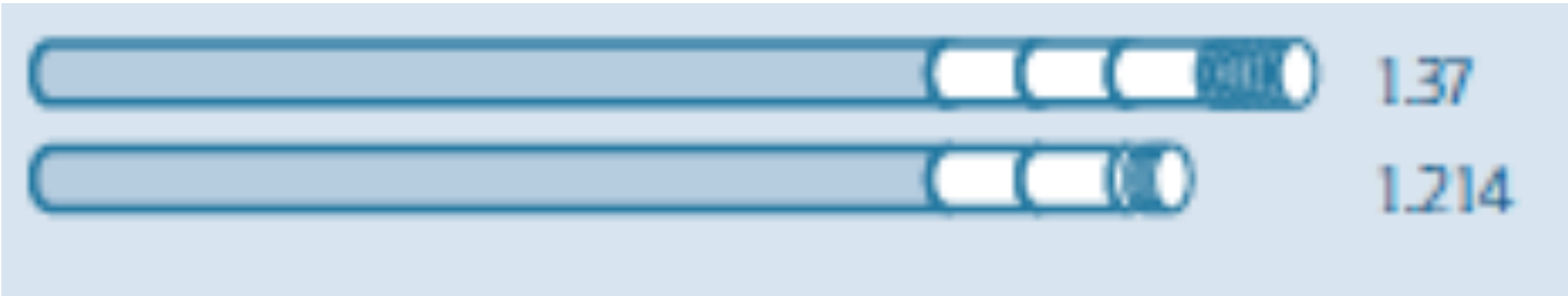
- What does one *hundredth* look like?



Classroom

Ten as the basis for our place value system

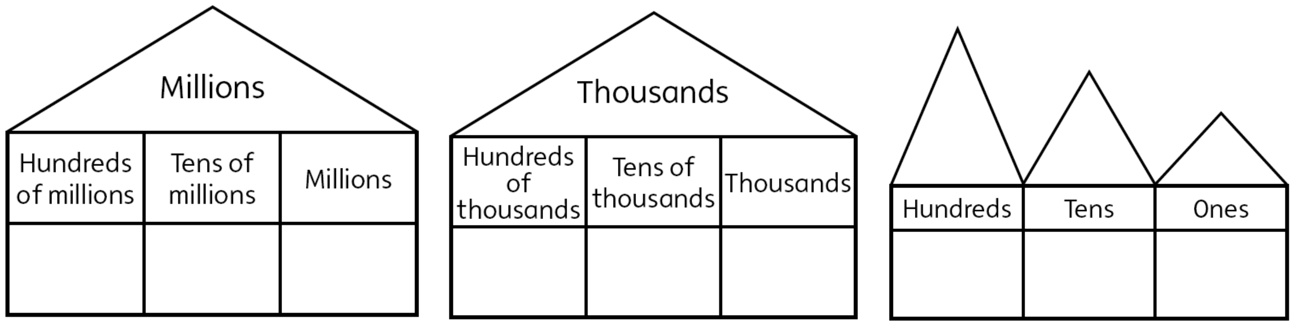
- What does one *thousandth* look like?



Classroom

Ten as the basis for our place value system

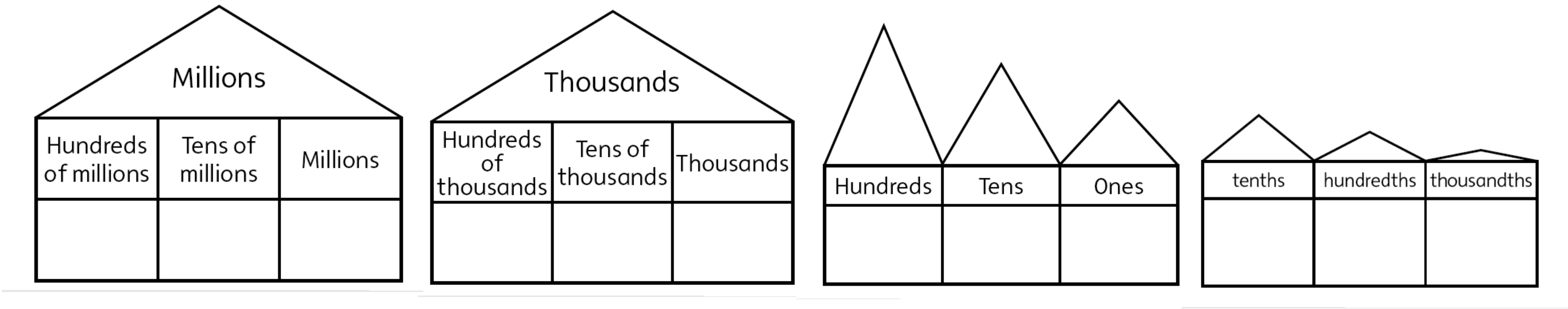
Place Value houses



Classroom

Ten as the basis for our place value system

Place Value houses



Classroom

Today's number

15.28

Tools session idea:
Today's number is… from MCTP

Classroom

Decimal misconceptions

Longer is larger

- whole number thinking
- column over flow thinkers take the name from the left most column so 0.35 is 35 tenths
- reverse thinkers believe the value of the column increases as we move to the right. 0.35 is larger than 0.41 because 53 is larger than 14.

Shorter is larger

- any number of tenths is more than any number of hundredths. So 0.4 is larger than 0.83
- reciprocal thinkers think that 0.3728 is like $^{1}/_{3728}$. They would choose 0.3 as larger than 0.4 because $^{1}/_{3}$ is larger than $^{1}/_{4}$

Money thinking

Zero does not matter

All decimals below zero (and past the negative numbers on the number line)

Classroom

Number Between

A good way to introduce the idea of the infinite nature of the place value system.

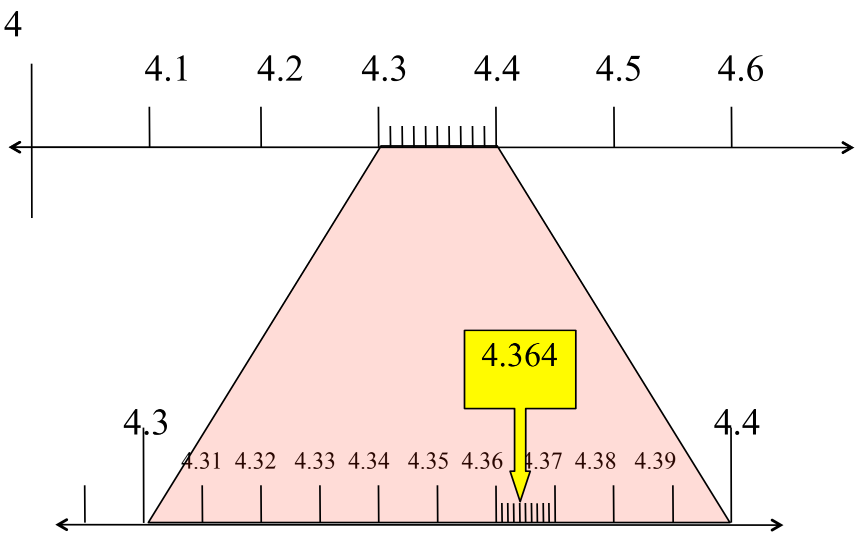
Hidden number

Used for building skills for comparing the relative size of decimal numbers

Classroom

The number line

Decimals on the number line



We want students to get the sense of being able to zoom in or out of the number line.

Classroom

The number line

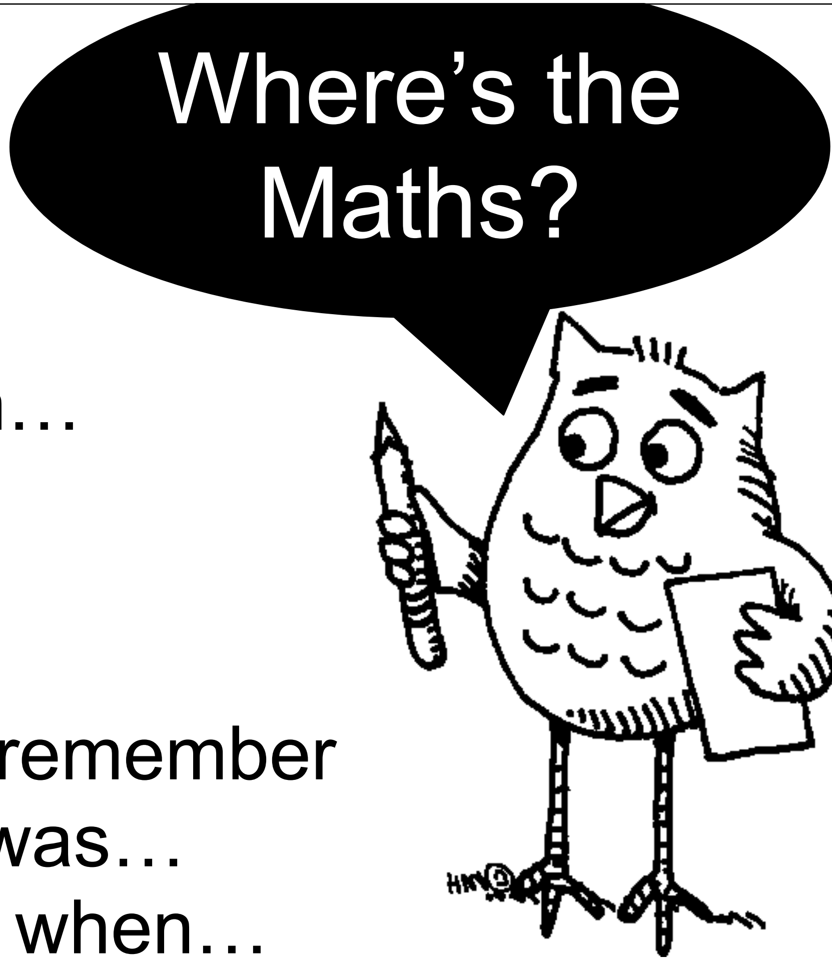
Number Trails

A game for developing number patterns and sequences.

Suitable for all students.

Begin with whole numbers and addition or subtraction then introduce multiplication and division and decimals and fractions.

We already knew…
We remembered…
We used equipment…
We need to find out…
It was interesting when…
The tricky bit was…
We didn't know that…
It was cool when…
The important thing to remember
A new word we learnt was…
Our group worked well when…
We discovered…
Congratulations to…
The strategy we used was…



We said…
We wrote…
We saw…
We heard…
We know…
We drew…
We said…
We asked…
We felt…
We liked…
We learnt…
We didn't like…
We found out…